EXAMPLES

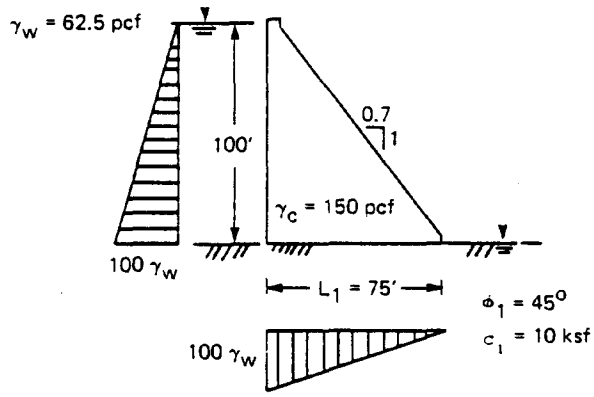
1. Examples of typical static loading conditions for single and multiple wedge systems are presented in this Inclosure.

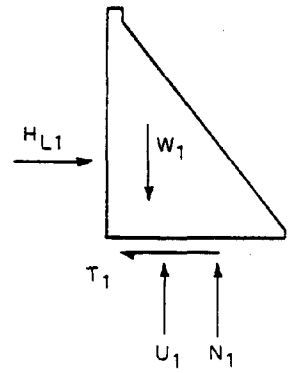
2. These examples are provided to clearly demonstrate the procedure for applying the general wedge equation to the sliding analysis of single and multiple wedge systems. The variation of uplift pressure, orientation of failure planes, etc., used in the examples were only selected to simplify the calculations, and are not intended to represent the only conditions to be considered during the design of a hydraulic structure.

81

Example 1: Single Wedge

Determine the factor of safety against sliding for the following single wedge system.





Free Body Diagram

General Wedge Equation

$$P_{i-1} - P_i = \frac{[(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i]\dfrac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + \dfrac{c_i}{FS_i}L_i}{\left(\cos\alpha_i - \sin\alpha_i \dfrac{\tan\phi_i}{FS_i}\right)}$$

Solve for Safety Factor (FS)

$i = 1 \quad H_{R1} = 0 \quad V_1 = 0 \quad P_o \quad P_1 = 0 \quad \alpha_1 = 0 \quad \cos\alpha_1 = 1 \quad \sin\alpha_1 = 0$

$$o = (W_1 - U_1)\frac{\tan 45}{FS} - H_{L1} + \frac{c_1}{FS}L_1$$

$$H_{L1} = \frac{1}{2}(100)^2\gamma_w = 312.5^k \qquad U_1 = \frac{1}{2}(75)(100)\gamma_w = 234.4^k\uparrow \qquad W_1 = 603.8^k\downarrow$$

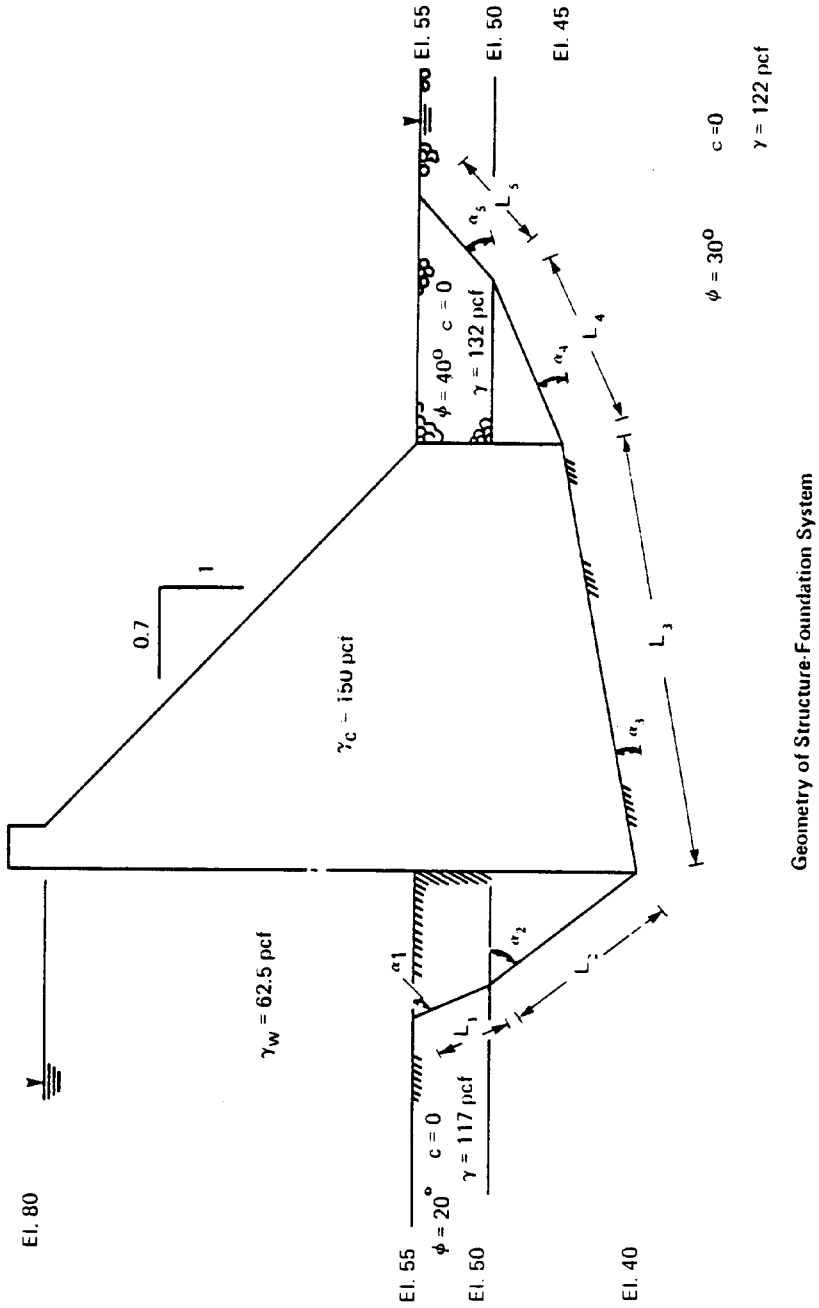
$$FS = \frac{(W_1 - U_1)\tan 45^o + c_1 L_1}{H_{L_1}}$$

$$FS = \frac{(603.8 - 234.4)(1) + 10(75)}{312.5} = \frac{(369.4 + 750)}{312.5} = 3.58$$

$\boxed{FS = 3.58}$

Example 2: Multiple Wedges

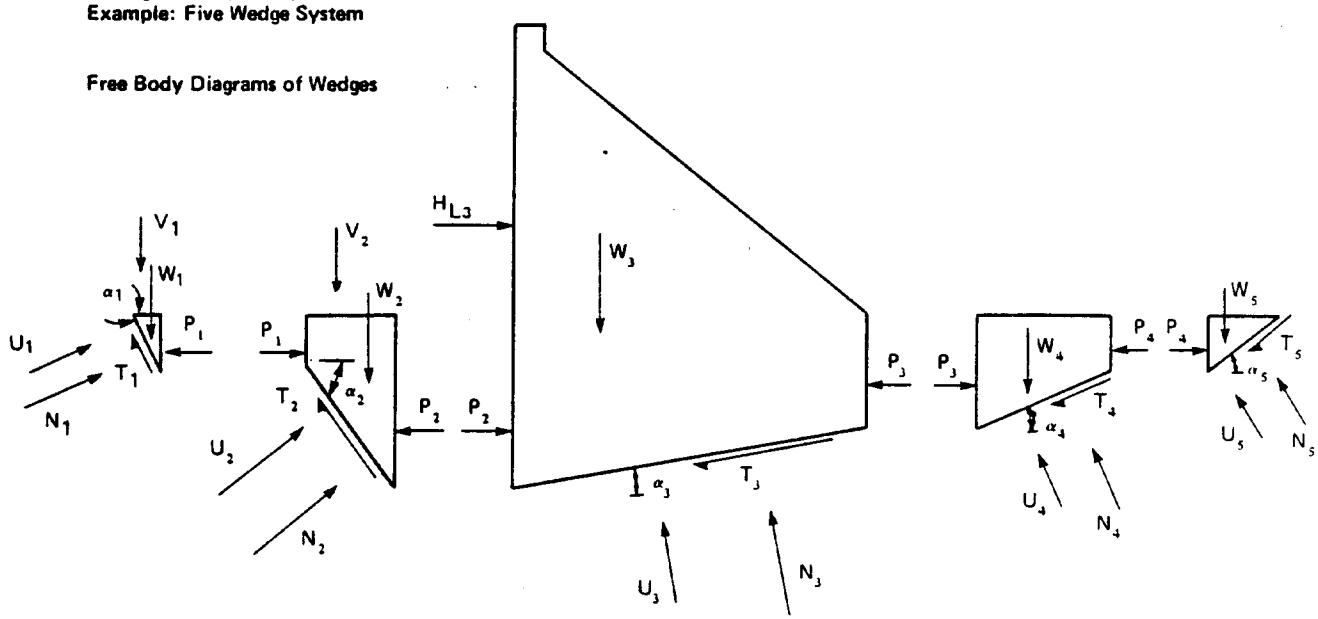
Determine the factor of safety against sliding for the following five wedge system



Geometry of Structure-Foundation System

Sliding Stability Analysis
Example: Five Wedge System

Free Body Diagrams of Wedges

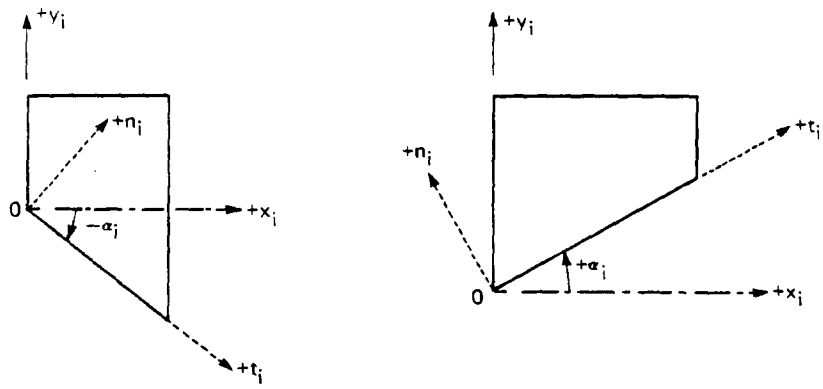


Wedge No. 1	Wedge No. 2	Wedge No. 3	Wedge No. 4	Wedge No. 5
(i = 1)	(i = 2)	(i = 3)	(i = 4)	(i = 5)

Sliding Stability Analysis
Example: Five Wedge System

General Wedge Equation

$$P_{i-1} - P_i = \frac{[(W_i + V_i)\cos\alpha_i - U_i + (H_{Li} - H_{Ri})\sin\alpha_i]\dfrac{\tan\phi_i}{FS_i} - (H_{Li} - H_{Ri})\cos\alpha_i + (W_i + V_i)\sin\alpha_i + \dfrac{c_i}{FS_i}L_i}{\left(\cos\alpha_i - \sin\alpha_i \dfrac{\tan\phi_i}{FS_i}\right)}$$



Sign Convention for General Equation

Wedge Forces for Trial Safety Factor of 1.5

$\underline{i = 1}$ $\quad H_{Li} = H_{Ri} = 0$

$$\tan\phi_d = \frac{\tan\phi_1}{FS_1} = \frac{\tan 20}{1.5} \qquad \phi_d = \tan^{-1}(0.243) = 13.64^\circ$$

$$\alpha_1 = -\left(45^\circ + \frac{\phi_d}{2}\right) = -51.82^\circ$$

$\sin(-51.82) = -0.786$

$\cos(-51.82) = 0.618$

This orientation of the failure path is only true if the stratification and surface are horizontal

$L_1 = 5/|\sin(-51.82)| = 5/0.786 = 6.36'$

Sliding Stability Analysis
Example: Five Wedge System

$$W_1 = \frac{1}{2}(0.117)\ (5)*6.36 \cos(-51.82) = 1.15^k \downarrow$$

$$V_1 = (25*.0625)\ 6.36 \cos(-51.82) = 6.14^k \downarrow$$

$$\Big\} 7.29^k \downarrow$$

$$U_1 = \frac{1}{2}(.0625)\ (25+30)\ 6.36 = 10.93^k \nearrow$$

$$(P_o - P_1) = \frac{[7.29\ (0.618) - 10.93]\ \dfrac{\tan 20}{1.5} + 7.29\ (-0.786)}{[0.618 - (-0.786)\ \dfrac{\tan 20}{1.5}]} = -9.01^k$$

$$\boxed{(P_o - P_1) = \overleftarrow{9.01}^k}$$

<u>i = 2</u> $\quad H_{L_2} = H_{R_2} = 0$

$$\tan \phi_d = \frac{\tan \phi_2}{FS_2} = \frac{\tan 30}{1.5} \qquad \phi_d = \tan^{-1}(0.385) = 21.05^o$$

$$\alpha_2 = -\left(45 + \frac{\phi_d}{2}\right) = -55.53^o$$

$$\sin(-55.53) = -0.8244 \qquad \cos(-55.53) = 0.566$$

$$L_2 = 10/\ |\sin(-55.53)| = 12.13'$$

$$W_2 = 0.117\ (5)\ (12.13^x 0.566) + \frac{1}{2}(.122)\ (10)\ (12.13^x 0.566) = 8.20^k \downarrow$$

$$V_2 = (25^x .0625)\ (12.13^x 0.566) = 10.73^k \downarrow$$

$$U_2 = \frac{1}{2}(0.0625)\ (30+40)\ (12.13) = 26.53^k \nearrow$$

Sliding Stability Analysis
Example: Five Wedge System

$$(P_1 - P_2) = \frac{[18.93\,(0.566) - 26.53]\,\dfrac{\tan 30}{1.5} + 18.93\,(-.8244)}{[0.566 - (-0.8244)\dfrac{\tan 30}{1.5}]} = -24.56^k$$

$$\boxed{(P_1 - P_2) = \overleftarrow{24.56}^k}$$

i = 3 $\alpha_3 = 9.5^o$ $L_3 = 5/\sin 9.5 = 30.3'$

$H_{L_3} = \frac{1}{2}(0.0625)\,(25)^2 = \overrightarrow{19.53}^k$ $H_{R_3} = 0$

$U_3 = \frac{1}{2}\,(0.0625)\,(40+10)\,(30.3) = 47.33^k \nwarrow$

$W_3 = 122.4^k \downarrow$

$\sin 9.5^o = 0.165$ $\cos 9.5 = 0.986$

$$(P_2 - P_3) = \frac{[122.4\,(.986) - 44.11]\,\dfrac{\tan 30}{1.5} - 19.53\,(0.986) + 122.4\,(.165)}{(.986 - .165 \times \dfrac{\tan 30}{1.5})} = 32.97^k$$

$$\boxed{(P_2 - P_3) = \overrightarrow{32.97}^k}$$

i = 4 $H_{L_4} = H_{R_4} = V_4 = 0$

$\tan \phi_d = \frac{\tan \phi_4}{FS_4} = \frac{\tan 30^o}{1.5}$ $\phi_d = \tan^{-1}(0.385) = 21.05^o$

$\alpha_4 = 45 - \frac{1}{2}\phi_d = 34.475^o$

$\sin(34.475) = 0.566$ $\cos(34.475^o) = 0.824$

Sliding Stability Analysis
Example: Five Wedge System

$L_4 = 5/\sin 34.475 = 8.83'$

$$W_4 = (0.132)\ (5)\ (8.83^{\times}.824) + \frac{1}{2}(0.122)\ (5)\ (8.83^{\times}0.824) = 7.02^k \downarrow$$

$$U_4 = \frac{1}{2}(0.0625)\ (5+10)\ (8.83) = 4.14^k \nwarrow$$

$$(P_3 - P_4) = \frac{[7.02\ (.824) - 4.14]\ \dfrac{\tan 30}{1.5} + 7.02\ (.566)}{[0.824 - 0.566\ \dfrac{\tan 30}{1.5}]} = 7.59^k$$

$$\boxed{(P_3 - P_4) = \overrightarrow{7.59}^k}$$

<u>i = 5</u> $H_{L5} = H_{R5} = V_5 = 0$

$$\tan\phi_d = \frac{\tan\phi_5}{FS_5} = \frac{\tan 40}{1.5} \qquad \phi_d = \tan^{-1}(0.559) = 29.22^o$$

$$\alpha_5 = (45 - \frac{1}{2}\phi_d) = 30.38 \qquad \sin 30.38 = 0.5058 \qquad \cos 30.38 = 0.8626$$

$L_5 = 5/\sin 30.38 = 9.89'$

$$W_5 = \frac{1}{2}(0.132)\ 5\ (9.89^{\times}0.8626) = 2.82^k \downarrow$$

$$U_5 = \frac{1}{2}(0.0625)\ (5)\ (9.89) = 1.545^k \nwarrow$$

$$(P_4 - P_5) = \frac{[2.82^{\times}.863 - 1.54]\ \dfrac{\tan 40}{1.5} + 2.82^{\times}.506}{[.863 - .506\ \dfrac{\tan 40}{1.5}]} = 3.32^k$$

$$\boxed{(P_4 - P_5) = \overrightarrow{3.32}^k}$$

24 Jun 81

..ng Stability Analysis
..xample: Five Wedge System

Summary: Wedge Forces for Trial Safety Factors

FS = 1.5

i	α_i	L_i	H_{Li}	H_{Ri}	V_i	W_i	U_i	$(P_{i-1} - P_i)$
1	−51.82	6.36	0	0	6.14	1.15	10.93	−9.01
2	−55.53	12.13	0	0	10.73	8.20	26.53	−24.56
3	9.5	30.3	19.53	0	0	122.4	47.33	32.97
4	34.47	8.83	0	0	0	7.02	4.14	7.59
5	30.38	9.89	0	0	0	2.82	1.54	3.32
							ΔP_R =	10.31

FS = 2.5

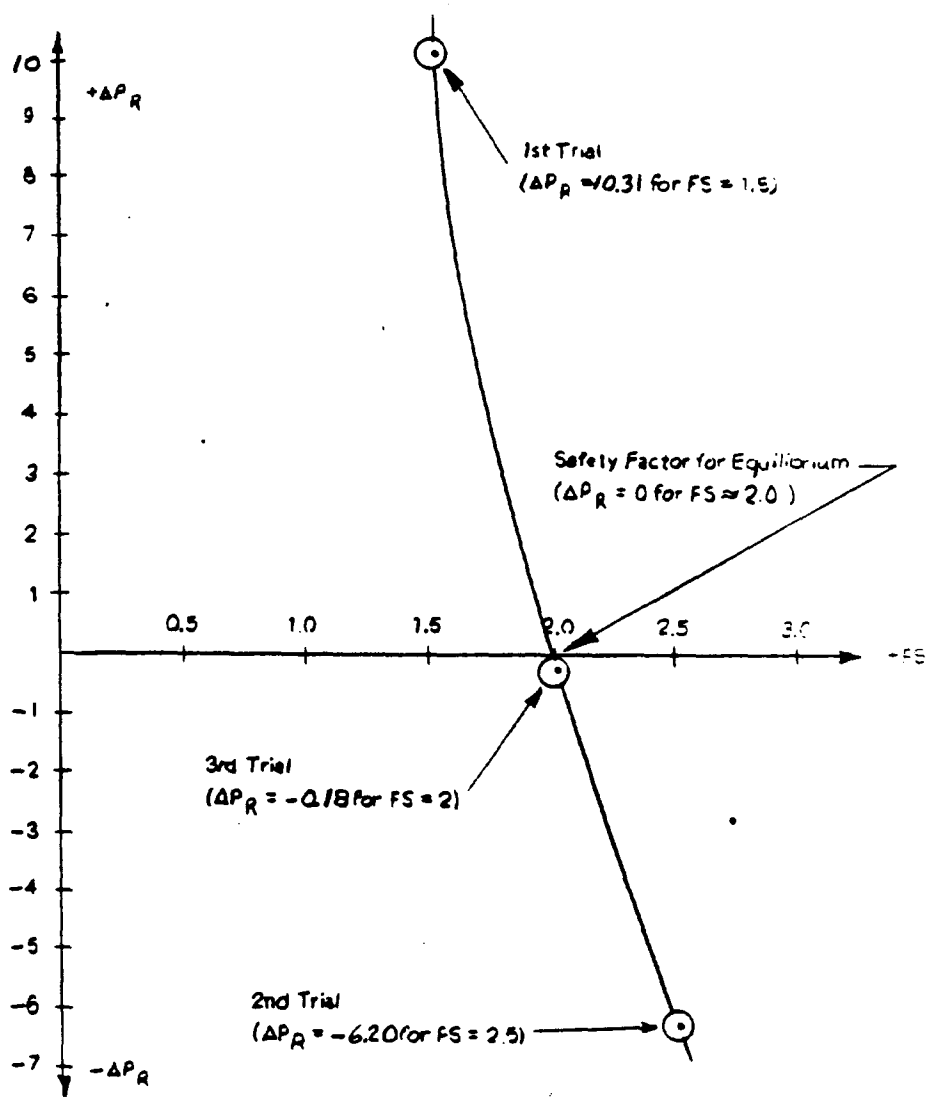
i	α_i	L_i	H_{Li}	H_{Ri}	V_i	W_i	U_i	$(P_{i-1} - P_i)$
1	−49.14	6.61	0	0	6.75	1.27	11.36	−9.10
2	−51.5	12.78.	0	0	12.43	9.50	27.95	−25.48
3	9.5	30.3	19.53	0	0	122.4	47.33	19.65
4	38.5	8.0	0	0	0	6.06	3.76	6.26
5	35.72	8.56	0	0	0	2.29	1.34	2.45
							ΔP_R =	-6.20

FS = 2.0

i	α_i	L_i	H_{Li}	H_{Ri}	V_i	W_i	U_i	$(P_{i-1} - P_i)$
1	−50.16	6.51	0	0	6.52	1.22	11.19	−9.06
2	−53.05	12.51	0	0	11.73	8.97	27.37	−25.13
3	9.5	30.3	19.53	0	0	122.4	47.33	24.53
4	36.95	8.33	0	0	0	6.43	3.9	6.73
5	33.62	9.03	0	0	0	2.48	1.41	2.75
							ΔP_R =	-0.18

Sliding Stability Analysis
Example: Five Wedge System

Graphical Solution for Safety Factor

The safety factor for sliding equilibrium of the five wedge system is determine from:

$$\sum_{i=1}^{5} (P_{i-1} - P_i) = \Delta P_R \begin{cases} \Delta P_R = 0 & \text{Safety factor for equilibrium} \\ \Delta P_R \neq 0 & \text{For trial safety factors} \end{cases}$$