APPENDIX C

AVAILABILITY AND OPERATIONAL READINESS

C-1. Availability

In general, availability is the ability of a product or service to be ready for use when a customer wants to use it. That is, it is available if it is in the customer's possession and works when it's turned on or used. A product that's "in the shop" or is in the customer's possession but doesn't work is not available. Measures of availability are shown in table C-1.

Table C-1. Quantitative measures of availability

Measure	Equation	Description
Inherent Availability: A_i	$\frac{MTBF}{MTBF + MTTR} \times 100\%$	• Where MTBF is the mean time between failure and MTTR is the mean time to repair • A probabilistic measure • Reflects the percent of time a product would be available if no delays due to maintenance, supply, etc. (i.e., not design-related) were encountered
Achieved Availability: A_a	$\frac{MTBM}{MTBM + MTTR_{Active}} \times 100\%$	• Where MTBM is the mean time between maintenance (preventive and corrective) and $MTTR_{Active}$ is the mean time to accomplish preventive and corrective maintenance tasks • A probabilistic measure • Similar to A_i except that preventive and corrective maintenance are included
Operational Availability: A_o	$\frac{MTBM}{MTBM + MDT} \times 100\%$	• Where MTBM is the mean time between maintenance (preventive and corrective) and MDT is the mean downtime, which includes MTTR and all other time involved with downtime, such as delays • A probabilistic measure • Similar to inherent availability but includes the effects of maintenance delays and other non-design factors • A_o reflects the totality of the inherent design of the product, the availability of maintenance personnel and spares, maintenance policy and concepts, and other non-design factors, whereas A_i reflects only the inherent design
Uptime Ratio: UR	$\frac{Uptime}{Uptime + Downtime} \times 100\%$	• Uptime is the time that the product is in the customer's possession and works; downtime is the total number of hours that the product is not operable/usable • A deterministic measure • Uptime Ratio is time-dependent; the time period over which the measurement is made must be known

MTBF = Mean Time Between Failure
MTBM = Mean Time Between Maintenance
MDT = Mean Downtime
MTTR = Mean Time to Repair (corrective only)

a. *Nature of the equations*. Note that the first three equations are time independent and probabilistic in nature. The value of availability yielded by each equation is the same whether the period of performance being considered is 1 hour or a year. However, the last equation is deterministic and not time independent. The period over which the measurement is made is very important.

b. *The Importance of the measurement period.* Consider the following example. A repairable product has an availability requirement of 99.5% over a year of operation. The predicted MTBF is 100 hours and the predicted MTTR is 0.5 hours.

(1) *System availability using equation 1.* Using the equation for inherent availability, the availability is predicted to be 99.5%, regardless of the time period of interest. The system is observed over a six-month period during which it operates for a total of 600 hours. The observed results are shown in figure C-1. Note that the number of operating hours per month varies.

Month	Hours	Failures	Downtime	Cum MTBF	Cum MTTR	A (Equation 1)	A (Equation 4)
1	20	1	2	20.000	2.0000	0.9090	0.9090
2	25	0	0	45.000	1.0000	0.9783	1.0000*
3	100	1	.5	72.500	1.2500	0.9831	0.9950*
4	355	1	1	167.667	1.1667	0.9902	0.9972*
5	10	1	.01	127.500	0.8775	0.9932	0.9990*
6	90	2	.02	100.00	0.5880	0.9942	0.9998*

*Meets or exceeds the availability requirement.

Figure C-1. Measuring availability using different measures.

(2) *System availability using equation* 4. Note that the availability as measured using equation 4 varies considerably depending on the length of the period and the number of failures. If equation 4 is used, the system "flunks" the test during the first period and surpasses the requirement in all of the other periods, reaching the theoretical maximum availability of 100% in the second period. Using equation 3, the availability approaches but never quite reaches the requirement. To calculate MTBF and MTTR, the cumulative failures, operating hours, and repair times are used. If the true MTBF and MTTR are equal to or are better than the predictions, then, in the long term (in the statistical sense), the availability will reach 99.5%.

c. *Derivation of steady state equation for availability.* The first three equations in table C-1 are actually steady state equations. The equation for inherent availability (equation C-1), for example, is the steady state equation derived from equation C-2, as time approaches infinity:

$$A_i = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \qquad \text{Equation C-1}$$

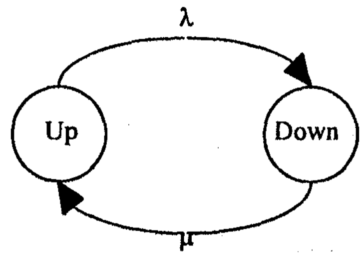
$$A_i = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} + \frac{\text{MTTR}}{\text{MTBF} + \text{MTTR}} e^{-\left(\frac{1}{\text{MTBF}} + \frac{1}{\text{MTTR}}\right)t} \qquad \text{Equation C-2}$$

1. Equation C-1 represents a limit for inherent availability. It represents the long-term proportion of time that a system will be operational.

2. Assuming that the times to failure and time to repair are both exponentially distributed, with rates λ and μ, respectively, equation C-1 can be expressed as:

$$A_i = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}+\frac{1}{\mu}} = \frac{\mu}{\mu+\lambda} \qquad \text{Equation C-3}$$

3. The derivation of equation C-1 now follows. A simple Markov model is used to evaluate availability. The probabilities of being in either the up state or the down state are determined using the Laplace transform. The model and equations are:



$$\frac{dP_{Up}(t)}{dt} = -\lambda P_{Up}(t) + \mu P_{Down}(t) \qquad \text{Equation C-4}$$

$$sL_{Up}(s) - P_{Up}(0) = sL_{Up}(s) - 1 = -\lambda L_{Up}(s) + \mu L_{Down}(s) \qquad \text{Equation C-5}$$

$$1 - sL_{Up}(s) = sL_{Down}(s) = \lambda L_{Up}(s) - \mu L_{Down}(s) \qquad \text{Equation C-6}$$

From equation C-4, $L_{Up}(s) = \dfrac{1+\mu L_{Down}(s)}{s+\lambda}$ Equation C-7

From equation C-5, $L_{Down}(s) = \dfrac{\lambda L_{Up}(s)}{s+\mu}$ Equation C-8

4. Substituting the expression for $L_{Down}(s)$ into equation C- 7,

$$L_{Up}(s) = \frac{1}{s+\mu+\lambda} + \frac{\mu}{s(s+\lambda+\mu)} \qquad \text{Equation C-9}$$

5. Then, availability = the inverse of the Laplace transform for $L_{Up}(s)$. To obtain the inverse,

$$\frac{1}{s+\mu+\lambda} + \frac{\mu}{s(s+\lambda+\mu)} = \frac{1}{\lambda+\mu}\left(\frac{\mu(s+\mu+\lambda)+\lambda s}{s(s+\mu+\lambda)}\right)$$

$$= \frac{1}{\lambda+\mu}\left(\frac{\mu}{s} + \frac{\lambda}{s+\mu+\lambda}\right)$$

$$= \frac{\mu}{\lambda+\mu}\,\frac{1}{s} + \frac{\lambda}{\mu+\lambda}\,\frac{1}{s+\mu+\lambda}$$

$$= \frac{1}{\lambda+\mu}\left(\frac{\mu}{s} + \frac{\lambda}{s+\mu+\lambda}\right)$$

$$= \frac{\mu}{\lambda+\mu} \times \frac{1}{s} + \frac{\lambda}{\mu+\lambda} \times \frac{1}{s+\mu+\lambda}$$

$$= \frac{\mu}{\lambda+\mu} \int_0^\infty e^{-st} dt + \frac{\lambda}{\mu+\lambda} \int_0^\infty e^{-(s+\mu+\lambda)t} dt$$

$$= \int_0^\infty \frac{\mu}{\lambda+\mu} e^{-st} + \frac{\lambda}{\mu+\lambda} e^{-(s+\mu+\lambda)t} dt$$

$$= \int_0^\infty e^{-st}\left(\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\mu+\lambda} e^{-(\mu+\lambda)t}\right) dt$$

$$= L\left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\mu+\lambda} e^{-(\mu+\lambda)t}\right]$$

$$\therefore A = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t} \qquad \text{Equation C-10}$$

6. Taking the limit of equation C-10 as t approaches infinity,

$$A_i = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\lambda+\mu} \times 0 = \frac{\mu}{\lambda+\mu}$$

$$A_i = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

Q.E.D.

C-2. Operational readiness

Closely related to the concept of operational availability but broader in scope is operational readiness. Operational readiness is defined as the ability of a military unit to respond to its operational plans upon receipt of an operations order. It is, therefore, a function not only of the product availability, but also of assigned numbers of operating and maintenance personnel, the supply, the adequacy of training, and so forth.

a. *Readiness in the commercial world.* Although operational readiness has traditionally been a military term, it is equally applicable in the commercial world. For example, a manufacturer may have designed and is capable of making very reliable, maintainable products. What if he has a poor distribution and transportation system or does not provide the service or stock the parts needed by customers to effectively use the product? Then, the readiness of this manufacturer to go to market with the product is low.

b. *Relationship of availability and operational readiness.* The concepts of availability and operational readiness are obviously related. Important to note, however, is that while the inherent design characteristics of a product totally determine inherent availability, other factors influence operational availability and operational readiness. The reliability and maintainability engineers directly influence the design of the product. Together, they can affect other factors by providing logistics planners with the information needed to identify required personnel, spares, and other resources. This information includes the identification of maintenance tasks, repair procedures, and needed support equipment.