CHAPTER 4

TWO-DIMENSIONAL RADIAL HEAT FLOW

4-1. General.

Radial flow of heat is considered in thermal problems involving the design of pile foundations in permafrost (TM 5-852-4/AFM 88-19, Chapter 4), the construction of utility supply lines for the transport of water and sewage in permafrost areas and seasonal frost areas (TM 5-852-5/AFR 88-19, Volume 5), and the design of artificially frozen ground for retaining structures during construction. A number of the basic concepts and techniques used to calculate radial heat flow from cylindrical surfaces are discussed below.

a. Thermal Resistance. In analyzing heat flow for areas with cylindrical cross sections, the effective thickness for radial flow from a unit length of the cylinder is

$$\frac{1}{2\pi} \ln \frac{r_2}{r_1} \quad \text{or } 0.367 \log \frac{r_2}{r_1}$$

where

r_1 = inside wall radius (ft)

r_2 = outside wall radius (ft).

The thermal resistance R is equal to the effective thickness divided by the conductivity of the material between the two radii. As an *example*, a concrete conduit with a wall thickness of 6 inches and an inside diameter of 10 feet is surrounded by 4 inches of cellular glass insulation and 4 feet of dry gravel. Calculate the thermal resistance between the inside concrete wall and the outer edge of the gravel material. The following thermal conductivities are given:

- Concrete, K = 1.00 Btu/ft hr °F.
- Insulation, K = 0.033 Btu/ft hr °F.
- Gravel, K = 1.5 Btu/ft hr °F.

Let (see fig. 4-1 for values of r_1 - r_4)

r_1 = radius to inner wall of conduit

r_2 = radius to outer wall of concrete

r_3 = radius to outer edge of insulation

r_4 = radius to outer edge of gravel

$K_{1\text{-}2}$ = thermal conductivity of concrete

$K_{2\text{-}3}$ = thermal conductivity of insulation

$K_{3\text{-}4}$ = thermal conductivity of gravel.

$$R = 0.367 \left(\frac{1}{k_{1\text{-}2}} \log \frac{r_2}{r_1} + \frac{1}{K_{2\text{-}3}} \log \frac{r_3}{r_2} + \frac{1}{K_{3\text{-}4}} \log \frac{r_4}{r_3} \right) \qquad \text{(eq 4-1)}$$

$$= 0.367 \left(\frac{1}{1.0} \log \frac{5.5}{5.0} + \frac{1}{0.033} \log \frac{5.83}{5.5} + \frac{1}{1.5} \log \frac{9.83}{5.83} \right)$$

$$= 0.367\,(0.041 + 0.767 + 0.151) = 0.352 \text{ ft.}^2 \text{ hr °F/Btu.}$$

If the temperature in the conduit were 45°F and the temperature at the outer face of the gravel were 35°F, the heat flow per linear foot of conduit would equal

$$\frac{1}{0.352}(45 - 35) = 28.4 \text{ Btu/hr.} \qquad \text{(eq 4-2)}$$

b. Temperature field surrounding a cylinder. The sudden or step change in surface temperature discussed for semi-infinite slabs in paragraph 3-6*a* has application to heat-flow problems associated with pile foudations in permafrost. A mathematical solution is available for the problem where the surface temperature of a cylinder is suddenly changed from the uniform temperature of the surrounding medium, as long as there is no phase change. Figure 4-2 is used to determine the temperature T at a distance r from the center of a cylinder of radius r_1, at a time t after the surface temperature of the cylinder is changed from T_o to T_s. The temperature T_o represents the uniform temperature of the medium prior to the sudden change in surface temperature.

Figure 4-1. Illustration for example in paragraph 4-1a.

4-2. Pile installation in permafrost.

At many arctic and subarctic sites, pile foundations are commonly placed in preaugered holes, and the annular space between the oversized hole and pile is backfilled with a slurry of soil and water. The tangential adfreeze strength of the pile is principally a function of the bond developed between the pile and the frozen slurry. Dissipation of the sensible and latent heat of the slurry into permafrost is a major design factor because construction scheduling depends upon the time required for slurry freeze-back. Pile spacing is important as each pile adds heat, i.e., piles spaced too closely may increase permafrost temperatures, with a reduction of pile adfreeze strength and an increase in freeze-back time. This is particularly true in relatively warm permafrost (above 30°F). The following discussion assumes first that the slurry will freeze back naturally because of heat transfer between the surrounding permafrost and the slurry, second that the time required for freeze-back at a particular depth is predominantly influenced by the permafrost temperature at that depth, and third that the permafrost does not thaw. Under certain conditions, artificial refrigeration may be required to ensure freeze-back within a reasonable time. The volumetric specific heat of the slurry and the effect of vertical heat flow are assumed to have a negligible effect in computing required freeze-back time. With proper pile spacing, the slurry temperature reaches that of the surrounding permafrost in time. Surrounding temperatures, natural freeze-back time, proper pile spacing, and heat removal by refrigeration, as well as heat transfer by thermal piles, are discussed below.

a. Surrounding temperatures. The increase in permafrost temperatures during slurry freeze-back is determined using the technique described in paragraph 4-1*b*.. For *example*, a preaugered hole for a pile installation is 16 inches in diameter and the slurry is placed at a temperature of 32°F. The surrounding permafrost has a thermal diffusivity of 0.06 ft²/hr and an initial temperature of 28°F. Calculate the ground temperature at a distance of 3 feet from the center of the pile after an elapsed time of 48 hours. Given

$r_1 = 0.667$ ft	$t = 48$ hr
$r = 3.00$ ft	$T_o = 28$°F
$a = 0.06$ ft²/hr	$T_s = 32$°F

Figure 4-2. Temperature around a cylinder having received a step change in temperature.

therefore,

$$\frac{r_1}{\sqrt{at}} = \frac{0.667}{\sqrt{(0.06)(48)}} = 0.39 \qquad \text{(eq 4.3)}$$

$$\frac{r}{r_1} = \frac{3.00}{0.667} = 4.50. \qquad \text{(eq 4-4)}$$

From figure 4-2,

$$\frac{T - T_o}{T_s - T_o} = 0.18 \qquad \text{(eq 4-5)}$$

$$T = 0.18\,(T_s - T_o) + T_o = 0.18\,(32-28)+28$$

$$= 28.7°F. \qquad \text{(eq 4-6)}$$

This technique is also used to predict the increase in permafrost temperature during slurry freeze-back. Since it assumes a constant surface temperature for a cylinder, it is applicable only to the time of freeze-back. After the slurry has frozen, the permafrost temperature decreases and the model is not valid.

b. Natural freeze-back time. This heat transfer problem assumes a slurried pile to be a finite heat source inside a semi-infinite medium, with a suddenly applied constant temperature source (32°F) that dissipates radially into frozen ground of known initial temperature. The general solution for the natural freeze-back problem, based upon the latent heat content of the slurry, is shown in figure 4-3.

(1) For *example*, calculate the time required to freeze back a 12.5-inch diameter timber pile placed in an 18-

Figure 4-3. General solution of slurry freeze-back.

inch hole preaugered in permafrost and backfilled with a slurry for the following conditions:

— Permafrost:	Silty sand	
	Initial temperature = 27°F	
	Dry unit weight = 94 lb/ft^3	
	Water content = 25%.	
— Slurry backfill:	Silt, water	
	Placement temperature = 33.5°F	
	Dry unit weight = 72 lb/ft^3	
	Water content = 45%.	

In figure 2-3, the thermal conductivity of the permafrost is determined to be 1.1 Btu/ft hr°F. The volumetric heat capacity is calculated to be

$$[94\,(0.17 + 0.5\,\frac{25}{100}\,)] = 27.7\ \mathrm{Btu/ft^3\,°F} \qquad \text{(eq 4-7)}$$

and the thermal diffusivity to be

$$\frac{1.1}{27.7} = 0.0397\ \mathrm{ft^2/hr.} \qquad \text{(eq 4-8)}$$

The volumetric latent heat of the slurry is

$$(144 \times 72 \times 0.45) = 4670\ \mathrm{Btu/ft^3\,°F} \qquad \text{(eq 4-9)}$$

and the latent heat per linear foot of backfill is

$$[\,\frac{\pi}{4}\,(1.5^2 - 1.04^2)]\ 4670 = 4280\ \mathrm{Btu.} \qquad \text{(eq 4-10)}$$

When figure 4-3 is entered with a value of

$$\frac{Q}{Cr_2^2\Delta t} = [\,\frac{4280}{(27.7)(0.75^2)(5)}\,] = 55 \qquad \text{(eq 4-11)}$$

then,

$$\frac{at}{r_2^2} = 12.4. \qquad \text{(eq 4-12)}$$

The time required to freeze back the slurry backfill is

$$\frac{12.4 \times 0.75^2}{0.0397} = 176 \text{ hours or about } 7.3 \text{ days.} \qquad \text{(eq 4-13)}$$

At this time the slurry temperature is 32°F. Subsequent to freeze-back, the temperature of the slurry will continue to decrease and will approach the permafrost temperature. Ninety percent of the temperature difference will disappear in about twice the time required for freeze-back to 32°F. In this example, after a period of [7.3 + (2 × 7.3) =] 22 days, the slurry temperature would be approximately [32 - 0.90 (32 - 27) =] 27.5°F. The time (22 days) should be increased by 50 percent to permit an element of safety in the design. (Note: the sensible heat introduced by the pile and slurry was negligible in comparison to the latent heat introduced by the slurry and was not considered in calculations.)

(2) Permafrost temperature variations with depth, as discussed in TM 5-852-4/AFM 88-19, Chap. 4, should be considered in calculating freeze-back time. Figure 4-4 illustrates the effect of permafrost temperature on freeze-back for the above example. Since heat input is governed principally by the latent heat of slurry backfill, which is a function of slurry volume, moisture content and dry unit weight, a family of curves relating volumetric latent heat of slurry, permafrost temperatures and freeze-back time may be developed for a specific site to account

Figure 4-4. Specific solution of slurry freeze-back for varying pile shapes and preaugered hole diameters. To minimize the heat introduced by the slurry, the water content should be the minimum required for complete saturation. This can be best accomplished by backfilling with the highest dry unit weight material that can be processed and placed, i.e., a well-graded concrete sand with a 6-inch slump.

c. Pile Spacing. The effect of pile spacing on the overall rise of permafrost temperature resulting from installation of piles in preaugered holes is found by equating the latent heat of slurry backfill with the allowable sensible heat (temperature) rise of the surrounding permafrost. For *example*, calculate the minimum allowable pile spacing in the preceding example so that the permafrost temperature will not rise above 31°F. The following equation, equating the latent heat of the slurry to the change of sensible heat in a permafrost prism of side S, is used to determine the pile spacing:

$$S = \sqrt{(\pi r_2^2) + \frac{Q}{C\,\Delta T}} \qquad \text{(eq 4-14)}$$

where

S = grid pile spacing (ft)

r_2 = radius of augered hole (ft)

Q = latent heat of slurry per lineal foot (Btu/ft)

C = volumetric heat capacity of permafrost (Btu/ft³ °F)

ΔT = temperature rise of permafrost (°F).

Substitution of appropriate values from the above example and a maximum allowable permafrost temperature rise ΔT of 4°F give a minimum pile spacing S of

$$\sqrt{(3.14)(0.75)^2 + \frac{4280}{(27.7)(4)}} = 6.4 \text{ ft.} \qquad \text{(eq 4-15)}$$

This spacing may not keep local temperature from rising to more than 31°F; however, it will keep the entire mass of permafrost from reaching that temperature.

(1) Numerical analysis of a number of pile installations indicates that pile spacing should be *at least* five diameters of the drill hole size. A plot, similar to that shown in figure 4-4, may be prepared to relate pile spacing and permafrost temperature rise for the volumetric latent heat of the slurry backfill introduced into the drill hole. A family of curves may be developed to account for variation of slurry volumetric latent heat.

(2) In this example the slurry backfill was placed at a temperature slightly above freezing (33.5°F) and, theoretically, the sensible heat of the slurry should be considered. The volumetric capacity of the unfrozen slurry was [72(0.17 + 1.0 × 0.45) =] 44.6 Btu/ft³ °F, and with a temperature difference of (33.5 - 32 =) 1.5°F, this represents a sensible heat of (1.5 × 44.6 =) 67 Btu/ft³. A comparison of this quantity with the volumetric latent heat of the slurry (4670 Btu/ft³) shows that its heat may be considered negligible, as long as it near the freezing point.

d. Artificial freeze-back time. If permafrost is temperatures are marginal, it may be necessary to refrigerate the pile to accelerate slurry freeze-back time and to have refrigeration available if permafrost temperatures rise after construction. The following *example* shows calculations required to determine the amount of heat to be extracted from the ground. The average volume of slurry backfill for a group of piles is 31 cubic feet each. The slurry is placed at a n average temperature of 48°F and must be frozen to 23°F. A silt-water slurry of 80 lb/ft³ dry weight and 40 percent water content is used as backfill material, and an available refrigeration unit is capable of removing 225,000 Btu/hr. Calculate the length of time required to freeze back a cluster of 20 piles.

—Volumetric latent heat of backfill:

$$L = (144 \times 80 \times 0.40) = 4600 \text{ Btu/ft}^3. \qquad \text{(eq 4-16)}$$

—Volumetric heat capacity of frozen backfill:

$$C_f = 80\ [0.17 + (0.5 \times 0.4)] = 29.6 \text{ Btu/ft}^3\ ^\circ\text{F}. \qquad \text{(eq 4-17)}$$

—Volumetric heat capacity of unfrozen backfill:

$$C_u = 80\ [0.17 + (1.0 \times 0.4)] = 45.6 \text{ Btu/ft}^3\ ^\circ\text{F}. \qquad \text{(eq 4-18)}$$

—Heat required to lower the slurry temperature to the freezing point:

$$45.6 \times 31\ (48 - 32) = 22{,}618 \text{ Btu/pile.} \qquad \text{(eq 4-19)}$$

—Heat required to freeze slurry:

$$31 \times 4600 = 142{,}600 \text{ Btu/pile.} \qquad \text{(eq 4-20)}$$

—Heat required to lower the slurry temperature from the freezing point to 23°F;

$$29.6 \times 31\,(32 - 23) = 8258 \text{ Btu/pile.} \qquad \text{(eq 4-21)}$$

—Total heat to be removed from the slurry:

$$20\,(22{,}600 + 142{,}700 + 8200) = 3{,}470{,}000 \text{ Btu.} \qquad \text{(eq 4-22)}$$

—Time required for artificial freeze-back, excluding allowances for system losses:

$$3{,}470{,}000/225{,}000 = 15.5 \text{ hours.} \qquad \text{(eq 4-23)}$$

e. Heat transfer by thermal piles. Artificial freeze-back may be accomplished also by use of two types of self-refrigerated heat exchangers: a single-phase liquid-convection heat transfer device and a two-phase boiling-liquid and vapor convection heat transfer device. TM 5-852-4/AFM 88-19, Chapter 4 presents heat transfer rates for the two-phase system. There are few heat transfer field data available for the single-phase system.

4-3. Utility distribution systems in frozen ground.

General considerations for the design of utility systems in cold regions are given in TM 5-852-5/AFR 88-19, Volume 5.

a. Burying water pipes in frozen ground. Water pipes that are buried in frozen ground may be kept from freezing by any one of the following methods: 1) placing the water line in an insulated utilidor, which is a continuous closed conduit with all lines, such as water, sewage and steamlines, installed away from direct contact with frozen ground, 2) providing a sufficient flow velocity such that the water temperature at the terminus of the pipeline does not reach the freezing point or 3) heating the water at the intake or at intermediate stations along the line. A layer of insulation around a pipeline will retard, but not prevent, freezing of standing water in a pipe. The thermal analysis of a pipeline buried in frozen ground is complicated by the changing thermal properties, ice content, seasonal and diurnal changes of temperature and the intermittent water flow. Some calculation techniques applicable to the problem of buried utilities are presented below for standing and for flowing water. Additional techniques are presented in TM 5-852-5/AFR 88-19, Volume 5.

b. Freezing of standing water in a buried pipe. Problems with freezeup of stationary water must take into account the initial time required to lower the water temperature to the freezing point and the amount of time required to form an annulus of ice in the pipe. In most instances the danger point is reached when water begins to freeze.

(1) The time required to lower the temperature of nonflowing water in an insulated pipe to the freezing point is given by the equation

$$t = \frac{31.2}{K_i}\left(r_p^2 \ln \frac{r_i}{r_p}\right) \ln \frac{T_w - T_s}{32 - T_s} \qquad \text{(eq 4-24)}$$

where

t = time (hr)

K_i = thermal conductivity of insulation (Btu/ft hr °F)

r_p = radius of pipe (ft)

r_i = radius to outer edge of insulation (ft)

T_w = initial water temperature (°F)

T_s = temperature of surrounding soil (°F).

For *example*, a 12-inch diameter iron pipe containing water at 42°F is buried in 28°F soil. Determine the time required to lower the water temperature to 32°F if the pipe is insulated with 3 inches of cellular glass (K_i = 0.033 Btu/ft hr°F).

$$t = \frac{31.2}{0.033}\left(0.50^2 \ln \frac{0.75}{0.50}\right) \ln \frac{42-28}{32-28} = 120 \text{ hours (5 days)} \qquad \text{eq 4-25}$$

(2) Once the water temperature has been lowered to the freezing point, ice begins to form in an annular ring inside the pipe. The following assumptions are made to solve this problem:

—The water is initially at 32°F.

—The heat released by the freezing of water does not affect the surrounding ground temperatures.

—The volumetric heat capacity of the ice may be ignored.

—The thermal resistance of the pipe wall is negligible.

The solution predicts the time required to form an annulus of ice around the inner wall of the pipe. Knowledge of pipe radius, insulation thickness and

and thermal properties, thermal conductivity of ice, latent heat of fusion of water, and surrounding ground temperatures are necessary to solve this problem. The temperature of ground surrounding the pipe and the time during which the ground remains below freezing is difficult to estimate. The relationship between time and the radius of ice formed inside an insulated pipe is given by the expression

$$t = \frac{L r_p^2}{2 K \Delta T} \left[\left(\frac{K}{K_i} \ln \frac{r_i}{r_p} + 1/2 \right) \left(1 - \frac{r^2}{r_p^2}\right) - \left(\frac{r}{r_p} \right)^2 \ln \frac{r_p}{r} \right]. \quad \text{(eq 4-26)}$$

where

- t = time (hr)
- L = latent heat of water (9000 Btu/ft^3)
- r_p = radius of pipe (ft)
- K = thermal conductivity of ice (1.33 Btu/ft hr °F)
- ΔT = temperature difference between water and surrounding soil (°F, assume water temperature is 32°F)
- K_i = thermal conductivity of insulation (Btu/ft hr °F)
- r_i = radius to outer edge of insulation (ft)
- r = inner radius of ice annulus (ft).

If the pipe is not protected by insulation, the equation is

$$t = \frac{L r_p^2}{2 K \Delta T} \left[1/2 \left(1 - \frac{r^2}{r_p^2}\right) - \left(\frac{r}{r_p} \right)^2 \ln \frac{r_p}{r} \right]. \quad \text{(eq 4-27)}$$

This expression for an insulated pipe may be simplified by rearrangement and substitution of numerical values for the latent heat and the thermal conductivity of ice. This yields

$$t = 1690 \frac{r_p^2}{\Delta T} \{y\} \quad \text{(eq 4-28)}$$

where

$$y = \left[1 - \frac{r^2}{r_p^2} \left(1 - \ln \frac{r^2}{r_p^2}\right)\right]. \quad \text{(eq 4-29)}$$

The relationship between y and r/r_p is given in figure 4-5.

Following is an *example.* A 12-inch iron pipe, insulated with 3 inches of cellular glass (K_i = 0.033 Btu/ft hr °F), is placed in 28°F soil. Calculate the time required to reduce the bore of the pipe to 6 inches and the time required to completely freeze the water. Assume the rate of flow does not influence freezing. The time required to reduce the bore to 6 inches will be

$$t = \frac{9000\,(0.5)^2}{2(1.33)(32-38)} \left[\left(\frac{1.33}{0.033} \ln \frac{0.75}{0.50} + 0.5 \right) \left(1 - \frac{0.25^2}{0.50^2}\right) - \left(\frac{0.25^2}{0.50} \right) \ln \frac{0.50}{0.25} \right]$$

$$= 2640 \text{ hours.} \quad \text{(eq 4-30)}$$

The time required to completely freeze the water in the pipe will be

$$t = \frac{9000\,(0.5)^2}{2(1.33)(32-38)} \left[\left(\frac{1.33}{0.033} \ln \frac{0.75}{0.50} + 0.5 \right) \left(1 - \frac{0}{0.50^2}\right) - \left(\frac{0}{0.50} \right)^2 \ln \frac{0.50}{0} \right]$$

$$= 3550 \text{ hours.} \quad \text{(eq 4-31)}$$

The calculation is simplified since the term "r" for the inner radius of the pipe goes to zero. For an uninsulated pipe, the calculations assume the simplified form of

$$t = \frac{(1690)(0.5)^2}{(32-28)} \{y\} \quad \text{(eq 4-32)}$$

where r/r_p = 0 and y = 1.0 (fig. 4-5). Therefore,

t = 106 hours.

This example illustrates the effectiveness of insulation in retarding the freezeup of water in pipes, but as stated above, the assumptions used to develop these equations are conservative and the actual length of freezing time would be greater.

c. Thawing of frozen soil around a suddenly warmed pipe. In the preceding example it was assumed that water initially at 32°F was placed in frozen ground and the time relationship for freezing of the water in the pipe was determined. If the water was maintained above freezing, the frozen soil surrounding the ppe would thaw. To formulate a mathematical expression relating the time with the radius of thaw, it is assumed that: 1) the volumetric heat capacity of the soil is negligible, 2) both the surrounding soil and pipe are initially at 32°F, and 3) the pipe temperature is suddenly raised to a temperature above 32°F. The formula for an insulated pipe is

Figure 4-5. Freezeup of stationary water in an uninsulated pipe.

$$t = \frac{L r^2}{2 K_u \Delta T} \left[\left(\frac{K_u}{K_i} \ln \frac{r_i}{r_p} - 0.5\right)\left[1 - \frac{r_i^2}{r^2}\right) + \ln \frac{r}{r_i}\right] \quad \text{(eq 4-33)}$$

where

- t = time (hr)
- L = latent heat of soil (Btu/ft^3)
- r = radius to outer edge of thawing soil (ft)
- K_u = thermal conductivity of unfrozen soil (Btu/ft hr °F)
- ΔT = temperature difference between pipe and surrounding soil (°F, assume soil at 32°F)
- K_i = thermal conductivity of insulation (Btu/ft hr °F)
- r_i = radius to outer edge of insulation (ft)
- r_p = radius of pipe (ft).

If the pipe is not protected by insulation, the expression is

$$t = \frac{L r^2}{2 K_u \Delta T} [-0.5 (1 - \frac{r_p^2}{r_2}) + \ln \frac{r}{r_p}]. \quad \text{(eq 4-34)}$$

For *example*, a 7- by 7-foot concrete utilidor has 9-inch concrete walls with an outer covering of 6 inches of insulation (K_i = 0.033 Btu/ft hr °F). The frozen soil around the utilidor is a sandy gravel with a dry density of 115 lb/ft^3 and a water content of 7.8 percent at a temperature of 32°F. Neglect the thermal resistance of the concrete, and determine the time required to thaw 1 foot of soil when the temperature of the utilidor walls is suddenly raised to 50°F.

K_u = 1.2 Btu/ft hr °F
L = 1290 Btu/ft^3
K_i = 0.033 Btu/ft hr °F.

For calculation, square sections may be treated as cylinders of the same perimeter.

Symbol	Dimension-square (ft)	Equivalent radius (ft)
r	9.5 + 24/12 = 11.5	7.33
r_i	8.5 + 12/12 = 9.5	6.04
r_p	7.0 + 18/12 = 8.5	5.42

$$t = \frac{(1290)(7.33)^2}{2(1.2)(50-32)} [(\frac{1.2}{0.033} \ln \frac{6.04}{5.42} - 0.5) (1 - \frac{6.04^2}{7.33^2}) + \ln \frac{7.33}{6.04}]$$

$$= 2060 \text{ hours.} \quad \text{(eq 4-35)}$$

d. Practical considerations.

(1) The above-mentioned formulas indicate the relationship between time, radius of freeze or thaw, and temperature difference between the water in the pipe and the ground. In sufficient time, standing water in the pipe will freeze or frozen ground will thaw, depending on temperature differentials. For practical problems the assumed constant temperature differentials will not exist for a long time but will vary with season and even with the hour at shallow depths. The freezing and thawing index concept considers the intensity of temperature differential from freezing (32°F) and the duration of this differential. In the preceding equations, the time t can be multiplied by the temperature differential ΔT to give either a freezing or thawing index, and the radius of ice formation or thawed ground radius can be determined by trial-and-error. It was shown that a temperature differential of (32-28) = 4°F lasting for 3150 hours would result in complete freezeup of the water in the pipe; this is equivalent to a freezing index of [(4 × 3150)/24 =] 524 degree-days. If the freezing index at the depth of pipe burial were less than 524, the standing water in the pipe would not completely freeze in that time. Thus, the freezing index at a particular depth can be used to forecast freezeup of stationary water in pipes located in the annual frost zone.

(2) Even insulated water lines located in frozen ground usually require an inlet water temperature significantly above freezing. Whether the water lines are insulated or not, this may thaw some of the surrounding frozen ground. This thawed annulus will retard water freezeup in the pipe if and when flow conditions change. The situation is practice is generally complicated by the intermittent character of water demand. In some northern communities the problems of irregular water demand are solved by constructing the water lines in a continuous loop with provisions for periodic flow reversals. Water temperatures should be closely monitored and water usage patterns considered in estimating water freezeup.

e. Freezing and flowing water in buried pipes. Problems involving freezing of flowing water in buried pipes require knowledge of the distance the water will flow before the temperature of the water lowers to the freezing point. By providing enough above-freezing water, the loss of heat to the surrounding frozen soil can be balanced to provide an outlet temperature slightly above freezing. The problems of freezing of flowing water in insulated and bare pipe are illustrated below.

(1) *Insulated pipe.* It is assumed that 1) the temperature of the frozen ground surrounding the pipe is constant for the period of flow over the

entire length of pipe, 2) the effect of friction heat developed by water flow is negligible, 3) the thermal resistance and heat capacity of the pipe wall are negligible, and 4) the temperature distribution of the water in the pipe is uniform at each cross-sectional area. The velocity required to prevent freeze-up of flowing water in a pipe is given by

$$V = \frac{s K_i}{112{,}000\, r_p^2 \left(\ln \frac{r_i}{r_p}\right)\left(\ln \frac{T_i - T_s}{T_2 - T_s}\right)} \quad \text{(eq 4-36)}$$

where

V = velocity of flow (ft/s)

s = length of pipeline (ft)

K_i = thermal conductivity of insulation (Btu/ft hr °F)

r_p = radius of pipe (ft)

r_i = radius to outer edge of insulation (ft)

T_i = inlet water temperature (°F)

T_2 = outlet water temperature (°F)

T_S = temperature of surrounding frozen soil (°F).

For *example*, an 11,000-ft long, 6-inch-diameter pipe is buried in 10°F soil. The pipe is covered with a 2-inch layer of insulation (K_i = 0.03 Btu/ft hr °F) and the inlet water temperature is 39°F. Calculate the velocity of flow required to keep the water from freezing.

$$V = \frac{(11{,}000)(0.03)}{112{,}000\,(0.25)^2 \left(\ln \frac{0.417}{0.25}\right)\left(\ln \frac{39 - 10}{32 - 10}\right)}$$

$$= 0.33 \text{ ft/s (20 ft/min).} \quad \text{(eq 4-37)}$$

To provide for temporary reductions in flow and in recognition of the uncertainties concerning the manner of ice formation within the pipe, it is recommended that the velocity of flow be doubled in design.

(2) Uninsulated pipe.

(a) When flowing water is first introduced into a bare pipe buried in frozen ground, the heat loss from water is greater than it is after the system has been in operation for a period of time. The initial heat loss is greater because the pipe wall and the soil immediately adjacent to the pipe are colder than they are after water has flowed over a time. Soil temperatures surrounding the pipe increase and eventually become reasonably stable with time. An expression relating these variables is

$$\frac{T_1 - T_S}{T_2 - T_S} = \exp \frac{s}{2r_p} \times \frac{h}{V} \times \frac{1}{5.6\times10^4} \quad \text{(eq 4-38)}$$

where

T_1 = inlet water temperature (°F)

T_S = frozen soil temperature (°F)

T_2 = outlet water temperature (°F)

s = length of pipeline (ft)

$2r_p$ = diameter of pipe (ft)

h = heat transfer coefficient (Btu/ft^2 hr °F)

V = velocity of flow (ft/s).

A nomogram of this equation is shown in figure 4-6.

(b) Limited field experiments in clay and sandy clay soils suggest values of h for metal pipelines subject to normal use (conditions or intermittent flow) of 6.0 for the initial period of operation an 2.0 thereafter. These values are not applicable for pipes smaller than 4-inches in diameter. The h value is dependent upon the thermal properties of the surrounding soil, the diameter of the pipe, the type of pipe material and the temperature gradient in the ground around the pipe's radius. The value for h given above provides a reasonable basis for design of pipelines in which the total quantity of water consumed per day is at least eight times the volume of pipes in the entire system. The time of operation required for the temperature distribution in the water to stabilize is approximately.

$$t_o = 0.005 \frac{s}{V} \quad \text{(eq 4-39)}$$

where

t_o = time (hr)

s = length of pipeline (ft)

V = velocity of flow (ft/s).

(c) For *example*, water at an inlet temperature of 40°F flows at 2 ft/s in a 12-inch iron pipeline, 2.2 miles long. The ground temperature surrounding the pipe is 25°F. Estimate the outlet water temperature during the initial period of flow (h = 6.0) and after surrounding temperatures have stabilized (h = 2.0).

—Initial period (see fig. 4-6):

$$\frac{s}{2r_p} = \frac{2.2 \times 5280}{1} = 1.16 \times 10^4 \qquad \text{(eq 4-40)}$$

$V = 2$ ft/s.

$h = 6.0$ Btu/ft^2 hr °F

thus

$$\frac{T_1 - T_S}{T_2 - T_S} = 1.77 = \frac{40 - 25}{T_2 - 25} \qquad \text{(eq 4-41)}$$

$T_2 = 33.5$°F.

—Stabilized period:

$$\frac{s}{2r_p} = 1.16 \times 10^4$$

$V = 2$ ft/s

$h = 2.0$ Btu/ft^2 hr °F

thus

$$\frac{T_1 - T_S}{T_2 - T_S} = 1.22 = \frac{40 - 25}{T_2 - 25} \qquad \text{(eq 4-42)}$$

$T_2 = 37.3$°F.

Figure 4-6. Temperature drop of flowing water in a pipeline.

To operate on the safe side, the outlet temperature T_2 should remain at or above 35°F. The calculated initial water temperature of 33.5°F would be considered unsafe and either the flow velocity should be increased to approximately 3 ft/s or the inlet water temperature should be raised to about 43°F. These precautions would be needed only for an initial period, i.e.,

$$t_o = 0.005 \frac{S}{V} = 0.005 \frac{11{,}600}{2}$$

$$= \text{approximately 29 hours.} \qquad \text{(eq 4-43)}$$

f. Design considerations. The calculation techniques presented above indicate the principal factors to be considered in design of water distribution lines placed in frozen ground. The use of these techniques together with recognition of the complexities of actual in-pipe ice formation and sound engineering judgment provides a basis for design of pipelines in areas of seasonal frost and permafrost. Changing the surface cover over an installed pipeline will affect the distribution of temperatures with depth and may result in depressing the temperatures adjacent to the pipe. This is particularly true if a natural vegetative cover is stripped and replaced by a snow-free pavement. The influence of new construction above an existing pipeline may require a change in operating procedures for the system, such as an increase in the velocity or flow or additional heating at the inlet.

4-4. Discussion of multidimensional heat flow.

a. The relatively simple analytical techniques discussed in this manual are not always sufficient for considering the concurrent thermal effects of multidimensional temperature change and soil water phase transformation. The one-dimensional and radial heat-flow computation techniques presented in this manual were based on field observations and the use of reasonable simplifying assumptions. The techniques are intended to facilitate analysis and to promote adequate design. The assumptions involved and technique limitations have been emphasized.

b. Heat flow beneath heated structures is multidimensional because of the finite boundaries of such structures. The ground surface temperature adjacent to the south side of the building is generally higher than that on the north side, and the ground surface temperature on the west side is generally higher than on the east side; in addition to this influence, the three-dimensional temperature distribution beneath the building will be affected by the plan dimensions of the floor. Three-dimensional solutions are available to the problem of heat flow in homogeneous materials beneath the surface of a heated finite area surrounded by an infinite area subject to a dissimilar surface temperature condition; however, the solutions consider only the effects of temperature change and not the effects of phase transformations. Such solutions tend to be rather complex and unwieldy, and their neglect of latent heat generally results in an over-estimation of the depth of freeze or thaw. The magnitude of this over-estimation is dependent on the quantity of moisture in the frozen or thawed soil.

c. The example given in paragraph 3-8*a* for calculating the depth of thaw beneath a heated slab-on-grade building considered only one-dimensional vertical heat flow and excluded lateral heat flow from the soil beneath the building to the surrounding soil mass in the winter. The amount of lateral heat flow would depend on the building dimensions and the wintertime soil temperature gradient. Slab-on-grade, heated structures usually prevent frost penetration under the center of the building and result in a thaw bulb in the foundation soil that may cause permafrost degradation with time. This type of construction is discussed in TM 5-852-4/AFM 88-19, Chapter 4.