# CHAPTER 12
# THERMAL CONSIDERATIONS

## 12-1. General considerations.

The thermal aspects of utility system designs are among the most critical elements for cold regions systems. The potential problems are failure of pipes due to freezing of water, thaw settlement or heaving of foundation soil, thermal strains and the associated stresses, the cost-effective selection of materials and insulation thicknesses, and economical operation. This chapter presents criteria and design examples for the most critical thermal calculations that might be required for design of pipes, utility structures and appurtenances.

## 12-2. Freezing of pipes and tanks.

Damage or failure occurs due to the expansion of water changing to ice. The hydrostatic pressure on the still-unfrozen liquid can reach several hundred atmospheres and it is this pressure, not the contact of the ice, that typically causes pipe failure. Prevention of freezing is accomplished via the most cost-effective combination of insulation, heat trace, circulation, etc., using the methods presented in this section. Insulation alone will not necessarily prevent freezing. It reduces the rate of heat loss and extends the freeze-up time. Small diameter service connections may have a freeze-up time measured in minutes or a few hours. These are the most vulnerable portion of the system and will usually freeze first. Thawing capability is mandatory for these small diameter pipes.

## 12-3. Thawing of frozen pipes.

Remote electrical thawing methods that can be incorporated in the original design include skin effect, impedance, and various resistance wire and commercial heating cable systems. Frozen wells have been thawed by applying a low voltage from a transformer to a copper wire located inside the riser. Once a small annulus is melted, the flow can be restarted and it will thaw the remaining ice. Chapter 6 contains details on thawing of frozen pipes.

## 12-4. Heat loss from pipes.

The total heat loss and the freeze-up time are dependent on the ambient and internal temperatures in the pipe system. Above-ground piping systems must be designed for the lowest expected air temperatures, which range from -40 to -75 degrees F. These extreme surface temperatures are attenuated by burial depending on the thermal properties of the soil. For example, the air temperature at the surface might have an annual range of 150 degrees F. At a depth of six feet the temperature may vary slightly with the season, and at 30 feet seasonal temperature fluctuations are negligible. Frost penetration will be greatest in rock or bare, dry soils. A snow cover will typically reduce the depth of frost penetration by an amount equal to the snow depth. Locating utility lines away from snow-plowed surfaces will take advantage of this potential. There is a time lag involved with frost penetration so that maximum depth of frost penetration will occur long after the extreme winter temperatures. At a depth of six feet the lag time may be one to five months after the onset of freezing conditions at the surface. The specific time depends on soil properties and moisture conditions.

*a. Direct burial.* Water and sewer mains are typically buried below the maximum depth of seasonal frost. In cold regions, the frost penetration is often greater than the common pipe burial depths of 6 to 10 feet, and may be 20 feet or more in exposed dry soil or rock. Deep frost penetration, high groundwater, hilly terrain, rock or other factors will make it more practical and economical to install all or portions of the utility system within the frost zone. In these cases, the degree of freeze protection necessary will depend upon the ground temperatures at the pipe depth. Where pipes are only intermittently or periodically within frost, conventional bare pipes will be adequate, provided a minimum flow can be maintained by circulation, bleeding or consumption. Frost-proof appurtenances, stable backfill and some heating will also be necessary. Heat loss and freeze danger are significantly reduced by insulating the pipes. Insulated pipes can be installed in shallower trenches or within berms at ground surface. In these cases, the minimum depth of cover would be 1.5 to 3.0 feet for exposed ground surfaces. Greater depths will be necessary if heavy surface traffic is expected.

*b. Insulation barrier far buried pipes.* Buried pipes within seasonal frost can also be protected by placing a layer of insulation board, usually polystyrene, above the pipe (fig 12-1). This method, using bare pipes and fittings and board insulation, is often less expensive (for materials) than use of prefabricated insulated pipe; however, the construction cost will be higher and the effectiveness of the insulation is lower than direct insulation on the pipes. The board method has been used where the soils underlying the pipe are frost-susceptible, since frost penetration beneath the pipes can be prevented by the insulation board. The necessary thickness and width of the board increases for shallower pipes and deeper frost penetration, such as dry soils or rock. The relative economics, compared to that for insulated pipes, is improved when pipes are placed in a common trench under a board and when warm sewer or central heating lines are included. Generally, the insulation should be a minimum of 4 feet wide for a single pipe, and the thickness will be determined by the proposed depth of burial and the expected or calculated frost penetration. In terms of reducing frost penetration, two inches of polystyrene foam insulation (k = 0.02 BTU/hr•ft•°F) is roughly equivalent to 4 feet of sand or silt or 3 feet of clay cover over the pipe. The heat loss and trench width will be reduced by placing the insulation in an inverted U. The design example in paragraph 12-9c illustrates frost penetration calculations beneath an insulation board.

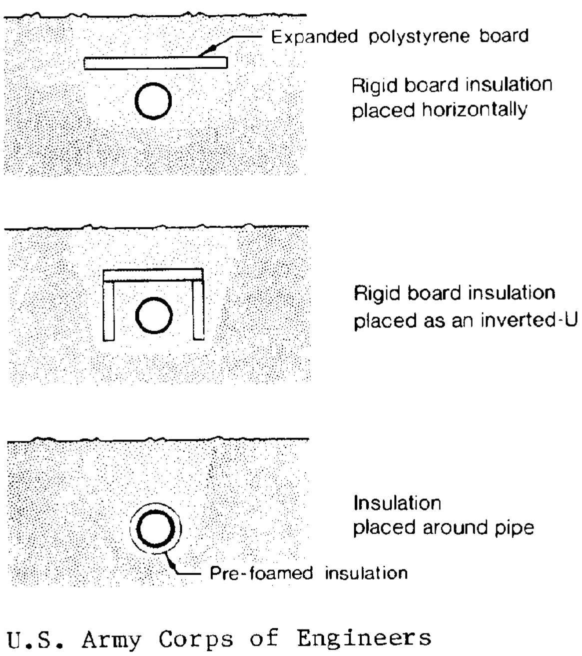


*Figure 12-1. Methods of insulating buried pipe.*

*c. Deep burial.* Deeper pipes will experience less extreme ambient temperatures, lower maximum rates of heat loss and a longer safety factor time for freezing. However, the heating period will be longer, and pipes installed in permafrost will require freeze-protection all year.

## 12-5. Physical methods for reducing heat loss.

The primary physical method of reducing heat loss is insulation. It is impractical to prevent gound moisture, humidity or water from pipe failures from reaching the insulation and, since moisture content is a key fctor in determining the thermal performance of insulations, only near-hydrophobic insulations will be used. Even these insulations will usually require some physical and moisture protection.

*a. Amount of insulation.* An economic analysis to balance heating and insulating costs must be performed to determine the minimum amount of insulation that is required (see para 12-9i) and will include factors, such as the freeze-up time, the maximum rate of heat loss and practical dimensional considerations. Heat loss estimates for pipe systems must consider exposed sections of pipes, joints and appurtenances, and thermal breaks such as at pipe anchors. For example, a 5-inch gate valve has a surface area equivalent to 3 feet of bare pipe. If this valve were left exposed it would lose as much heat as about 200 feet of 5-inch pipe insulated with 2 inches of polyurethane insulation and freezing would occur at the valve first. To ensure a safe design, the thermal resistance around appurtenances must be 1.5 times that required around the adjacent pipe sections.

*b. Location of insulation.* Heat loss and the volume of materials will be reduced by minimizing exposed surface area. This is most important for above-ground pipes and facilities. Insulation is most effective when it is placed directly around the source of heat. These characteristics are illustrated by the simple shapes in figures 12-2 and 12-3. Where there is an air space, the thermal resistance of the pipe air film can be quite significant and must be considered. For a single pipe, insulation is best applied in an annulus directly around the pipe. Heat loss from several pipes in a compact utilidor is less than that of the same number of separate pipes insulated with the same total volume of insulation. Heat loss will also be reduced and freeze protection improved by installing one water pipe inside of a larger one, rather than using two separate pipes. This technique is applicable for freeze protection of small-diameter recirculation pipes used to maintain a flow in supply lines, or dead ends within a water distribution system.

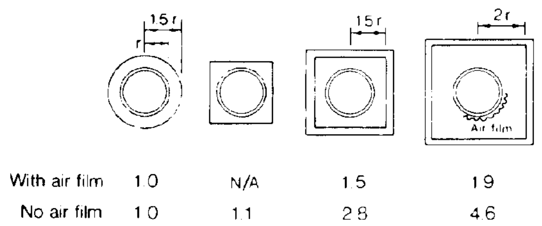


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*Figure 12-2. Relative heat loss from single pipes insulated with same volume of insulation.*

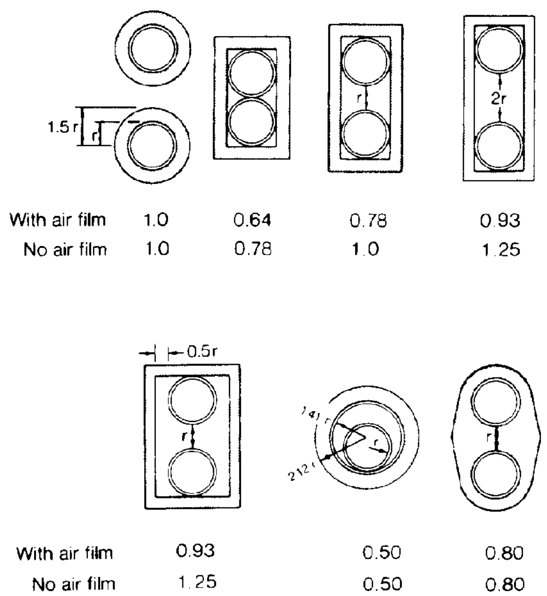


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*Figure 12-3. Relative heat loss from two pipes insulated with same volume of insulation.*

## 12-6. Heat loss replacement.

If ambient temperatures are below 32 degrees F, freezing Will eventually occur in the pipe unless heat is added to the fluid. Heat can be added either continuously or at point sources.

*a. Fluid replacement.* Freezing will not occur if the liquid residence time in the pipeline is less than the time necessary for it to cool to the freezing point. The quantity and temperature of the replacement water must be sufficient and the flow must be reliable. Operation without additional heating is restricted to situations where relatively warm water supplies, such as ground-water, are used or where the flow rate is reliable and high, such as in some water supply pipelines or mains. Bleeding of water has been used to maintain or enhance the flow in service lines, dead ends and intermittent flowing pipelines, but the wasting of large quantities of water is inefficient and results in water supply and wastewater treatment problems. Recirculation will maintain a flow and a uniform temperature within the system, and prevent premature freezing at locations with lower-than-average ambient temperatures or at poorly insulated sections. However, the water temperature will still decline unless warmer water is added or the recirculating water is heated.

*b. Point sources of heat.* Water may be heated at the source, treatment plant, pumping stations, or along the pipeline or within distribution systems as required. Heat is commonly obtained from oil-fired boilers; however, simple electric water heaters have been used where the heat requirements are very low. The heating of water can be practical with low-temperature waste heat, such as from electric power generation. There must also be sufficient flow within the piping system to distribute the heat. If the normal water demand is too low or is intermittent, then bleeding or recirculation is necessary. A minimum water temperature will be maintained with the piping system by increasing either the flow rate or the input water temperature while keeping the other parameters constant, or by adjusting them simultaneously. As a general rule, the temperature drop along a pipeline must always be kept to less than 10 degrees F, and preferably less than 5 degrees F, by insulation, higher flow rates, or intermediate heating along the pipeline. Velocities greater than 0.3 foot per second for 6-inch pipes and 1.5 feet per second for 2-inch pipes are of little benefit in reducing total energy input to maintain a specified minimum water temperature. Higher velocities must be balanced with the electrical energy requirements for pumping and are not usually practical for large diameter mains.

*c. Heat tracing.* Replacement of heat losses and maintenance of a minimum temperature can also be accomplished with heat tracing systems. Circulation of warm air has been used in large, open utilidors but the most common heat tracing systems are either separate fluid or electrical lines as the heat source.

(1) *Fluid tracers.* For pipe heat tracing, hot water, generally between 175 and 200 degrees F, is much simpler to use than either steam or higher temperature water. The use of an antifreeze solution protects the heat trace piping, allows start-up during winter and provides a means of thawing frozen pipes. The viscosity of low freezing point glycol and water mixtures is greater than that of water; therefore, the required pumping capacity and friction losses will be higher. The heat transfer

characteristics are also poorer than for water. For example, a 50% mixture of glycol and water would require a 15% increase in flow rate to achieve the same heat transfer. Design information on these heating systems is available from the American Society of Heating, Refrigeration and Air-Conditioning Engineers (ASHRAE) (ethylene glycol is toxic and cross-contamination must be prevented). Propylene glycol, which is non-toxic, but more expensive, can be used. Glycol solutions are corrosive to zinc and can leak through joints and pump seals that will not leak water at the same pressures. Some boiler manufacturers void their warranty if glycol solutions are used. Mechanical seal pumps should be utilized to prevent leakage. Special organic fluids can also be used instead of water-glycol solutions.

(2) *Electric heat tracing.* Electric heat tracing systems are relatively easily installed and controlled. They can be installed continuously on water and sewer pipelines, or only at freeze-susceptible locations, such as road crossings, service connections or at appurtenances such as fire hydrants. Because of the relatively high cost of electrical energy, these systems are usually installed for freeze-prevention or system restoration in the event of operating upset, such as a prolonged no-flow condition, rather than as the primary method of maintaining a minimum operating temperature in the system.

*(a) Types*. A variety of electric heat tracing systems and products are available from a number of manufacturers. Resistance-type cables and wires are available for installation with pipes or for exterior tracing. Small-diameter metal pipes, such as service lines, can be heated or thawed by induction heating from an alternating current in a wire wrapped around the pipe which induces eddy currents within the pipe. The most common electric heat tracing systems used are zone-parallel and self-limiting continuous parallel heating cables and strips. Since they contain separate conductor and resistance buss wires or conducting material in the same casing, they produce a constant heat output per unit length, and can be conveniently cut to the desired length. Maximum lengths are usually 250 feet to 650 feet. One type has a carbon-filled polymeric heating element with self-adjusting properties that decreases heat output as the temperature increases. This cable will not burn itself out or overheat plastic pipes, and the heat output modulates, to some extent, with temperatures along the pipe.

*(b) Location.* Maximum heat transfer efficiency will be obtained if the heating cables are installed inside the pipe. However, the coating and joints of only a few cables, such as mineral insulant (M.I.) resistance cable, are approved for installation within water pipes or for submerged conditions. In-line cables are more practical for long water supply transmission mains, but are impractical in the distribution network because valves and other fittings must be bypassed. They may be subject to vibration damage when fluid velocity is greater than 4 feet per second, and the cables must be removed to clean the pipes and when some types of pipe repairs are made. Heating cables are more conveniently and commonly located on the outside pipe surface. The required capacity for heating cables must be increased by a factor of 1.5 unless flat or wide heating strips or adequate contact between the cable and the pipe, preferably with heat transfer cement, is maintained. Exterior cables for pre-insulated pipe are commonly installed within a raceway or conduit attached to the pipe surface, which facilitates fabrication, installation, removal and replacement. In this configuration, the air space and poor contact of the heating cable with the pipe can further reduce the heat transfer efficiency and the heat input for plastic pipes may need to be two to six times that for a heating cable within the pipe. It is difficult to make the joints in the exterior heating cable channel watertight, as is required for most cables or their joints when used undergound.

*(c) Overheat protection.* Plastic pipes, insulations and the electric heat tracing system itself must be protected from overheating unless the self-limiting heating cable is used. For conventional cables, a high temperature thermostat cut-off is usually installed and set at about 85 degrees F and the sensor is placed on the surface of the heat cable.

*(d) Controls.* To provide freeze protection, automatic control systems must activate the electric heat tracing system at a set point above 32 degrees F to provide some lead time and allow for variances in the temperature detection sensitivity of the thermostat and sensor. To provide economical operation, the controls also cut off the power supply when heating is not required. These controls are often a major cause of malfunction and wasted energy. Mechanical thermostats with capillary tube sensing bulbs are limited to about 16 feet in length and temperature control is only possible within a few degrees. Electronic thermostats are much more sensitive but they are expensive. The resistance sensors they use can be located any practical distance from the controller and the system can be selected to maintain fluid temperatures within 0.1 degree F. This type of system, which is commonly used in Greenland, allows the utility system to be reliably operated at near-freezing temperatures. The sensors must be located with care to provide proper control, freeze-protection, and prevent the waste of

energy. To accurately measure the fluid temperature, they should be put in a pipe well or attached to the pipe surface with heat transfer cement, particularly for plastic pipes. They should be located where the lowest pipe temperatures within the section being controlled are expected, such as at exposed windswept areas or shallow buried sections.

(3) Pipe friction. Friction heating is negligible for smooth pipes with fluid velocities less than 6 ft/s (feet per second), which is about the desirable upper limit for flow in pipes. At high velocities frictional heat is significant, but deliberately increasing the velocity for this purpose is an inefficient method of heating since the energy is supplied by pumping. The equations for frictional heat input are presented in figure 12-4.

## 12-7. Insulation materials.

Common insulating materials are plastics, minerals and natural fibers, or composite materials. For design purposes, the structural and thermal properties for the worst conditions must be used. These conditions occur after aging, compaction, saturation and freeze-thaw cycles. Other selection considerations are ease of installation, vapor transmission, burning characteristics, and susceptibility to damage by vandals, animals, chemicals and the environment. The insulating value of a material depends more or less directly on the volume of entrapped gas in the material. If the material becomes wet and the voids filled with water, the insulating properties are lost since the thermal resistance of air is about 25 times that of water and 100 times that of ice. In the past, the lack of a near- hydrophobic insulation made the design of piping in moist environments very difficult and is a major reason for the development of aboveground utilidors.

*a*. Polyurethane foam is used extensively in cold regions to insulate pipes and storage tanks, and is also used in some buildings and foundations. Urethane will bond to most materials. Piping or other components can be pre-insulated or polyurethane can be applied on-site from the raw chemicals, which are about 1/30th the final volume. Field applications are restricted by climatic conditions, and the density and thermal conductivity will often be higher than values attainable under factory conditions. The foam must be protected from ultraviolet radiation. A metal skin has proven effective to prevent the loss of entrapped heavy gas which can increase the thermal conductivity by about 30% above the theoretical minimum value. Densities over 6 pounds per cubic foot are essentially impermeable, but lighter foams, which are better insulators, require coatings to prevent water absorption, since freeze-thaw cycles of the moisture can lead to deterioration of the insulation.

*b*. Extruded polystyrene, particularly the high density products (3 pounds per cubic foot), suffer the least from moisture absorption and freeze-thaw but the outer 0.25 inch of unprotected buried insulation should be disregarded in thermal analyses. Molded polystyrene will absorb some moisture and should not be used in moist conditions. Polystyrene is available in board stock or beads. The former has been extensively used to reduce frost penetration. Beads are useful for filling voids in utilidors while retaining easy access to pipes. Although the thermal conductivity of polystyrene is higher than that of urethanes, the volumetric cost is usually lower.

*c*. Glass fiber batt insulation is the most common building insulation, primarily because it is fire-resistant and relatively inexpensive. Its insulating value is significantly reduced when wet, and is reduced by half if 8% by volume is water. For this reason, glass fiber should not be used underground but may be considered wherever dry conditions can be ensured. Cellular glass is very water-resistant but is seldom used because it is brittle and difficult to work with. Lightweight insulating concrete made with polystyrene beads, pumice or expanded shale can be formulated with relatively high strength and thermal resistance. It can be poured in place around piping but must be protected from moisture to prevent freeze-thaw deterioration.

## 12-8. Thermal calculations.

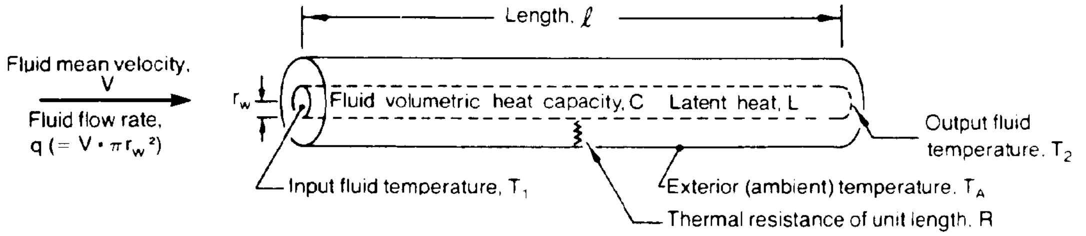
The analytical thermal equations presented below use a number of simplifying approximations. The user must determine their applicability for particular problems and consider the various models and a range of values for physical and temperature conditions. This chapter includes time-independent steady-state heat flow procedures as well as calculations to determine ground temperature and the depth of freezing and thawing. The symbols used are defined in table 12-1 and the thermal conductivity of common materials in table 12-2. Solutions to typical utility system problems are given to illustrate the procedures involved.

*a*. *Steady-state pipeline heat loss*. These include typical cases for bare and insulated pipes, and single and multiple pipes in above- and below-ground configurations. These methods are presented in figures 12-5,12-6 and 12-7.

(1) Figure 12-5 deals with heat flow from a bare pipe, an insulated pipe, a single pipe in an insulated box, and a utilidor carrying multiple pipes. In each case, some of the major approxima-

## Heat Loss and Temperature Drop in a Fluid Flowing Through a Pipe

## Freeze-Up Time For a Full Pipe Under No-Flow Conditions ($V = o$)




**Comments:** The above sketch is schematic. R and $T_A$ appearing in these equations can be replaced by the thermal resistance and corresponding exterior temperature for any shape or configuration.

$D = \pi r_w^2 \; V \cdot C \cdot R$

**Calculate $T_1$ or $T_2$, Given R, $T_1$ or $T_2$, $T_A$**

$T_1 = T_A \cdot (T_2 \;\; T_A) \;\; \exp(-\ell/D)$

$\;\; T_A \cdot (T_2 \;\; T_A) \;\; (1 + \ell/D) \;\; \text{if } \ell/D \cdot 0.1$

$T_2 = T_A + (T_1 - T_A) \exp(-\ell/D)$

**Calculate R, Given $T_1$, $T_2$, $T_A$**

$R = -\ell \;\; \pi r_w^2 \cdot V \cdot \;\; C \;\; \cdot \ln[(T_2 - T_A) \;\; (T_1 - T_A)]$

$\;\; \ell(T_1 \;\; T_A) \;\; \pi r_w^2 \cdot R \cdot C \;\; (T_1 - T_2) \;\text{if } \ell/D \cdot 0.1$

**Calculate V, Given $T_1$, $T_2$, $T_A$, R**

$V = -\ell \;\; \pi r_w^2 \cdot R \cdot \;\; C \;\; \cdot \ln[(T_2 \;\; T_A) \;\; (T_1 \;\; T_A)]$

$\;\; \ell(T_1 \;\; T_A) \;\; \pi r_w^2 \cdot V \cdot C(T_1 \;\; T_2) \;\; \text{if } \ell/D \cdot 0.1$

**Calculate Heat Loss (Q), Given $T_1$ or $T_2$, $T_A$, V, R**

$Q \;\; (D \; R)(T \;\; T_A)[1 \;\; \exp(\;\ell \; D)]$

$\;\; (\ell \; R)(T_1 \;\; T_A) \;\; \text{for } \ell \; D \cdot 0.1$

$= D/R \;(T_2 - T_A)\left[\exp(\ell/D) - 1\right]$

**Calculate Friction Heating, Given V, f**

$Q_f \;\; F \cdot r_w^2 \cdot V \cdot f$

Where
- $Q_f$ BTU h • ft
- F 0.2515 BTU ft⁴
- r ft
- V ft h
- f friction head loss, ft ft length
- Not significant for V · 2.3x10⁴ ft h

or
- $Q_f$ J s • m
- F 3.074 x 10⁴ J m⁴
- r m
- V m s
- f friction head loss, m m length
- Not significant for V · 2 m s

**Freeze-Up Times; Given R, $T_1$, $T_A$**

Assume that thermal resistance of the ice, as it forms, and the heat capacity of the pipe and insulation are negligible.

Design Time (Recommended)

$t_D$ = Time for the fluid temperature to drop to the freezing point.

$\pi \cdot r_w^2 \cdot R \cdot \;\; C \;\; \cdot \ln[(T_1 - T_A) \;\; (T_2 - T_A)]$

Safety Factor Time

$t_{SF}$ Time for the fluid to drop to the nucleation temperature. Same as $t_D$ but with $T_C$ replaced by 27°F

Complete Freezing Time

$t_F$ = Time for the fluid at freezing point, 32°F, to completely freeze solid.

$\pi \cdot r_w^2 \cdot R \cdot L \cdot \;\; (T_C \;\; T_A)$

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*Figure 12-4. Temperature drop and freeze-up time in pipes.*

*Table 12-1. List of symbols used in thermal calculations*

Symbols

A = Amplitude

A = Thermal factor = T′ arccosh $H_p/r_p$

B = $[\pi C_\ell / k \cdot p]^{1/2}$

C = $[H^2 - r^2]^{1/2}$ ft

$c_m$ = Mass heat capacity, BTU/lb·°F

C = Volumetric heat capacity, BTU/$ft^3$·°F

d & t = Thickness, ft
D = Scaling parameter, ft
E = Young's modulus, lb/$ft^2$
F = arccosh (H/r)
h = Thermal film coefficient (or surface conductance), BTU/h·$ft^2$·°F
H = Depth of burial, ft
I = Freezing or thawing index, °F·h
k = Thermal conductivity, BTU/h·ft·°F
$\ell$ = Length, ft
L = Volumetric latent heat, BTU/$ft^3$
p = Period, sec, hr
P = Perimeter (mean), ft
q = Fluid flow rate, $ft^3$/sec

Q = Rate of heat loss per unit longitudinal length, BTU/ft·hr

r = Radius, ft

R = Thermal resistance of unit longitudinal length, hr·ft·°F/BTU

t = Time, sec, hr

T = Temperature, °F

T* = $(T_I - T_G)/(T_0 - T_G)$

u = Coefficient of thermal expansion, ft/ft·°F

w = Moisture content of dry weight, %

V = Velocity ft/hr
x = Depth
X = Depth to freezing (32°F) plane, ft
$\alpha$ = Thermal diffusivity, $ft^2$/hr
$\gamma$ = Unit weight (density), lb/$ft^3$

a, $\mu$, $\lambda$ = Coefficients in modified Berggren equation

Subscripts

A - refers to air

$\alpha$ = thermal diffusivity, $ft^2$/hr

$\gamma$ = unit weight (density), lb/$ft^3$
a, $\mu$, $\lambda$o ground freezing index
G - refers to ground
h = refers to heating index
I - refers to insulation
j - denotes 1,2,3, . . . .
L = refers to thermal lining (of utilidor)
m - refers to mean
0 - refers to (zero) freezing point of water
P - refers to pipe
S - refers to soil
t - refers to thawed soil
U - refers to utilidor

W - refers to water (fluid) within a pipe

x - refers to depth

Z - refers to zone of thaw

*Table 12-2. Thermal conductivities of common materials*

| Material | Unit Weight (dry) lb/ft³ | Specific Heat Capacity | BTU/ ft·h·°F |
|---|---|---|---|
| Air, no convection (32°) | | 0.24 | 0.014 |
| Air film, outside, 15 miles·hr wind (per air film) | | | 0.50 |
| Air film, outside (per air film) | | | 0.14 |
| Polyurethane foam | 2 | 0.4 | 0.014 |
| Polystyrene foam | 1.9 | 0.3 | 0.020 |
| Rock wool, glass wool | 3.4 | 0.2 | 0.023 |
| Snow, new, loose | 5.3 | 0.5 | 0.05 |
| Snow, on ground | 19 | 0.5 | 0.13 |
| Snow, drifted and compacted | 31 | 0.5 | 0.4 |
| Ice at -40°F | 56 | 0.5 | 1.54 |
| Ice at 32°F | 56 | 0.5 | 1.28 |
| Water at 32°F | 62.4 | 1.0 | 0.34 |
| Peat, dry | 16 | 0.5 | 0.04 |
| Peat, thawed, 80% moisture | 16 | 0.32 | 0.08 |
| Peat, frozen, 80% ice | 16 | 0.22 | 1.0 |
| Peat, pressed, moist | 71 | 0.4 | 0.40 |
| Clay, dry | 106 | 0.22 | 0.5 |
| Clay, thawed, saturated (20%) | 106 | 0.42 | 1.0 |
| Clay, frozen, saturated (20%) | 106 | 0.32 | 1.2 |
| Sand, dry | 125 | 0.19 | 0.06 |
| Sand, thawed, saturated (10%) | 125 | 0.29 | 1.9 |
| Sand, frozen, saturated (10%) | 125 | 0.24 | 2.4 |
| Rock typical | 156 | 0.20 | 1.3 |
| Wood, plywood, dry | 37 | 0.65 | 0.10 |
| Wood, fir or pine, dry | 31 | 0.6 | 0.07 |
| Wood, maple or oak, dry | 44 | 0.5 | 0.10 |
| Insulating concrete (varies) | 12 to 94 | | 0.04 to 0.35 |
| Concrete | 156 | 0.16 | 1.0 |
| Asphalt | 156 | | 0.42 |
| Polyethelene, high density | 59 | 0.54 | 0.21 |
| PVC | 87 | 0.25 | 0.11 |
| Asbestos cement | 119 | | 0.38 |
| Wood stave (varies) | — | | 0.15 |
| Steel | 486 | 0.12 | 25 |
| Ductile iron | 468 | | 30 |
| Aluminum | 169 | 0.21 | 115 |
| Copper | 550 | 0.1 | 220 |

(1) Values are representative of materials but most materials have variable properties.

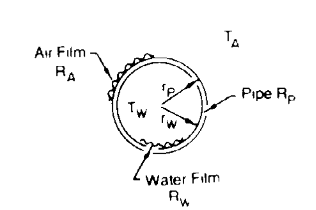
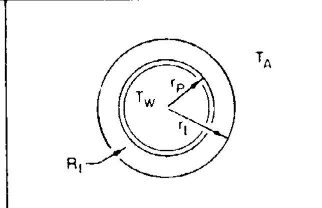
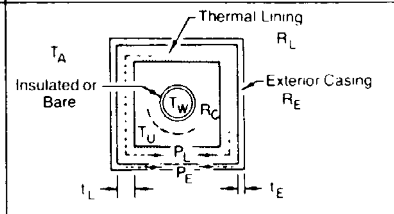
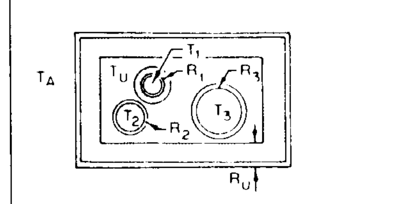
| | (a) Bare Pipe | (b) Insulated Pipe | (c) Single Pipe in a Box | (d) Multiple Pipe Utilidor |
|---|---|---|---|---|
| Sketch |   |   |   |   |
| Assumptions | Thin walled pipe (i.e. $r_p < 2r_w$)<br>$R_W$ is negligible, $R_p < R_A$. | All thermal resistances but that of the insulation are neglected. | Convection ensures the temperature inside the utilidor. $T_U$ is uniform. Utilidor air films neglected. | Same as (c). |
| Thermal Resistance | $R_p = (r_p - r_w)(r_p - r_w) \pi k_p$<br>$h_A = N\left(\frac{T_W - T_A}{r_p}\right)^{0.25} W$<br>$R_A = 1/2 \pi r_p h_A$<br>$R_C = R_p + R_A$<br>$N = 0.23$ Btu h · ft$^{7/4}$ · F$^{5/4}$<br>$W = \sqrt{12.5 V + 1}$ for $V$ = miles h<br>$N = 1.12$ J s · m$^{7/4}$ · C$^{5/4}$<br>$W = \sqrt{0.56 V - 1}$ for $V$ = m s | $R_C = R_I = \frac{\ln(r_I/r_p)}{2 \pi k_I}$<br>$= \left(\frac{r_I - r_p}{r_I + r_p}\right) / \pi k_I$ if $r_I < 2 r_p$<br>Or given $r_I/r_p$ and $k_j$; read off $R_I$ from graph, Fig. 12-8. | Calculate $R_C$, the thermal resistance of the interior conduit by:<br>using (b) if insulated or using (a) if bare and replacing $T_A$ in the formula for $h_A$ by an estimate for $T_U (< T_W)$<br>$R_L = t_L/P_L k_L \quad R_E = t_E/P_E k_E$<br>$R_U = R_L + R_E$<br>$R = R_C + R_U$<br>$T_U = \frac{(T_W/R_C) + (T_A/R_U)}{(1/R_C) + (1/R_U)}$<br>if bare pipe, iterate $T_U$ | Calculate R for each pipe as in (c) to get $R_j$, ($j = 1,2,3,....$)<br>Calculate $R_U$ as in (c)<br>$T_U = \frac{\sum_j (T_j/R_j) + (T_A/R_U)}{\sum_j (1/R_j) + (1/R_U)}$<br>If bare pipes present, iterate $T_U$ |
| Rate of Heat Loss | $Q = (T_W - T_A) R_C$ | $Q = (T_W - T_A)/R_I$ | $Q = (T_W - T_A)/R$ | $Q_j = (T_j - T_U)/R_j$ (per pipe)<br>$Q = \sum_j Q_j - (T_U - T_A)/R_U$ |
| Insulation Thickness (given Q) | N/A | $r_I - r_p = r_p \left\{\exp\left[2 \pi k_I (T_W - T_A)/Q\right] - 1\right\}$<br>$= \pi k_j (T_W - T_A) Q$ if $r_I < 2 r_p$<br>Or given $R_I$ and $k_I$; read off $r_I/r_p$ from Figure 12-8. | Obtain $R_E$ and $R_C$ as above<br>$t_U = P_L k_L \left[\frac{(T_W - T_A)}{Q} R_E - R_C\right]$<br>If bare interior pipe, iterate $T_U$, $R_C$ and hence $t_L$ | Given acceptable $Q_j$, calculate $R_j$ as above and evaluate $T_U = T_j - R_j Q_j$ for each pipe for which Q is known. Using the maximum $T_U$ found, calculate new Q as above.<br>Using these $Q_j$ and the same $T_U$, evaluate<br>$t_L = P_L k_L \left[\frac{T_U - T_A}{\Sigma Q_j} R_E\right]$<br>If bare pipes present, iterate $T_U$, $R_j$ and hence $t_L$ |
| Comments | Often, for metal pipes, $R_p$ may be neglected. If $R_p$ is significant, the expression above for $h_A$ will generate an overestimate of Q. If $T_A > T_W$ switch $T_A$ and $T_W$ in the expression for $h_A$. | The neglected thermal resistances given in (a) may be included if desired. Estimate a value for the insulation surface temperature and calculate $h_A$ and $R_A$. Interate. | The value of $h_A$, and hence $R_A$, is fairly insensitive to the choice of $T_U$, and so one iteration on $T_U$ is usually sufficient. Often $R_E$ may be neglected. Similar calculational procedure may be performed for pipes and utilidors of different cross-section. | as (c).<br>If it is clear that one pipe dominates the heat loss process, (c) may be used to estimate $T_U$. It is wise to consider the heat loss from the various pipes if certain other pipes cease to function. |

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*Figure 12-5. Steady-state thermal equations for above-surface pipes.*

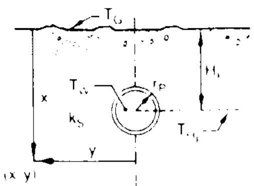
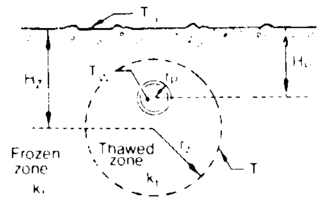
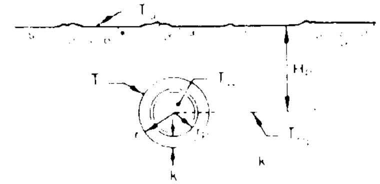
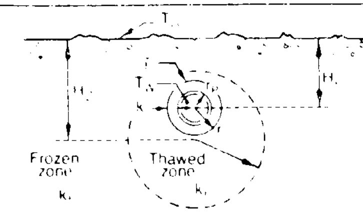
| | (a) Bare, No Thaw | (b) Bare, With Thaw Zone | (c) Insulated, No Thaw | (d) Insulated, With Thaw Zone |
|---|---|---|---|---|
| Sketch |  |  |  |  |
| Assumptions | Neglect all thermal resistances except that of the soil | Same as (a), but accounting for the different conductivities of thawed and frozen soil | Neglecting all thermal resistances except those of the soil and insulation. Outer surface of insulation assumed to be isothermal. $r_I = r_p - H_p$. | Same as (c) but accounting for the different thermal conductivities of thawed and frozen soil. |
| Thermal Resistance and Thaw Zone Parameters | $R_S = \frac{\text{arccosh}(H_p r_p)}{2\pi k_S}$<br>$= \ln \frac{(H_p r_p) + \sqrt{(H_p r_p)^2 - 1}}{2\pi k_S}$<br>$\simeq \frac{\ln(2 H_p r_p)}{2\pi k_S}$ if $H_p > 2r_p$ | $T'_W = \frac{k_t}{k_f}(T_W - T_O) \cdot T_O$<br>$T' = \frac{T_p - T_G}{T'_W - T_G}$<br>$c = \sqrt{H_p^2 - r_p^2} \simeq H_p$ if $H_p r_p > 7$<br>$A = T' \text{arccosh}(H_p r_p)$<br>$= T' \ln(2 H_p r_p)$ if $H_p > 2 r_p$<br>Use Fig. 12-9 to read off arccosh $(H/r_p)$, then solve for A, then use Fig. 12-10 for $H_z$ and $r_z$.<br>$H_z = c \coth A$ $r_z = c \,\text{csch} A$<br>$R_t$, $R_f$ and $R_S$ (= $R_t - R_f$) as given in (d) but with $r_I$ replaced by $r_p$. | $R_I$ as given in Figure 12-5 (b).<br>$R_S$ as given in (a), but with $r_p$ replaced by $r_I$.<br>$T_I = T_W - \frac{R_I(T_W - T_G)}{R_S + R_I}$<br>For known $T_W$, $T_G$, and $R_S$, the minimum insulation thickness to prevent thaw (i.e. $T_I = T_O$) is given by:<br>$R'_I = \frac{T_W - T_O}{T_O - T_G} R_S$ | $R_I$ as given in Figure 12-5 (b).<br>$T'_W$, $T'$, c, $H_Z$, $r_Z$ and $R'_S$ as in (b) but with $r_p$ replaced by $r_I$ and using:<br>$A = T' [\text{arccosh}(H_p r_p) + 2\pi k_t R_I$<br>$T_I = T_W - \frac{R_I(T'_W - T_G)}{R'_S + (k_t k_f) R_I}$<br>Also:<br>$R_t = [\text{arccosh}(H_p r_I] [\text{arccosh}(H_Z r_Z)]/2\pi k_t$<br>$= [\ln(H_p \cdot r_Z r_I \cdot H_Z]/2\pi k_t$ if $H_Z \geqslant 2r_Z$<br>$R_f = [\text{arccosh}(H_Z r_Z)]/2\pi k_f$<br>$= [\ln(2 H_Z r_Z)]/2\pi k_f$ if $H_Z \geqslant 2r_Z$<br>$R_S = R_f + R_t$ |
| Rate of Heat Loss | $Q = \frac{T_W - T_G}{R_S}$ | $Q = \frac{T'_W - T_G}{R'_S}$ where $R'_S = \frac{\text{arccosh}(H_p r_p)}{2\pi k_f}$<br>Or: To evaluate $R'_S$: use Figure 12-9. | $Q = \frac{T_W - T_G}{R_I + R_S}$ | $Q = \frac{T'_W - T_G}{R'_S + (k_t k_f) R_I}$ |
| Insulation Thickness | N/A | N/A | For no thawing outside the insulation the minimum insulation thickness is given by:<br>$r_I - r_p = (r_p [\exp(2\pi k_I R'_I) - 1])$<br>Or: Given $R_I$ and $k_I$:<br>Read off $r_I/r_p$ from Figure 12-8. | $R_I = [(A/T') + \text{arccosh}(H_p/r_I)]/2\pi k_t$<br>$r_I = r_p$ as in (c) but with $R'_I$ replaced by $R_I$ from above. |
| Comments | For calculations of heat loss when there is a temperature gradient in the soil and $H_p > 2r_p$, $T_G$ may be replaced by $T_{Hp}$, the undisturbed ground temperature at the pipe axis depth. For an upper limit on heat loss use $k_S = k_f$, otherwise use $k_S = (k_f + k_t)/2$. | The thawed zone is a circle in cross-section. | May be used to approximate (d) if $k_t = k_f$ and/or $r_Z \cong r_I$, and thaw zone parameters are not required. Use $k_S = k_f$ or $k_S = (k_f + k_t)/2$ as in (a). | Often the above expressions for $R_t$, $R_f$ and $R_S$ are not required. |

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*Figure 12-6. Steady-state thermal equations for below-surface pipes.*

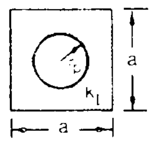
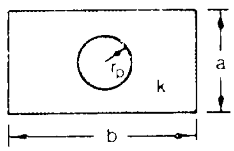
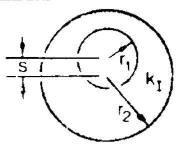
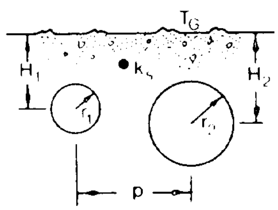
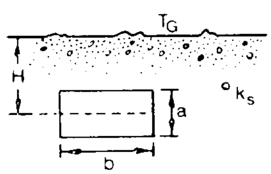
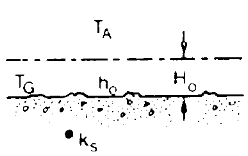
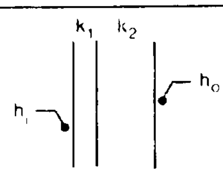
| Condition | Sketch | Thermal resistance |
|---|---|---|
| Square insulation |  | $R = \frac{1}{2\pi k_i} \ln 1.08 \frac{a}{2r_i}$ |
| Rectangular insulation |  | $R = \frac{1}{2\pi k} \ln\left(\frac{4a}{\pi r} - 2S\right)$ <br> b/a, S: 1.00, 0.08290; 1.25, 0.03963; 1.50, 0.01781; 1.75, 0.00816; 2.00, 0.00373; 2.25, 0.00170; 2.50, 0.00078; 3.00, 0.00016; 4.00, $6.97 \times 10^{-6}$; 5.00, $3.01 \times 10^{-7}$; ⋮ ; ∞, 0 |
| Eccentric cylindrical insulation |  | $R = \frac{1}{2\pi k_i} \ln \frac{\sqrt{(r_2+r_1)^2 - s^2} + \sqrt{(r_2-r_1)^2 - s^2}}{\sqrt{(r_2+r_1)^2 - s^2} - \sqrt{(r_2-r_1)^2 - s^2}}$ <br> $= \frac{1}{2\pi k_i} \operatorname{arccosh} \frac{r_1^2 + r_2^2 - s^2}{2r_1 \cdot r_2}$ |
| Two buried pipes |  | Where $H_1 \geq 3r_1$, $H_2 \geq 3r_2$ and $p \geq 3(r_1+r_2)$ <br> $R_{1-2} = \frac{1}{2\pi k_s} \cdot \frac{\ln\frac{2H_1}{r_1} \cdot \ln\frac{2H_2}{r_2} - \left[\ln\sqrt{\frac{(h_1+h_2)^2+p^2}{(h_1-h_2)^2+p^2}}\right]^2}{\ln\sqrt{\frac{(h_1+h_2)^2+p^2}{(h_1-h_2)^2+p^2}}}$ <br> $R_{1-T_G} = \frac{1}{2\pi k_s} \cdot \frac{\ln\frac{2H_1}{r_1} \cdot \ln\frac{2H_2}{r_2} - \left[\ln\sqrt{\frac{(h_1+h_2)^2+p^2}{(h_1-h_2)^2+p^2}}\right]^2}{\ln\frac{2H_2}{r_1} = \ln\sqrt{\frac{(h_1+h_2)^2+p^2}{(h_1-h_2)^2+p^2}}}$ |
| Buried rectangular duct |  | $R = \frac{1}{k_s\left(5.7 + \frac{b}{2a}\right)} \ln \frac{3.5H}{b^{0.25} \cdot a^{0.75}}$ |
| Surface thermal resistance |  | Surface thermal resistance between ground and air can be approximated as the equivalent thickness of the underlying soil equal to <br> $H_o = \frac{k_s}{h_o}$ |
| Composite wall |  | $R = \frac{1}{h_i} + \frac{1}{h_o} + \frac{x_1}{k_1} + \frac{x_2}{k_2}$ |

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*Figure 12-7. Steady-state thermal resistance of various shapes and bodies.*

tions, in addition to the implied time-independent steady-state assumptions, are indicated. Some comments intended to facilitate application of the equations are also included. Where applicable, procedures are given for relevant thermal resistance, rates of heat flow and insulation thicknesses.

(2) Figure 12-6 gives similar information for uninsulated and insulated buried pipes. In each of these two cases, the presence of thawed ground around the pipe is considered, and equations are included to determine the dimensions of the resulting thaw cylinder.

(3) Figure 12-7 presents procedures for calculating the thermal resistance of typical shapes and bodies. Equations are given in figure 12-4 for estimating the temperature drop (or gain) along a pipeline system, and simple procedures to determine freeze-up times under no-flow conditions are included.

(4) To facilitate computation, numerical values for certain variables in some of the calculations may be read directly from figures 12-8, 12-9 and 12-10. These curves present a partial solution for some of the heat loss equations. Their use is demonstrated in paragraphs 12-9*d* and *f*.

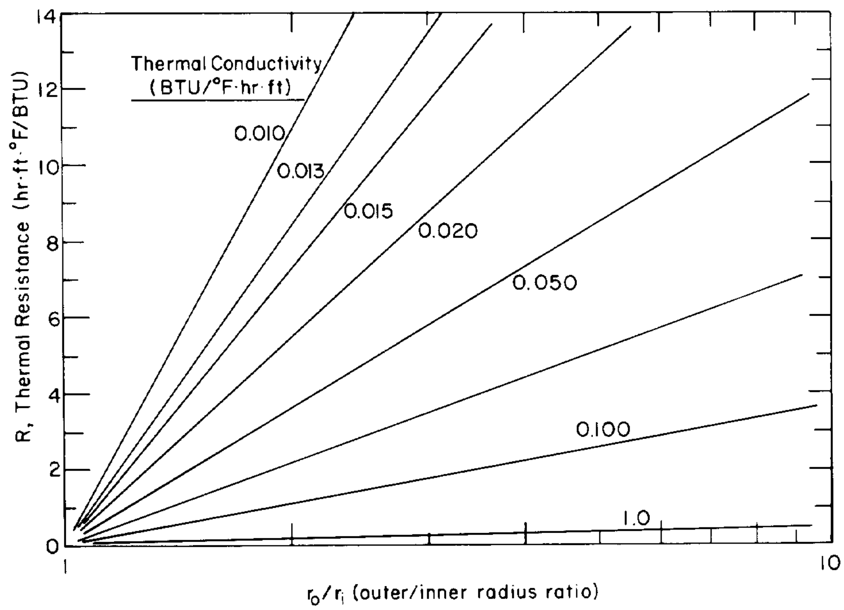


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*Figure 12-8. Thermal resistance of a hollow cylinder, solution to the equation $R = \ln(r_o/r_i)/2\pi k$.*

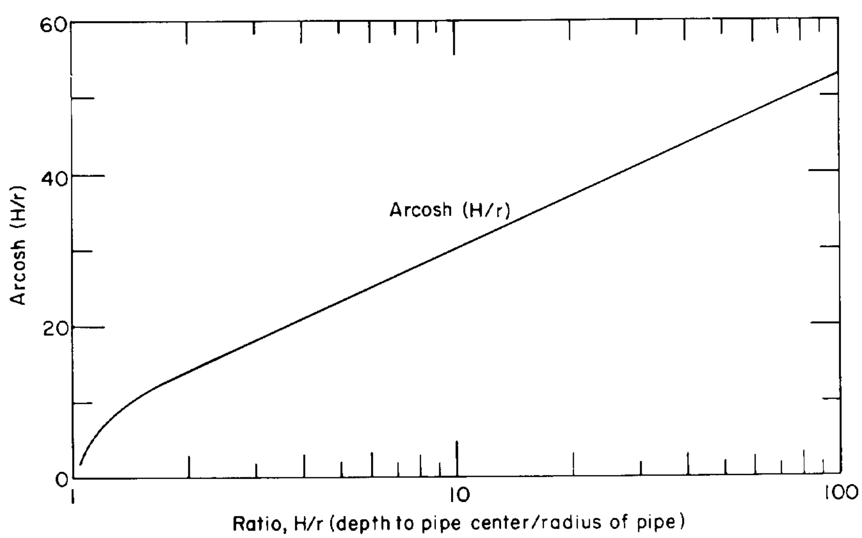


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*Figure 12-9. Arccosh (H/r) versus $H/r_\perp$ for solution of thaw zone equations.*

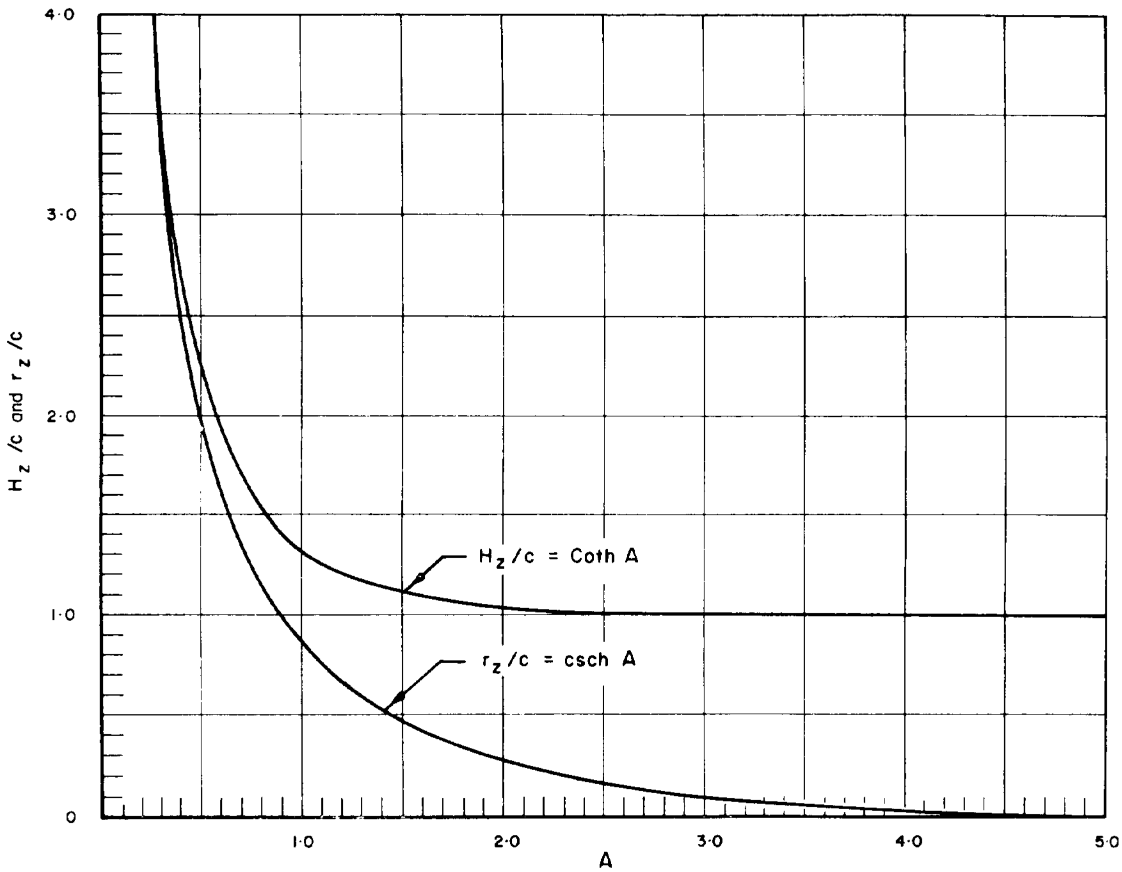


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*Figure 12-10. ($H_{\perp}/c$) and ($r_z/c$) versus A factor for solution of thaw zone equations.*

(5) Steady-state thermal influences in isotropic, homogenous soils can be summed and geometric modifications and approximations can be made to the basic steady-state equations. For example, a layered soil can be represented by an "effective" soil thickness with the same total thermal resistance as the layered soil. When pipes are buried below the area influenced by short-term air temperature fluctuations, the ground temperatures around the pipeline resemble a slowly changing series of steady-state conditions. The heat loss from deeply buried pipes can be calculated from steady-state equations for a cylinder of material around a pipe if the fluid temperature and the soil temperature at a known distance from the pipe are measured, and the soil and insulation thermal conductivities are known. Heat loss from deep pipes can also be conveniently estimated by replacing the ground surface temperature in the steady-state equations with the undisturbed ground temperature at the pipe depth. Heat loss from a buried pipe over a time period can be calculated from the heating index during that period (see paragraph 12.9*h*):

$$\text{Heat loss} = \frac{I}{R} \qquad \text{(eq 12-1)}$$

$$\text{or} = \frac{\Sigma\,(\text{Pipe temperature - Ambient temperature})}{\text{Thermal resistance}}\,. \qquad \text{(eq 12-2)}$$

where

I = heating index °F (time period).
R = thermal resistance hr•ft•°F/BTU.

*b. Depth of freezing or thawing.* The depth of freezing or thawing of soil and the ice thickness on water bodies are best obtained by field measurements, but they can be estimated using one of the many analytical solutions available. Because of the assumptions necessary in these analytical solutions, such as assuming a step change in surface temperature or neglecting the soil temperature changes, they generally overestimate the maximum freezing isotherm depths for the given conditions and are, therefore, conservative for engineering applications. They are generally Neumann or Stefan-based solutions which have the basic form:

$$X = m(I_g)^{1/2} \qquad \text{(eq 12-3)}$$

where

- $X$ = depth of freezing or thawing, feet
- $m$ = coefficient of proportionality
- $I_g$ = ground surface freezing ($I_f$) or thawing ($I_t$) index, °F•hr

The following equations incorporate various assumptions, and are useful for specific conditions:

$$X = \left( \frac{2k \cdot I_g}{L} \right)^{1/2} \qquad \text{(eq 12-4)}$$

$$X = \left( \frac{2k \cdot I_g}{L + C\left( T_m - T_o + \frac{I_g}{2_t} \right)} \right)^{1/2} \qquad \text{(eq 12-5)}$$

$$X = \left( \left( \frac{k_2}{k_1} d_1 \right)^2 + \frac{2k_2 \cdot I_g - \frac{d_1^2 \cdot L_1}{2k_1}}{L_2} \right)^{1/2} - \left( \frac{K_2 - 1}{K_1} \right) ( d_1 ) \qquad \text{(eq 12-6)}$$

$$X = \left( \frac{2k \cdot I_g}{L} \right)^{1/2} \left( 1 - \frac{C \cdot I_g}{8L \cdot t} \right) \qquad \text{(eq 12-7)}$$

$$X = \lambda \left( \frac{2k \cdot I_g}{L} \right)^{1/2} \qquad \text{(eq 12-8)}$$

where

- $k$ = thermal conductivity of the material above the freezing isotherm, $k_f$ for frost penetration and $k_t$ for thawing calculations, BTU/hr•ft•°F.
- $L$ = volumetric latent heat of the material undergoing phase change, BTU/cf (for water L = [(144 BTU/lb) (62.4 pcf) ] = 8986 BTU/cf)
- $C$ = volumetric heat capacity of the material above the freezing isotherm, $C_f$ or $C_1$. For thawed soil: $C_t = \gamma\,[\,C_s + C_w\,(w/100)\,]$ and for frozen soil: $C_f = \gamma[\,C_s + C_i(w/100)]$
- $\gamma$ = dry unit weight of soil, pcf
- $C_s$ = mass heat capacity of mineral matter in soil; assume a value of 0.2 BTU/lb
- $C_w$ = mass heat capacity of water = 1.0 BTU/lb
- $C_i$ = mass heat capacity of ice, assumed value of 0.5 BTU/lb
- $w$ = moisture content of soil, %
- $T_m$ = mean annual site temperature, °F
- $t$ = freezing or thawing period, consistent units
- $T_o$ = freezing point, 32°F for water
- $d$ = thickness of layer of material, feet
- $\lambda$ = a correction coefficient which takes into consideration the effect of temperature change in the soil, and primarily accounts for the volumetric specific heat effects. It is a function of two parameters. the thermal ratio (a) and the fusion parameter (μ), and is determined from figure 12-11:

$$a = \frac{(T_m - T_o)}{T_m} = \frac{(T_m - T_o) \cdot t}{I_g}$$

$$\mu = \frac{C \cdot I_g}{L \cdot t}$$

- $T_s$ = $I_g/t$, surface freezing or thawing index divided by the time period t, °F.

Subscripts f and t refer to freezing and thawing, and subscripts 1 and 2 refer to the surface layer and the underlying material (all symbols are also defined in table 12-1).

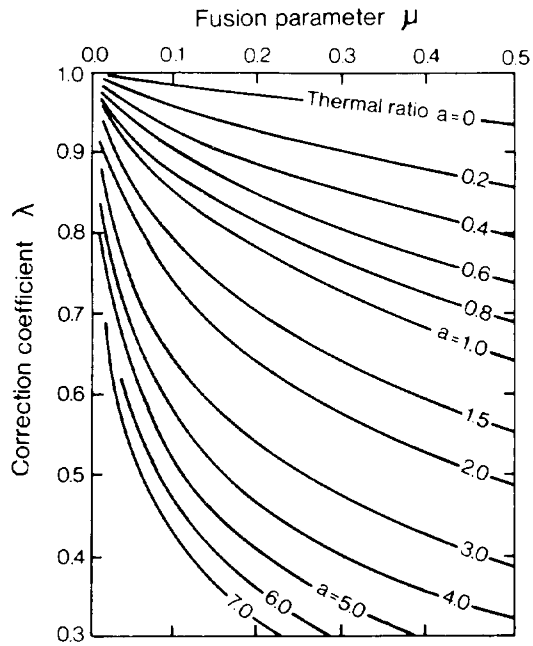


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*Figure 12-11. Correction coefficient λ, for use in Berggren equation.*

(1) Equation 12-4 is the Stefan solution for a homogeneous material with a step change in surface temperature (see example 12-9a). This is modified in equation 12-5 to account for the temperature change in the freezing or thawing soil. Equation 12-6 is a two-layer solution of the Stefan equation that is useful for calculations involving snow cover, a gravel pad or a board of thermal insulation, in which the surface layer has no latent heat and the equation is simplified (see para 12-9*b* and *c*). Equation 12-7 is a close approximation of the Neumann solution when the ground temperatures are near freezing. Equation 12-8, the modified Berggren equation, is perhaps the most commonly used approach for determining thermal responses of soils. When the soil has a high moisture content the *X* coefficient approaches unity, and the equation is identical to the Stefan approach (equation 12-4). In climates where the mean annual temperature is near or below freezing, the thermal ratio approaches zero and the *[A]* coefficient is greater than 0.9. In very dry soils, the soil warming or cooling can be significant and should be included. Multilayered soil systems can be solved by determining that portion of the surface freezing or thawing index required to penetrate each layer. The sum of the thicknesses of the frozen or thawed layers whose indices equal the total index is equal to the depth of freeze or thaw. The partial freezing or thawing index to penetrate the $n^{th}$ layer is

$$I_n = \frac{L_n \cdot d_n}{\lambda^2} \sum_{1}^{n-1} R + \frac{R_n}{2} \qquad \text{(eq 12-9)}$$

where

$1_n$ = the partial freezing or thawing index required to penetrate the $n^{th}$ layer, °F•hr

$L_n$ = volumetric latent heat in the $n^{th}$ layer, $BTU/ft^3$

$d^n$ = thickness of the $n^{th}$ layer, ft

$\lambda$ = the coefficient based on the weighted average values for μ down to and including the $n^{th}$ layer (see figure 12-13)

$\sum_{1}^{n-1} R + \frac{R_n}{2}$ = the sum of the thermal resistances of the layers above the $n^{th}$ layer

$R_n$ = $d_n/k_n$, the thermal resistance to the $n^{th}$ layer, hr • ft • °F/BTU.

(2) The solution for multi-layered systems is facilitated by tabular arrangement of the intermediate values. The penetration into the last layer must be solved by trial and error to match the total freezing or thawing index at the site. It is necessary to determine the temperature condition at the ground surface to determine subsurface thermal effects, including the depth of freezing and thawing. Since air temperatures are readily available, but surface temperatures are not, a correlation factor which combines the effects of radiation, and convective and conductive heat exchange at the air-ground surface is used:

$$I_g = n \cdot I_a \qquad \text{(eq 12-10)}$$

where

$I_g$ = ground surface freezing or thawing index, °F • hr
$I_a$ = air freezing or thawing index, °F • hr
n = n-factor, ratio of the surface and air temperature indices.

The n-factor is very significant in analytical ground thermal considerations. It is highly variable and is usually estimated from published observations such as the values listed in Table 12-3.

*Table 12-3. Typical values of the n-factor for correlation of air temperature with surface temperature of various materials (see eq 12-10).*

| Surface | n-factors Thawing | Freezing | |
|---|---|---|---|
| Snow | ............... | 1.0 | General application |
| Pavement free of snow and ice | ............... | 0.9 | General application |
| Sand and gravel | 2.0 | 0.9 | General application |
| Turf | 1.0 | 0.5 | General application |
| Spruce | 0.35 to 0.53 | 0.55 to 0.9 | Thompson, Manitoba |
| Spruce trees, brush | 0.37 to 0.41 | 0.28 | Fairbanks, Alaska |
| Above site, cleared, moss surface | 0.73 to 0.78 | 0.25 | Fairbanks, Alaska |
| Stripped, mineral soil surface | 1.72 to 1.26 | 0.33 | Fairbanks, Alaska |
| Spruce | 0.76 | ............... | Inuvik, NWT |
| Willows | 0.82 | ............... | Inuvik, NWT |
| Weeds | 0.86 | ............... | Inuvik, NWT |
| Gravel fill slope | 1.38 | 0.7 | Fairbanks, Alaska |
| Gravel road | 1.99 | ............... | Fairbanks, Alaska |
| Concrete road | 2.03 | ............... | Fairbanks, Alaska |
| Asphalt road | 1.74 to 2.70 | ............... | Fairbanks, Alaska |
| White painted surface | 0.76 to 1.25 | ............... | Fairbanks, Alaska |
| Peat bales on road | 1.44 to 2.28 | ............... | Fairbanks, Alaska |
| Dark gravel | 1.15 to 1.73 | ............... | Fairbanks, Alaska |

(3) Ice thickness on water bodies can be estimated from the previous depth of freezing equations or from equations 12-3 with the m values in table 12-4 (see example 12-9a). Snow cover has a significant insulting effect and can significantly reduce the maximum ice thickness (see example 12-9b). The ice formation can be greater than calculted if the weight of snow or the lowering of the water level causes cracks in the ice and water overflows onto the surface. This water is drawn into the snow and the mixture refreezes and bonds to the original ice.

*Table 12-4. m-factors for ice thickness estimation*

| m-factor inch [°F·d] $^{1/2}$ | Conditions |
|---|---|
| 0.9 - 0.95 | Practical maximum for ice not covered with snow |
| 0.8 | Windy lakes with no snow |
| 0.7 - 0.8 | Medium-sized lakes with moderate snow cover |
| 0.6 - 0.65 | Rivers with moderate flow |
| 0.4 - 0.5 | River with snow |
| 0.2 - 0.4 | Small river with rapid flow |

## 12-9. Design examples.

Eight typical examples are given below to illustrate the utilization of the calculation procedures described above.

*a.* Estimate the practical maximum ice thickness on a water reservoir with no snow cover when the annual air freezing index ($I_g$) is 3000 °F • d. Use equation 12-3:

$$x = m(I_g)^{1/2}$$

—From table 12-4, m = 0.95 inch/(°F)½(d)½

$$x = (0.95)(3000)^{1/2}$$
$$= 52 \text{ inches}$$
$$= 4.33 \text{ feet.}$$

—The Stefan equation (equation 12-4) can also be used:

$$x = \left(\frac{(2k)(I_g)}{L}\right)^{1/2}$$

where

k = thermal conductivity of material above the freezing isotherm, ice in this case, so from table 12-2:

$$k_{ice} = 1.28 \text{ BTU/ft} \cdot \text{hr} \cdot {}^\circ\text{F}$$

L = volumetric latent heat of material undergoing phase change, in this case water, so:
Latent heat of water at 32°F = 144 BTU/lb
Density of water at 32°F = 62.4 lb/ft$^3$

L = (144 BTU/lb)(62.4 lb/ft$^3$) = 8985.6 BTU/ft$^3$

$I_g$ = (3000°F • d)(24 hr/day) = 72,000 °F • hr

$$X = \left[\frac{(2)(1.28 \text{ BTU/ft} \cdot \text{hr} \cdot {}^\circ\text{F} \cdot \text{hr})(72{,}000\ {}^\circ\text{F} \cdot \text{hr})}{8986 \text{ BTU/ft}^3}\right]^{1/2}$$

= 4.5 ft.

*b.* Estimate the ice thickness on the reservoir when there is an 8-inch snow cover on top of the ice and $I_g$ = 3000 °F • d.

—From equation 12-3, and Table 12-4:

$$X = m(Ig)^{1/2} = 0.7(3000)^{1/2} = 38 \text{ inches} = 3.2 \text{ ft.}$$

—Or, use the Stefan equation (equation 12-6) for a two-layer system:

$$X = \left[ \left(\frac{k_2}{k_1} d_1\right)^2 + \frac{(2k_2)(I_g) - \dfrac{(d_1)^2(L_1)}{2k_1}}{L_2} \right]^{1/2} - \left(\frac{k_2}{k_1} - 1\right)(d_1)$$

—The first layer is snow, $d_1$ = 8 inches = 0.667 feet, assumed to be drifted and compact. From table 12-2, $k_1$ = 0.4 BTU/ft • hr • °F. Since no phase change occurs in the snow, $L_1$ = 0.

—Ice: $k_2$ = 1.28 BTU/ft • hr • °F

$L_2$ = (144)(62.4) = 8986 BTU/ft$^3$

$$X = \left[ \left[\left(\frac{1.28}{0.4}\right)(0.667)\right]^2 + \frac{2(1.28)\cdot(3000)(24)}{8986} \right]^{1/2} - \left(\frac{1.28}{0.4} - 1\right)(0.667)$$

$$= \sqrt{4.55+20.5} - 1.47$$

= 3.5 ft (includes the 8 in. of snow).

*c.* The Stefan equation (equation 12-6) can also be used to estimate the depth of frost penetration beneath a gravel pad or an insulation board. The $L_1$ in either case would be zero. The $L_2$ in this example would be the latent heat of fusion for the soil and would be dependent on the moisture content in the soil.

—Assume: sandy soil, dry density 125 pcf, moisture content 6% and a freezing index ($I_g$) = 3000 °F • d

—Find depth of frost penetration under 3-inch-thick polystyrene board. From table 12-2:
$k_1$ = 0.020 BTU/ft • °F • hr (for polystyrene),
$k_2$ = 1.0 BTU/ft • °F • hr (for sand) and thus
$d_1$ = 3/12 = 0.25 feet. The moisture content in the soil = (0.06)(125 pcf) = 7.5 lb water/ft$^3$ soil.

Latent heat of water = 144 BTU/lb

$L_2$ = (144 BTU/lb)(7.5 lb/ft$^3$) = 1080 BTU/ft$^3$ of soil

$L_1$ = 0

$$X = \left[ \left(\frac{k_2}{k_1} d_1\right)^2 + \frac{2k_2 I_g}{L_2} \right]^{1/2} - \left(\frac{k_2}{k_1} - 1\right)(d_1)$$

$$X = \left[ \left[\left(\frac{1.00}{0.020}\right)(0.25)\right]^2 + \frac{(2)(1.00)(3000)(24)}{1080} \right]^{1/2} - \left(\frac{1.00}{0.020} - 1\right)(0.25).$$

$$= \sqrt{156+133} - 12.25$$

$$= 4.75 \text{ ft.}$$

—The depth of frost penetration would be 11.5 feet in the same soil, under the same conditions, if the insulation board were not in place.

*d.* Determine the rate of heat loss per linear foot of above-ground pipe from a 5-inch-ID (wall thickness ½ inch) plastic pipe encased in a 2-inch thickness of polyurethane insulation. Water inside the pipe is maintained at 40°F, ambient air temperature is -40°F, and wind speed is 15 mph. Thermal conductivity of pipe material is 0.208BTU/°F • ft • hr and thermal conductivity q of the insulation material is 0.0133 BTU/°F • ft • hr.

— Use equations a and b from figure 12-4:

$$\text{Thermal resistance of pipe } R_p = \frac{\ln(r_{out}/r_{in})}{(2)(\pi)(k_p)}$$

Inside radius = $r_{in}$ = 2.5 inch

Outside radius = $r_{out}$ = 2.5 + 0.5 = 3.0 inch

$$R_p = \frac{\ln(3.0/2.5)}{(2)(3.14)(0.208)} = \frac{0.182}{1.306}$$

$= 0.139$ hr ft F/BTU

— Thermal resistance of insulation
$R_I = \ln(r_{out}/r_{in})2\pi k_I$:
Inside radius, $r_{in} = 3$ inches
Outside radius, $r_{out} = 5$ inches

$$R_I = \frac{\ln(5/3)}{(2)(3.14)(0.0133)}$$

$= 6.115$ hr • ft • °F/BTU.

— To determine the thermal resistance of the air film ($R_A$) it is necessary to estimate the surface conductance ($h_a$). From figure 12-4a:

$$h_a = N \left[\frac{T_s - T_A}{r_p}\right] W.$$

— From figure 12-4a, N = 0.23 and w = $(12.5V+1)^{½}$ where V + windspeed, mph:

$W = ((12.5)(15)=1)^{½}$

$= 13.73$

— In this case $R_p$ is to the outer surface of the insulation = 5/12 = 0.417 ft. For the first iteration one must assume a surface temperature ($T_s$). This will be close to air temperature. Assuming $T_s$ = -39°F, then:

$$h_a = (0.23) \left[\frac{-39-(-40)}{0.417}\right](13.73)$$

$= 7.573$ BTU/hr • °F•ft

— Then, to calculate thermal resistance of air film:

$$R_A = \frac{1}{2\pi\, rp\, h_a}$$

$$= \frac{1}{(2)(3.14)(0.417)(7.573)}$$

$= 0.0504$ hr·ft·°F/BTU

— Then, check the assumed air film temperature:

$$T_S = T_A + [T_w - T_A]\left[\frac{R_A}{R_A + R_I + R_P}\right]$$

$$= -40 + [40-(-40)]\left[\frac{0.0504}{0.504 + 6.116 + 0.1396}\right]$$

$= -40 + (80)(0.00799)$

$= -39.4$ vs assumed $-39.0$, which is close enough.

— If the values did not check it would be necessary to repeat the calculation with another assumed $T_s$ until a reasonable check is attained. The combined thermal resistance ($R_c$) is

$R_c = R_A + R_I + R_p$

$= 0.0504 + 6.116 + 0.1296$

$= 6.306$ hr•ft•°F/BTU.

—The rate of heat loss (Q) is

$$Q = \frac{(T_w - T_A)}{R_c}$$

$$= \frac{(40-(-40))}{6.306}$$

$= 12.7$ BTU/hr·linear foot of pipe.

—Figure 12-8 can be used to obtain an estimate of heat loss if it is assumed that the thermal resistance of the air film and of the pipe material are negligible:

$$\frac{r_{out}}{r_{in}} = \frac{5}{3} = 1.667$$

$K_1 = 0.0133$ BTU/f•°F•hr.

—Then, enter figure 12-8 with these values and

R =6.4.

—So: $Q = \dfrac{[40-(-40)]}{6.4}$

=12.5 BTU/hr linear foot (LF) of pipe.

*e.* Compare the heat losses for the water pipe in the example above if installed at Barrow, Alaska, above ground or at a depth of 4 feet. Assume the minimum air temperature is -58°F, and the minimum mean daily soil temperature at a depth of 4 ft is 1.4°F. Thermal conductivity of soil is 1.2 BTU/ft•hr•°F.

(1) *Above-ground installation.* Assume a 5-inch ID plastic pipe, with 2 inches of polyurethane insulation:

$R_c$ = 6.306 hr•ft•°F/BTU (from previous example).

—The water inside pipe will be maintained at 40°F, so that the maximum rate of heat loss

$$Q = \frac{(T_w - T_A)}{R_c}$$

$$= \frac{[40 - (-58)]}{6.306}$$

= 15.5 BTU/hr•LF pipe.

(2) *Buried installation.* Assume that the top of the pipe is 4 feet below the surface, the radius to the outer surface = 5 in. = 0.416 feet, the depth to center of the pipe $H_p$ = 4.416 ft, and the radius of pipe = 0.416 feet (see equations, fig 12-5a) and $H_p$ is > 2rp. So the thermal resistance of the soil ($R_s$) is

$$R_s = \frac{1n\ (2Hp/rp)}{2\pi\ k_k}$$

$$R_s = \frac{1n\ [(2)(4.416/0.416)]}{(2)(3.14)(1.2)}$$

= 0.405 hr·ft·°F/BTU.

—The air film is not a factor for a buried pipe of this type so the combined resistance $R_c$ equals

$R_c = R_p + R_I + R_s$

= 0.1396 + 6.116 + 0.405 ($R_p$ and $R_I$ from previous example)

= 6.66 1 hr~ft~0F/BTU.

—So, the heat loss Q equals

$$Q = \frac{(T_w - T_A)}{R_c} \text{ (} T_A \text{ in this case is soil temperature 1.4°F)}$$

$$= \frac{(40 - 1.4)}{6.661}$$

= 5.79 BTU/hr•linear foot (LF) of pipe.

—This is about one-third the heat loss rate calculated for an above-ground installation in the same location. The responsible factor is the attenuation of the extreme surface temperature at the 4-foot depth.

*f.* Determine the mean size of the thaw zone and the average rate of heat loss from a 6-inch steel pipe buried 4 feet below the surface in a clay soil, where the soil thermal conductivities are $k_f$ (thawed) = 0.60 BTU/hr•ft•°F and $k_f$ (frozen) = 1.0 BTU/hr•ft•°F. Mean soil temperature at the ground surface is 27.5°F and the water in the pipe is maintained at 45°F. (See figure 12-sb, for schematic, symbols and equations.) A bare steel pipe has negligible thermal resistance so

$R_p$ = 0.

—Outer pipe radius $r_p$ = 6 inch/(2)(12) = 0.25 ft

—Depth to center of pipe $H_p$ = 4.0 ft.

$$T'_w = \frac{k_t}{k_f}(T_w - T_o) + T_o$$

where

$T_w$ = water temperature inside pipe

$T_o$ = soil temperature at interface of thawed zone

= 32°F

$$T'_w = \left(\frac{0.6}{1.0}\right)(45 - 32) + 32 = 39.8°F$$

$$T' = \frac{T_o - T_G}{T'_w - T_G}$$

$T_G$ = temperture and ground surface

$$= \frac{(32 - 27.5)}{(39.8 - 27.5)} = \frac{4.5}{12.3} = 0.366°F.$$

—Depth to center of thawed zone, $H_2$:

$H_2$ = (c)(coth A)

—Radius of thawed zone, $r_2$:

$r_2 = (c)(\text{csch } A)$

$c = (H^2_p = r^2_p)^{½} = [(f)^2 - (0.25_)2]^{1/2} = 399$ ft

$A = T'$ arccosh $(H_p/r_p)$

—If $H_p \geq 2r_p$

$$A \simeq T' \text{1n}(2H_p/r_p)$$
$$= (0.366)(\text{1n } 8/0.25)$$
$$= (0.366)(3.466)$$
$$= 1.268$$

$$H_2 = (3.99)(\coth 1.268)$$
$$= (3.99)(1.172)$$
$$= 4.68 \text{ ft}$$

$$r_2 = (3.99)(\text{csch } 1.268)$$
$$= (3.99)(0.5215)$$
$$= 2.08 \text{ ft.}$$

—The thaw zone, under steady state conditions will be a cylinder of soil enclosing, and parallel to, the pipe. The radius of this zone will be about 2 feet and the axis will be about 5 inches below the bottom of the pipe:

The axis = $H_2 - (H_p + r_p) = 4.68 - 4.25 = 0.43$ ft = 5.2 in. below pipe

The heat loss (Q) from this pipe would be

$$Q = \frac{T'_w - T_o}{R_s}$$

$$R'_s = \frac{\text{arccosh}(hp/r_p)}{(2)(k_f)(\pi)}.$$

If $H_p \geq 2r_p$:

$$R' = \frac{\text{1n}(2h\ p/rp)}{(2)(k_f)(\pi)}$$

$$= \frac{\text{1n }(8/0.25)}{(2)(1.0)(3.14)}$$

$$= \frac{3.466}{6.28} = 0.552 \text{ hr·ft·°F/BTU}$$

$$Q = \frac{(39.8 - 27.5)}{0.552}$$

$$= 22.3 \text{ BTU/hr LF of pipe.}$$

—Figures 12-9 and 12-10 can be used to assist in calculations of this type:

$\frac{H_p}{r_p} = \frac{4}{0.25} = 16$, From figure 12-9, arccosh $(\frac{H}{r})$ = 3.48

$$A = T' (\text{arccosh } (\frac{H_p}{r_p}))$$
$$= (0.366)(3.48)$$
$$= 1.27.$$

—Then from figure 12-10, with A = 1.27

$$\frac{H_2}{c} = 1.18$$

$$\frac{r_2}{c} = 0.6$$

$$c = [(H_p)^2 - (r_p)^2]^{1/2} = [16 - 0.0625]^{1/2}$$
$$= 3.99 \text{ ft.}$$

—So:

$$H_2 = (1.18)(3.992) = 4.7 \text{ ft}$$

$$r_2 = (0.6)(3.992) = 2.39 \text{ ft}$$

and

$$R'_s = \frac{\text{arccosh}(H_p/r_p)}{(2)(k_f)(\pi)}$$

$$= \frac{3.48}{(2)(1)(3.14)}.$$

—So:

$$Q = \frac{(39.8 - 27.5)}{0.554}$$

$$= 22.2 \text{ BTU/hr LF of pipe.}$$

This agrees closely with the 22.3 BTU/hr value determined previously.

*g*. Determine the design time, the safety factor time and the complete freezing time for the pipe designed in paragraph 12-9*d* if the water stopped flowing. From paragraph 12-9*d*, assume a 5-inch ID plastic pipe with 2-inch polyurethane insulation, constructed above ground. Water temperature $t_w$ = 40°F, air temperature $T_A$ = -40°F, and wind speed 15 mph. Use equations from figure 12-4.

—Design time = time for water in pipe to drop to freezing temperature (32°F); see figure 12-4 for definition of terms.

$$t_0 = \pi r^2_w \bullet R \bullet C \bullet 1n\ [(T_1 - T_A)(T_2 - T_A)]$$

Design time = $(3.14)(0.208)^2(6.306)(62.4)$

$$[1n\ (\frac{40 - (-40)}{32 - (-40)})]$$

= (53.5)(0.105)

= 5.6 hr

—Safety factor time = time for water in pipe to reach nucleation temperature for ice formation. Assume 27°F.

Substitute 27°F for $T_2$ in previous equation:

Safety factor time = $(3.14)(0.208)^2(6.306)(62.4)$

$$[1n\ (\frac{40 - (-40)}{27 - (-40)})]$$

= (53. 5)(0.177)

= 9.5 hr.

— Complete freezing time = time for water at 32°F in pipe to freeze completely solid.

$$= \frac{(\pi)(r_w)^2(R_c)(L)}{(T_2 - T_A)}$$

L = volumetric latent heat of water

= (144 BTU/lb)(62.4 lb/ft$^3$)

= 8986 BTU/ft$^3$.

Other factors are as defined above.

—Complete freezing time

$$= \frac{(3.14)(0.208)^2(6.306)(8986)}{[32 - (-40)\ ]}$$

$$= \frac{7698}{72}$$

= 107 hours.

*h.* Estimate the total annual heat loss from the above-surface pipe design described in paragraphs 12-9d and e if it were located at Barrow, Alaska. The maximum rate of heat loss (Q) was calculated in paragraph 12-9*e* for extreme winter conditions. Extrapolation of that rate to an annual value would seriously overestimate the total heat losses. An estimate is possible by determining the annual heating index (equation 12-1). Mean monthly temperatures (°F) for Barrow, Alaska, are:

| J | F | M | A | M | J | J | A | S | O | N | D |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -9 | -13 | -11 | 2 | 11 | 30 | 40 | 37 | 32 | 16 | 5 | -6 |

—The temperature of water in pipe $T_w$ = +40°F
—Heating index = $\Sigma(T_w - T_A)$ (equation 12-2)
—Tabulate heating index by months:

| | | |
|---|---|---|
| Jan | [40 - (-40)] | = 80 |
| Feb | [40 - (-13)] | = 53 |
| Mar | [40 - (-22)] | = 62 |
| Apr | (40 - 2) | = 38 |
| May | (40 - 11) | = 29 |
| June | (40 - 30) | = 10 |
| July | (40 - 40) | = 0 |
| Aug | (40 - 37) | = 3 |
| Sept | (40 - 32) | = 8 |
| Oct | (40 - 16) | = 24 |
| Nov | (40 - 5) | = 35 |
| Dec | [40 - (- 6)] | = 46 |

—Heating index = 388°F•month

$$= (388°\text{F}\cdot\text{month})\ (30.4\ \frac{\text{average days}}{\text{month}})$$

(24 hr/day)

= 283,0850Fhr

$R_c$ = 6.306 hr•ft•°F/BTU (see example 12-9d)

—Annual heat loss = heating index/$R_c$
= 44,891 BTU/yr•linear foot of pipe.

This is considerably less than extrapolation of the maximum heat loss rate calculated in paragraph 12-9e (134,904 BTU/yr LF pipe).

*i.* Determine the economical thickness of insulation for an above-ground 6-inch-OD water main to be located at Barrow, Alaska (temperature conditions as defined in paragraph 12-9*f*). Assume fuel oil cost (140,000 BTU/gallon) at $1.05/gallon, and an 85% efficient heating plant. Assumed installed costs of polyurethane foam are tabulated below for various available thicknesses. Thermal conductivity for polyurethane is 0.014 BTU/ft•hr•°F. Assume a 20-year design life at an interest rate of 8%.

—Neglecting air film and steel pipe material, the combined thermal resistance is

$$R_c = \frac{\ln (r_{out}/0.25)}{(2)(\pi)(k)}$$

$$= \frac{\ln (r/0.25)}{(2)(3.14)(0.014)}$$

$$= (11.37)\ [\ln(r/0.25)$$

—The heating index for Barrow, Alaska, would be 275,059°F•hr (see example 12-9g).

—The cost of heat $= \dfrac{\$1.05/\text{gal.}}{(140{,}000\ \text{BTU/gal})(0.85)}$

$$= \$0.0000088/\text{BTU}$$

—The economical thickness will have the lowest total cost for construction plus the present worth of the annual heating costs:

Present worth factor

$$(PWF) = \frac{[(1+i)^n - 1]}{(i)(1+i)^n}$$

where i = interest rate (as a decimal)
n = design period
for i =0.8 and n = 2O

$$PWF = \frac{[1.08)^{20} - 1]}{(0.08)(1.08)^{20}} = 9.818.$$

—Present worth of heating costs = heating index/$R_c$ x (PWF)(cost of heat)

$$= \frac{275{,}059}{(11.37)\ [\ln (r/0.25)]}\ (0.0000088)(9.818)$$

$$= \frac{2.09}{\ln (r/0.25)}\ \$/\text{ft}$$

at an insulation thickness of 1 in., r = (1/12 in.) + (0.25 ft) = 0.33 ft. Therefore, the present worth of heating costs

$$= \frac{2.09}{\ln (0.33/0.25)}$$

$$= \$7.53/\text{linear foot.}$$

—Repeat calculations, and tabulate, for typical insulation thickness:

| *Insulation thickness (in.)* | *Present worth of heat ($/ft)* |
|---|---|
| 1 | 7.53 |
| 2 | 4.09 |
| 3 | 3.02 |
| 4 | 2.47 |
| 5 | 2.13 |

—Installation costs for polyurethane insulation (assumed for this example only, obtain from suppliers in actual case):

| *Insulation thickness (in.)* | *Installation Costs ($/ft)* |
|---|---|
| 1 | 3.60 |
| 2 | 4.70 |
| 3 | 5.40 |
| 4 | 6.20 |
| 5 | 7.10 |

—Combining heating and construction costs give:

| *Insulation thickness (in.)* | *Total Present Worth of Costs ($/ft)* |
|---|---|
| 1 | 11.13 |
| 2 | 8.79 |
| 3 | 8.42 |
| 4 | 8.67 |
| 5 | 9.23 |

—Plot these results on arithmetic graph paper to determine the lowest cost. Then select the available nominal thickness of insulation that is closest to the graphical value. In this case a 3-inch thickness would be the most cost effective for the conditions assumed.