# CHAPTER 7

# REINFORCED MASONRY SHEAR WALLS

**7-1. Introduction.** This chapter contains design requirements for reinforced masonry shear walls, not including seismic requirements. Requirements for shear walls in buildings located in seismic zones 1 through 4 are given in TM 5-809-10/NAVFAC P-355/AFM 88-3, Chapter 13, Seismic Design for Buildings. Except as contained herein, design criteria, section properties, material properties, design equations, and allowable stresses are contained in chapter 5.

**7-2. General**. A masonry shear wall is any masonry wall, external or internal, which resists externally applied in-plane horizontal forces. A shear wall is a vertical element in the building lateral load resisting system. It transfers horizontal forces vertically downward from a diaphragm above to a diaphragm or a foundation below. Thus, horizontal wind or seismic forces are collected at floor or roof diaphragm levels and transferred to the building foundation by the strength and rigidity of the shear walls. A shear wall may be considered analogous to a plate girder cantilevered off the foundation in a vertical plane. The wall performs the function of a plate girder web and the integral vertical reinforcement at the ends of wall panels, between control joints, function as the beam flanges. Pilasters or floor diaphragms, if present, function as web stiffeners. Axial, flexural, and shear forces must be considered in the design of shear walls, including the tensile and compressive axial stresses resulting from loads tending to overturn the wall.

**7-3. Allowable shear stresses**. The allowable shear stress in a shear wall is dependent upon the magnitude of the ratio of M/(Vd), where M is the maximum moment applied to the wall due to the in-plane shear force, V, and d is the effective length of the wall. Therefore, if the shear wall is assumed fixed at the top and bottom (a multistory shear wall), M = ½hV, and M/(Vd) becomes h/2d, where h is the height of wall. However, if the shear wall is assumed fixed at the bottom only, (a single-story cantilevered shear wall), M = hV, and M/Vd becomes h/d. Figure 7-1 illustrates these conditions.
The allowable shear stress is also dependent upon whether or not shear reinforcement is provided. If the calculated shear stress, $f_{vm}$, exceeds the allowable shear stress, $F_{vm}$, then shear reinforcement will be provided. The shear reinforcement will be designed to carry the entire shear force. The following equations illustrate the limitations and requirements of determining the allowable shear stress in a shear wall.

*a. No shear reinforcement provided.* The calculated shear stress, fvm, shall not exceed the allowable shear stress, $F_{vm}$.

If, $\frac{M}{Vd} < 1.0$

Then,

$$F_{vm} = \frac{1}{3}\left[4 - \frac{M}{Vd}\right](f'_m)^{1/2}\ \text{(psi)} \qquad \text{(eq 7–1)}$$

But,

$$F_{vm} \le 80 - 45\left[\frac{M}{d}\right]\ \text{(psi)} \qquad \text{(eq 7–1a)}$$

If, $\frac{M}{Vd} \ge 1.0$

Then,

$$F_{vm} = 1.0(f'_m)^{1/2} \qquad \text{(eq 7–2)}$$

But,

$$F_{vm} \le 35\ \text{psi} \qquad \text{(eq 7–2a)}$$

*b. Shear reinforcement provided.* When firm exceeds Fvm, shear reinforcement will be provided and designed to carry the entire shear force. The calculated shear stress in the reinforcement, fvm, shall not exceed the allowable shear stress, $F_{vs}$.

If, $\frac{M}{Vd} < 1.0$

Then,

$$F_{vs} = \frac{1}{2}\left[4 - \frac{M}{Vd}\right](f'_m)^{1/2}\ \text{(psi)} \qquad \text{(eq 7–3)}$$

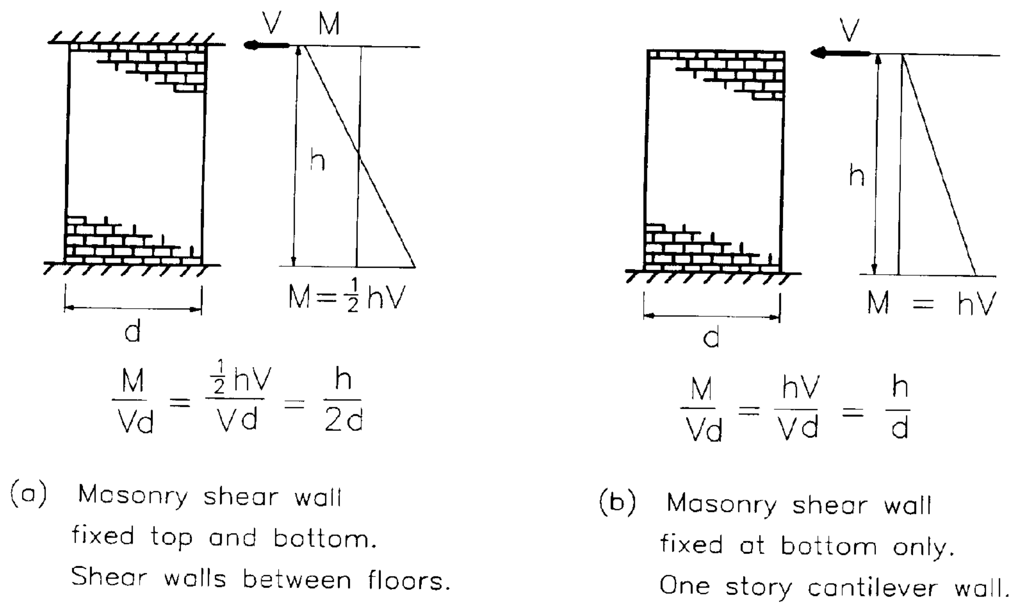


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*Figure 7-1. M/Vd ratios for shear walls.*

But,

$$F_{vs} \leq 120 - 45\left[\frac{M}{Vd}\right] \text{ (psi)} \qquad \text{(eq 7–3a)}$$

If, $\dfrac{M}{Vd} \geq 1.0$

Then,

$$F_{vs} = 1.5(f'_m)^{1/2} \text{ (psi)} \qquad \text{(eq 7–4)}$$

But,

$$F_{vs} \leq 75 \text{ psi} \qquad \text{(eq 7–4a)}$$

The ratio of M/(Vd) will always be taken as a positive number. The values of $F_{vm}$ and $F_{vs}$ may be increased by a factor of 1.33 when wind or seismic loads are considered in the loading combination.

## 7-4. Design Considerations.

*a. Shear Stresses.* The calculated shear stress, $f_{vm}$, will be determined as follows:

$$f_{vm} = \frac{V}{td} \qquad \text{(eq 7–5)}$$

Where:

V = The total shear load, pounds.

t = The actual thickness of shear wall section for solid grouted masonry or the equivalent thickness of a partially grouted hollow masonry wall, inches. (See Chapter 5 for the equivalent thicknesses).

d = The actual length of the shear wall element, inches.

(To be more exact, the actual wall panel length minus the tension reinforcement cover distance may be used). When the allowable shear stress, $F_{vm}$, is exceeded, horizontal and vertical shear reinforcement must be provided. The horizontal shear steel will be designed to carry the entire in-plane shear force. The area of shear reinforcement, $A_v$, will be determined as follows:

$$A_v = \frac{Vs}{F_s d} \text{ (in}^2\text{)} \qquad \text{(eq 7–6)}$$

Where:

s = The spacing of the shear reinforcement, inches.

$F_s$ = the allowable tensile stress in the reinforcement, psi.

Horizontal shear reinforcement will be uniformly distributed over the full height of the wall. Shear reinforcement will consist of deformed bars, thus joint reinforcement that is in the wall to control cracking will not be considered as shear reinforcement. The vertical spacing of shear reinforcement will not exceed the lessor of d/2 or 48 inches. Shear reinforcement will be terminated with a standard hook or will have an embedment length beyond the vertical reinforcing at the end of the wall panel. The hook or embedded extension will be turned up, down, or extended horizontally. Vertical deformed bar reinforcement that is at least equal to one-third $A_v$ will be provided in all walls requiring shear reinforcement. This vertical reinforcement will be uniformly distributed and will not exceed a spacing of 48 inches.

*b. Shear stresses from seismic loadings.* When designing shear walls for buildings in seismic zones 1 through 4, the increase in the seismic shear forces required in TM 5-809-10/NAVFAC P-355/AFM 88-3, Chapter 13 will be included.

*c. Other shear wall stresses.*

(1) The axial stresses caused by dead and live loads from roofs and floors will be considered in design of shear walls.

(2) The flexural stresses caused by moments from lateral in-plane shear force applied to the top of the wall or by the diaphragm will also be consideration in design. This in-plane moment is Vh for cantilever shear walls with fixed ends.

(3) The combined effects of axial and bending stresses must be considered. The unity equation or other methods using accepted principles of mechanics will be used.

**7-5. Rigidity.**

*a. General.* The magnitude of the total lateral forces at any story level depends upon the structural system as a whole. Also, the proportion of the total horizontal load that is carried by a particular shear wall element is based on the rigidity of the wall element relative the combined rigidities of all the wall elements on that same level. The relative rigidities of shear wall elements are inversely proportional to their deflections when loaded with a unit horizontal force. The total deflection at the top of a shear wall element is the sum of the shear deformation and flexural deflection (Figure 7-2) plus any additional displacement that may occur due to rotation at the base. For most shear walls in ordinary buildings, shear deformation is the major contributor to in-plane deflection.

*b. Factors affecting rigidity.*

(1) Control joints are complete structural separations that break the shear wall into elements. The elements must be considered as isolated structural members during shear wall rigidity analyses. The number

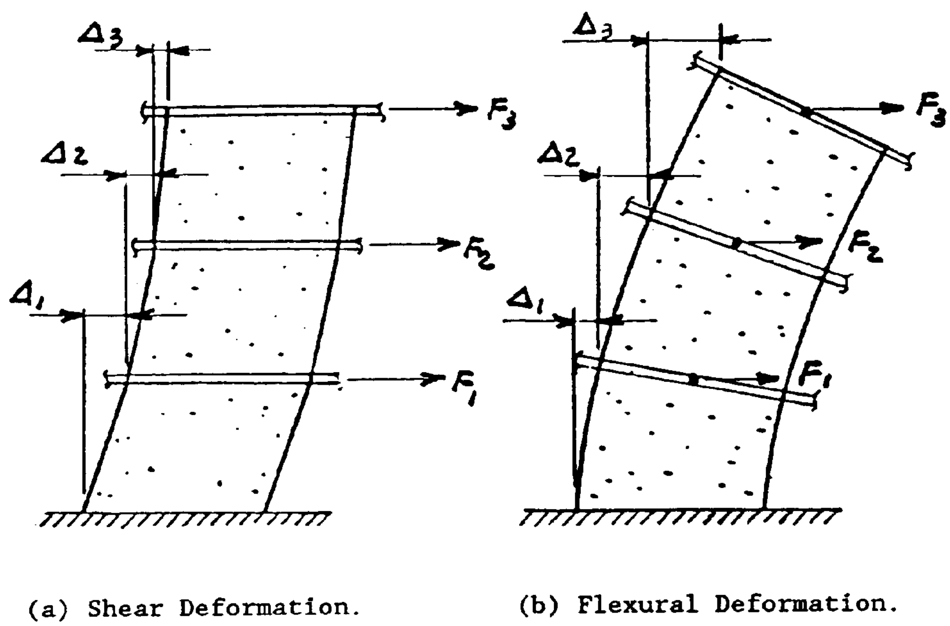


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*Figure 7-2. Shear wall deformation.*

and location of control joints within the total length of a wall may significantly affect element rigidities, especially flexural deformation.

(2) Openings for doors, windows, etc., reduce the rigidity of shear wall elements. If openings are significantly large or are significantly large in number, they should be considered in rigidity analyses as given in paragraph 7-7.

(3) A shear wall element which is structurally intern-al at its end with a shear wall that is normal to the element, forming an “L” or “T” in-plan shape, is called a corner element. The rigidity of a corner element is greater than that of a straight element. The amount of increase in rigidity is difficult to quantify but may be taken into account empirically when rigidity analyses are done using the method given in this chapter.

(4) Since shear walls are by nature, very rigid, rotation of the foundation can greatly influence the overall rigidity of a wall. However, the rotational influence on relative rigidities of walls for purposes of horizontal force distribution may not be as significant. Considering the complexities of soil behavior, a quantitative evaluation of the foundation rotation is generally not practical, but a qualitative evaluation, recognizing the limitations and using good judgment, should be a design consideration. It is usually assumed either that the foundation soil is unyielding or that the soil pressure varies linearly under the wall when the wall is subjected to overturning. These may not always be realistic assumptions, but are generally adequate for obtaining the relative rigidities required for design purposes.

**7-6. Distribution of Forces to Shear Walls.**

*a. General.* The distribution of lateral forces by different types of diaphragms is discussed in TM 5-809-10/NAVFAC P-355/AFM 88-3, Chap. 13, Seismic Design For Buildings. A brief description is provided herein.

*b. Translational shears.* The distribution of lateral story level shears from a diaphragm to the vertical resisting elements (in this case, masonry walls acting as shear walls) is dependent upon the relative stiffness of the diaphragm and the shear walls. A rigid diaphragm is assumed to distribute horizontal forces to the masonry shear walls in direct proportion to the relative rigidities of the shear walls. Under symmetrical loading, a rigid diaphragm will cause all vertical shear wall elements to deflect equally with the result being that each element will resist the same proportion of lateral force as the proportion of rigidity that element provides to the total rigidity of all the elements in the same level and direction. Flexible diaphragms, on the other hand, are considered to be less rigid than shear walls and will distribute the lateral forces to the wall elements in a manner analogous to a continuous beam without regard to the rigidity of the walls. A flexible diaphragm is considered incapable of resisting torsional rotational moments (see below).

*c. Rotational shears.* In a rigid diaphragm, when the center of gravity of the lateral forces fails to coincide with the center of rigidity of the supporting shear wall elements, a torsional moment will be generated within the rigid diaphragm. Provisions will be made to account for this torsional moment in accordance with TM 5-809-10/NAVFAC P-355/AFM 88-3, Chap. 13, Seismic Design For Buildings.

*d. Maximum shear wall deflection.* Roof and floor diaphragms, must be capable of transmitting horizontal shear forces to the shear walls without exceeding a deflection that which would damage the vertical elements. The maximum allowable deflection for horizontal diaphragms in buildings utilizing masonry shear walls will be as follows:

$$\text{Deflection} = \frac{h^2 F_b}{0.01 E_m t} \qquad \text{(eq 7-7)}$$

Where:

$F_b$ = The allowable flexural compressive stress in masonry, psi.
= $(0.33)f'_m$
$E_m$ = The modulus of elasticity of masonry, psi.
= $(1000)f'_m$ for CMU
$t$ = The effective thickness of the wall, inches.

This equation is neither exact nor technically correct. However, its primary function is to force the designer to think about limiting the deflection of the diaphragm to a value that will not adversely affect, architecturally, the completed wall.

**7-7. Effects of Openings in Shear Walls.** The effects of openings on the ability of shear walls to resist lateral forces must be considered. If openings are very small, their effect on the overall state of stress in a shear wall will be minor. Large openings will have a more pronounced effect. When the openings in a shear wall become so large that the resulting wall approaches an assembly similar to a rigid frame or a series of elements linked by connecting beams, the wall will be analyzed accordingly. It is common for openings to occur in regularly spaced vertical rows (or piers) throughout the height of the wall with the connections between the wall sections within the element being provided by either connecting beams (or spandrels) which form a part of the wall, or floor slabs, or a combination of both. If the openings do not line up vertically and/or horizontally, the complexity of the analysis is greatly increased. In most cases, a rigorous analysis of a wall with openings is not required. When designing a wall with openings, the deformations must be visualized in order to establish some approximate method to analyze the stress distribution of the wall. Figure 7-3 gives a visual description of such deformations. The major points that must be considered are; the lengthening and shortening of the extreme sides (boundaries) due to deep beam action, the stress concentration at the corner junctions of the horizontal and vertical components between openings, and the shear and diagonal tension in both the horizontal and vertical components.

*a. Relative rigidities of piers and spandrels.* The ease of analysis for walls with openings is greatly dependent upon the relative rigidities of the piers and spandrels, as well as the general geometry of the building. Figure 7-4 shows two extreme examples of relative rigidities of exterior walls of a building. In figure 7-4(a) the piers are very rigid relative to the spandrels. Assuming a rigid base, the shear walls act as vertical cantilevers. When a lateral force is applied, the spandrels act as struts with end moments-thus the flexural deformation of the struts must be compatible with the deformation of the cantilever piers. It is relatively simple to determine the forces on the cantilever piers by ignoring the deformation characteristics of the spandrels. The spandrels are then designed to be compatible with the pier deformations. In figure 7-4(b), the piers are flexible relative to the spandrels. In this case, the spandrels are assumed to be infinitely rigid and the piers are analyzed as fixed-end columns. The spandrels are then designed for the forces induced by the columns. The calculations of relative rigidities for both cases shown in figure 7-4 can be aided by the use of the wall deflection charts given later in this chapter. For cases of relative spandrel and pier rigidities other than those shown, the analysis and design becomes more complex.

*b. Methods of Analysis.* As stated above, approximate methods of analyzing walls with openings are generally acceptable. A common method of determining the relative rigidity of a shear wall with openings is given in the design example in this chapter. For the extreme cases shown in figure 7-4, the procedure is straight-forward. For other cases, a variation of assumptions may be used to determine the most critical

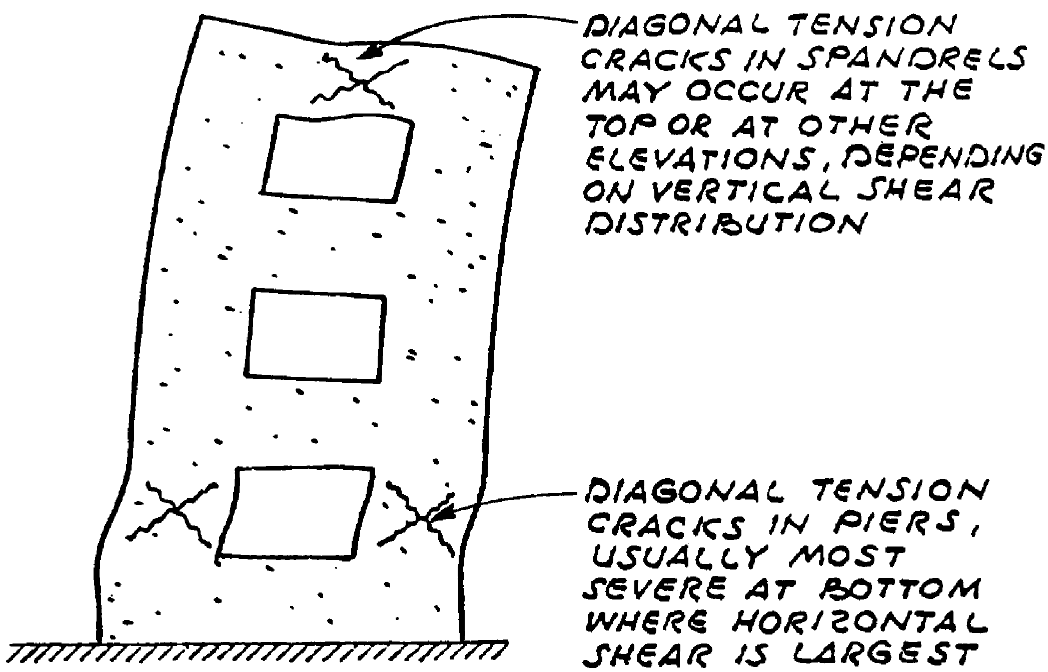


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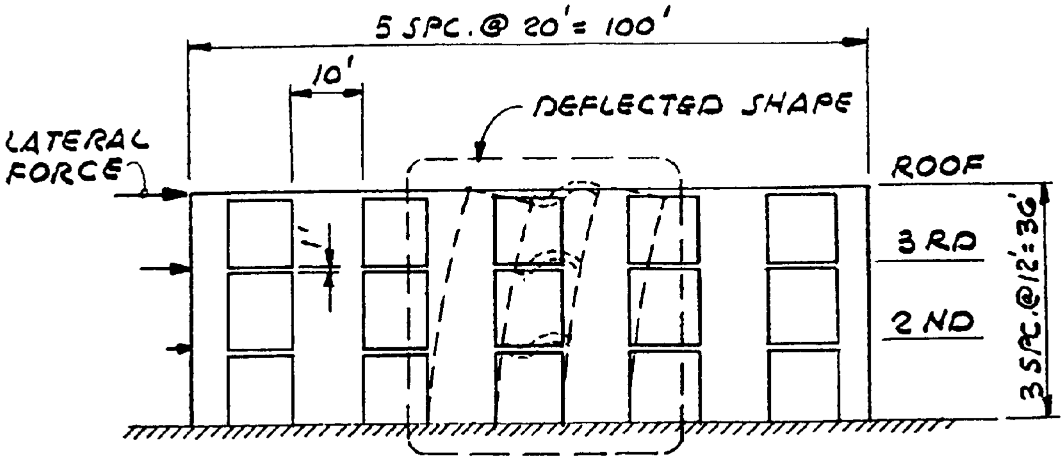
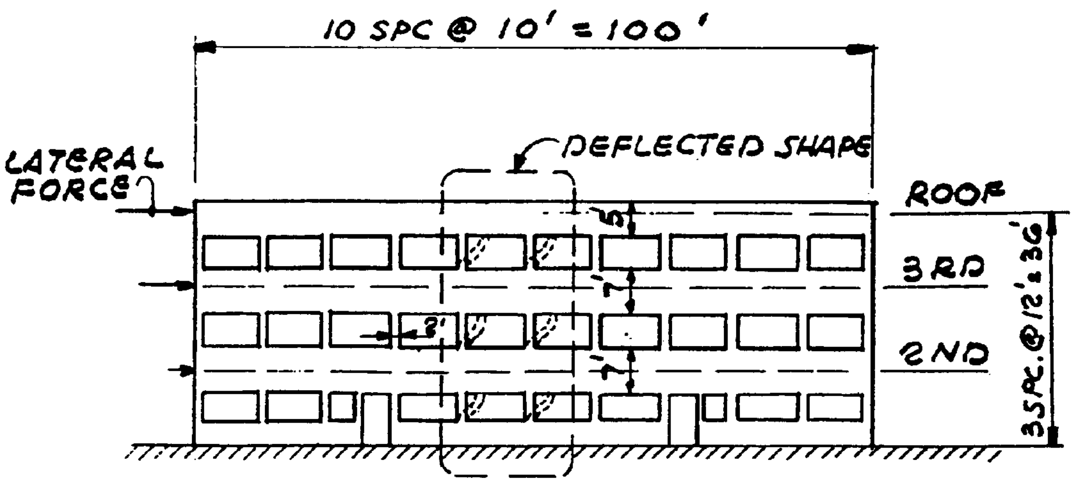
*Figure 7–3. Deformation of shear wall with openings.*




(a) Rigid Piers and Flexible Spandrel.




(b) Flexible Piers and Rigid Spandrel.

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*Figure 7–4. Relative rigidities of piers and spandrels.*

loads on various elements, thus resulting in a conservative design. In some cases a few additional reinforcing bars, at little additional cost, can greatly increase the strength of shear walls with openings. However, when the reinforcement requirements or the resulting stresses of this approach appear excessively large, a rigorous analysis may be justified.

**7-8. Shear Wall Rigidity Analysis.** The rigidity of a shear wall element is inversely proportional to its deflection, thus rigidity has units of kips per inch. The relative rigidity of a wall element is usually obtained by inverting the deflection caused by a unit horizontal load. The parameters in the rigidity equations for shear wall elements are: the dimensions of height, length, and thickness; the modulus of elasticity, $E_m$; the modulus of rigidity or shear modulus, $E_v$; and the fixity conditions of support of the wall element at top and bottom.

*a. Wall Deflections.* When a horizontal shear force is applied at the top of a masonry wall or pier element, it will produce a deflection. This deflection is the sum of the deflection due to flexure plus the deformation due to shear. When both ends (top and bottom) of the element are fixed, the total deflection, $\Delta_f$, is defined as follows:

$$\Delta_f = \Delta_b + \Delta_v = \frac{Vh^3}{12E_mI} + \frac{1.2Vh}{E_vA} \qquad \text{(eq 7–8)}$$

Where:

$\Delta_b$ = The flexural deflection, inches.
$\Delta_v$ = The shear deformation, inches.
A = The horizontal cross sectional area of the wall element, $in^2$
I = The horizontal cross sectional moment of inertia of the wall element in direction of bending, $in^4$.
$E_v$ = The shear modulus of masonry, psi.
= $0.4E_m$

When the wall or pier element is fixed at the bottom only, creating a cantilever condition, the total deflection, is defined by the following equation:

$$\Delta_c = \Delta_b + \Delta_v = \frac{Vh^3}{3E_mI} + \frac{1.2\ Vh}{E_vA} \qquad \text{(eq 7–9)}$$

The rigidity or stiffness of the shear wall, usually expressed as, k, is defined as the inverse of the total deflection of the wall as stated in the following equation:

$$k = \frac{1}{\Delta_b + \Delta_v} \qquad \text{(eq 7–10)}$$

In the case of a solid wall with no openings, the computations of deflection are quite simple. However, where the shear wall has openings for doors, windows, etc., the computations for deflection and rigidity are much more complex. Since an exact analysis which considers angular rotation of elements, rib shortening, etc., is not necessary, several short cut approximate methods, involving more or less valid assumptions, have been developed. Any simplified method of determining shear wall rigidity can give inconsistent or unsatisfactory results; therefore, a conservative approach and judgment must be used.

*b. Wall deflection charts.* The recommended approximate method of determining deflections and rigidities of shear wall elements, including walls with openings is the wall deflection charts given in figure 7-5. The charts are based upon equations 7-8 and 7-9. When openings are present, a solid wall is assumed and subtractions and additions of the rigidities of pier increments are done to determine the relative rigidity of the panel. By substituting "$td^3/12$" for "I", "td" for "A", and "$0.4E_m$" for "$E_v$" equations 7-8 and 7-9 can be simplified to equations 7-11 and 7-12, respectively, as follows:

$$\Delta_f = \frac{V}{E_mt}\left[\left[\frac{h}{d}\right]^3 + 3\left[\frac{h}{d}\right]\right] \qquad \text{(eq 7–11)}$$

$$\Delta_c = \frac{V}{E_mt}\left[4\left[\frac{h}{d}\right]^3 + 3\left[\frac{h}{d}\right]\right] \qquad \text{(eq 7–12)}$$

Since only relative rigidity values are required, any value could be used for $E_m$, and t as long as walls with differing moduli of elasticity and thickness are not being compared. V could also be arbitrary, as long as it is consistently used throughout the comparative process. The charts in figure 7-5 are based on values of; V = 1,000,000 pounds, $E_m$ = 1,350,000 psi, and t = 12 inches. Using these values, equations 7-11 and 7-12 can be simplified to equations 7-13 and 7-14, respectively, as follows:

$$\Delta_f = 0.0617\left[\frac{h}{d}\right]^3 + 0.1852\left[\frac{h}{d}\right] \qquad \text{(eq 7–13)}$$

$$\Delta_c = 0.2469\left[\frac{h}{d}\right]^3 + 0.1852\left[\frac{h}{d}\right] \qquad \text{(eq 7–14)}$$

The thickness value used assumes a solid 12" thick masonry wall which is not equal to the actual standard masonry unit thickness of 11.62" but suffices for the purposes of these equations. Curves 2 and 4 of figure

7-5 provide a graphical solution for equations 7-13 (for fixed ended rectangular piers) and 7-14 (for cantilever rectangular piers), respectively. When walls of different moduli of elasticity, $E_m$, are being compared, the deflection values shall be multiplied by the ratio of 1.35 x $10^6/E_m$. When walls of different actual thicknesses or equivalent thicknesses, t, are being compared, the deflection values shall be multiplied by the ratio of 12/t. In corner pier curves (1 and 3) the corner pier moment of inertia, I, is assumed to be 1.5 times that of the rectangular pier. The equations for the corner piers are derived by the procedure given above (using equation 7-8 and 7-9) except that (1.5)I is substituted for I in the bending term of the equations, and the correction factor of 1.2 in the shear term of the equations is replaced by 1.0, since the section can no longer be considered rectangular. These substitutions result in equations 7-15 (for a fixed ended corner pier) and equation 7-16 (for a cantilever corner pier) as follows:

$$\Delta_f = 0.0412 \left[ \frac{h}{d} \right]^3 + 0.1543 \left[ \frac{h}{d} \right] \qquad \text{(eq 7-15)}$$

$$\Delta_c = 0.1646 \left[ \frac{h}{d} \right]^3 + 0.1543 \left[ \frac{h}{d} \right] \qquad \text{(eq 7-16)}$$

For other values of I, the flexural portion of the deflection curves would be proportional. The deflections shown on the charts are reasonably accurate. The formulas written on the curves can be used to check the results. However, the charts will give no better results than the assumptions made in the shear wall analysis. For instance, the point of contraflexure of a vertical pier may not be in the center of the pier height. In some cases the point of contraflexure may be selected by judgment and an interpolation made between the cantilever and fixed conditions.

**7-9. Design examples.** The following design examples illustrate the procedure for determining the rigidity of a shear wall section with one opening and give the complete design of a shear wall with two openings.

*a. Design example 1.*

(1) Given:

*(a)* 12-inch normal weight CMU
*(b)* Wall height (h) = 12 feet
*(c)* Wall length (d) = 20 feet
*(d)* Reinforcement = #5 bars @ 24” o.c.
*(e)* Type S mortar is used with:
$f'_m$ = 1350 psi
$E_m$ = l000f'm = 1,350,000 psi
$E_v$ = 0.4$E_m$ = 540,000 psi.
*(f)* There is a 4-feet by 4-feet window opening centered in wall.

(2) *Problem.* Determine the rigidity of the wall.

(3) *Solution.* The procedure involves determining the rigidities or stiffnesses of each segment within the shear wall element. The method is based on the deflection charts of figure 7-5. In this method; the deflection of the solid wall is determined, the deflection of the horizontal strip of the wall containing all the openings is deducted from the solid wall deflection, and then the deflections of the piers within this opening strip are added to this modified wall deflection to obtain the total deflection of the actual wall with openings. The reciprocal of this deflection value becomes the relative rigidity of that wall. Note that the following solution is carried out, in some instances, to four significant figures. This was done for calculation purposes and does not imply that the deflections would actually be accurate to the degree of precision since the procedure used is only approximate with simplified assumptions made.

*(a)* A solid wall containing ABCD with no openings is assumed fixed at the bottom only (use rectangular pier cantilever curve #4 from figure 7-5). Note that the equivalent wall thickness for 12-inch CMU with grouted cells @ 24” o.c. from table 5-2 is 5.7 inches.

$$\text{Solid Wall: } \frac{h}{d} = \frac{12}{20} = 0.60$$

$$\Delta_{Solid} = 0.1644 \times \frac{12''}{5.7''} = 0.3461''$$

$$k_{Solid} = \frac{1}{\Delta_{Solid}} = \frac{1}{0.3461} = 2.89 \text{ (Stiffness of solid wall)}$$

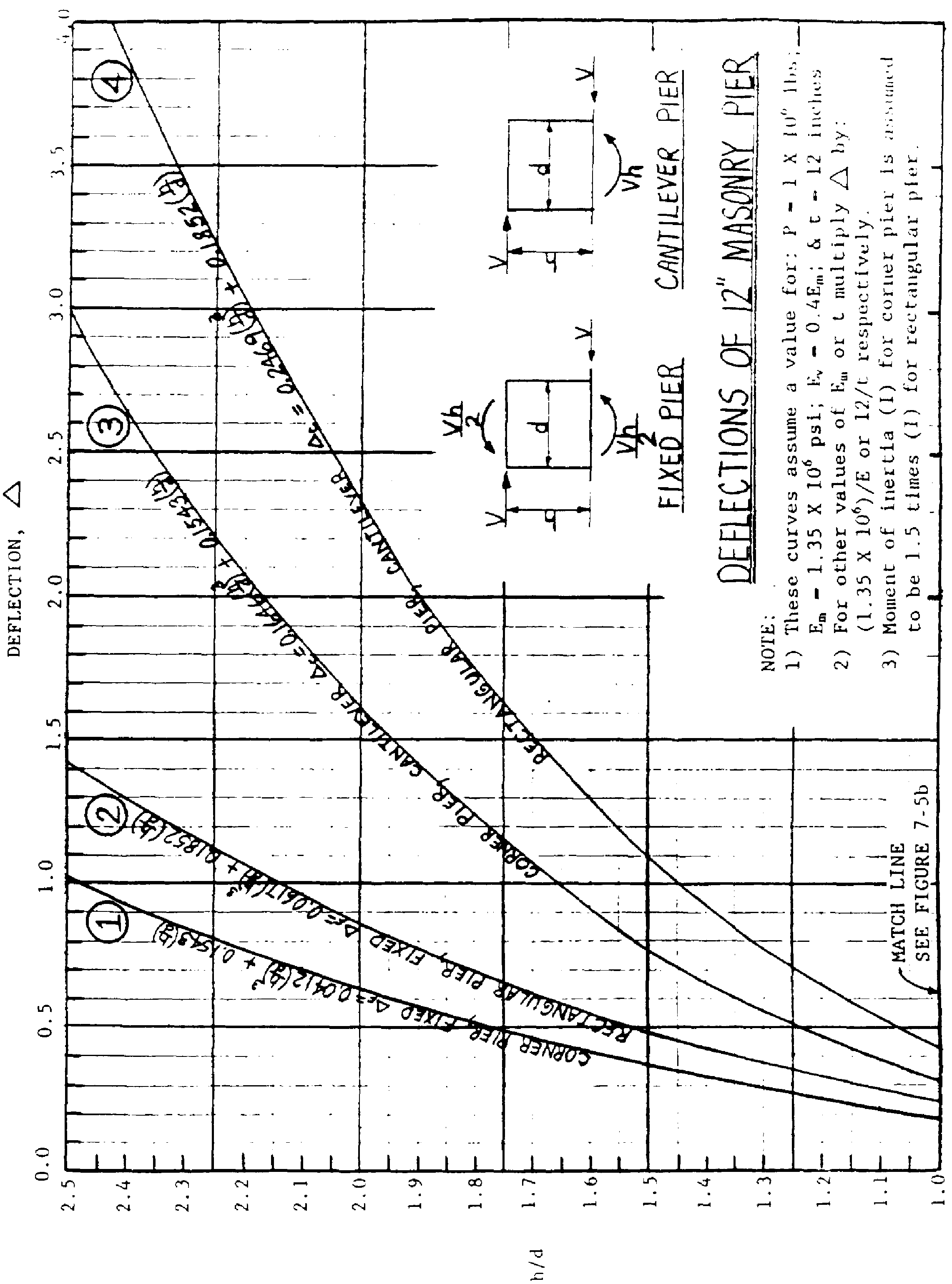


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*Figure 7–5a. Wall deflection chart.*

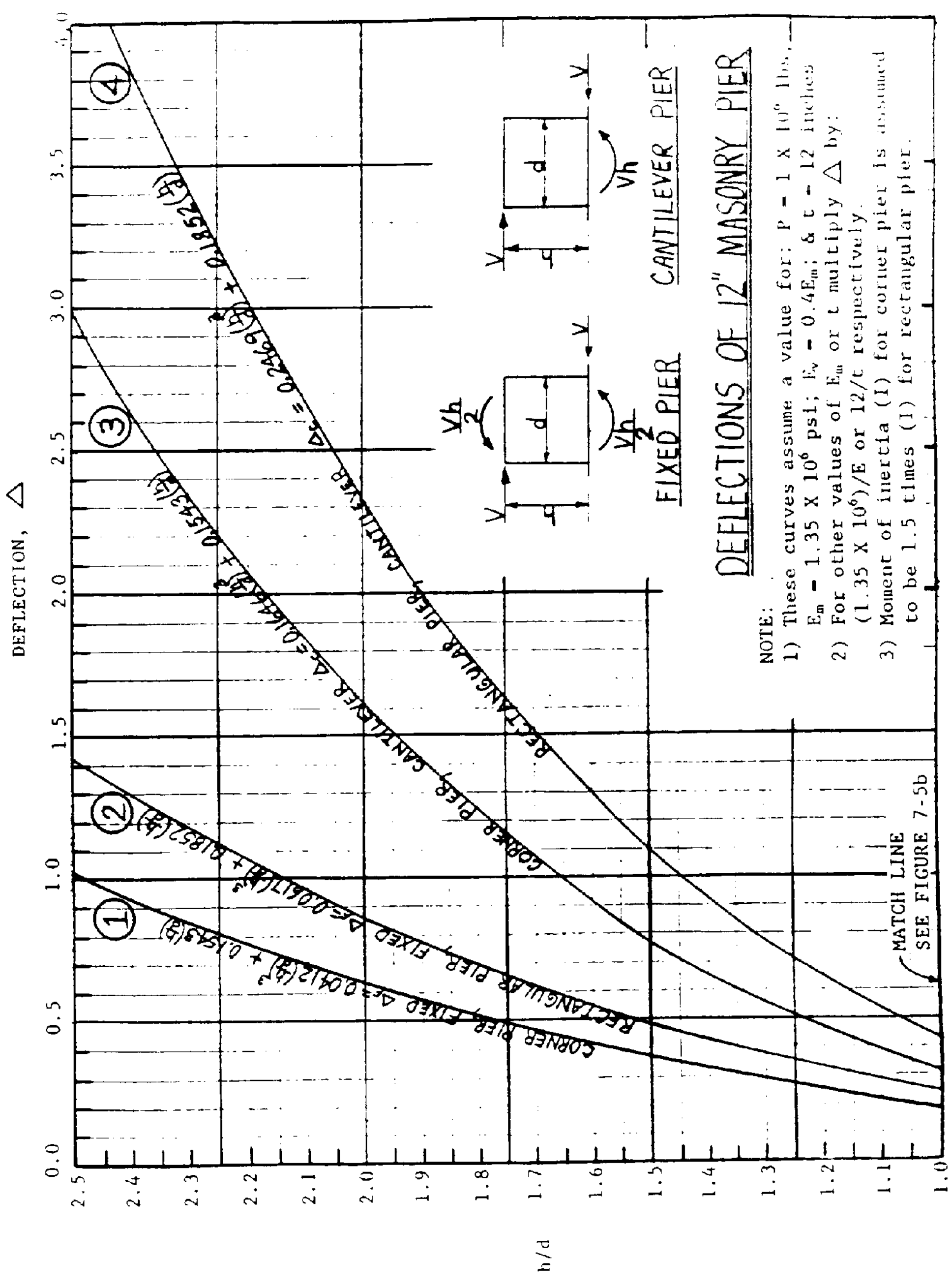


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*Figure 7-5a. Wall deflection chart.*

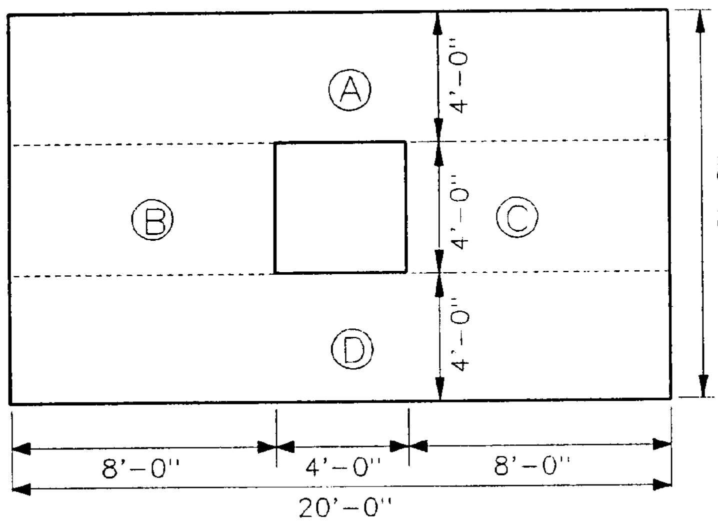


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*Figure 7–6. Design example 1 wall elevation.*

*(b)* The deflection of the solid middle 4'-0" strip containing B and C is determined assuming it is a fixed pier (use rectangular pier fixed curve #2 from figure 7-5) as follows:

$$\frac{h}{d} = \frac{4}{20} = 0.20$$

$$\Delta_{Solid} = 0.03753'' \times \frac{12''}{5.7''} = 0.0790''$$

*(c)* The individual deflections of piers B and C are determined assuming fixed top and bottom (use rectangular pier fixed curve #2 from figure 7-5) as follows:

$$\text{Pier B \& C: } \frac{h}{d} = \frac{4}{8} = 0.50$$

$$\Delta_B = \Delta_C = 0.10'' \times \frac{12''}{5.7''} = 0.211''$$

$$k_B = k_C = \frac{1}{0.211} = 4.739$$

$$k_{BC} = k_B + k_C = 4.739 + 4.739 = 9.478$$

$$\Delta_{BC} = \frac{1}{K_{BC}} = \frac{1}{9.478} = 0.1055''$$

*(d)* The total shear wall deflection and stiffness can now be found as follows:

$\Delta_{TOTAL} = \Delta_{Solid} = \Delta_{Strip} = \Delta_{BC}$

$\Delta_{TOTAL} = 0.3461'' - 0.0790'' + 0.1055'' = 0.3726''$

$$k_{TOTAL} = \frac{1}{\Delta_{TOTAL}} = \frac{1}{0.3726} = 2.68 < k_{Solid} = 2.88 \quad \text{. . . O.K.}$$

(4) *Summary.* The design example solution provided above illustrates the recommended procedure for determining the relative rigidity of a masonry shear wall element. Note that the relative rigidity of this wall element with one opening is about 93% of the solid wall element rigidity. Thus, it can be concluded that the opening has not significantly reduced the rigidity of the shear wall.

*b. Design example 2.*

(1) Given:

*(a)* 8-inch normal weight CMU
*(b)* Wall height (h) = 12 feet
*(c)* Wall length (d) = 20 feet
*(d)* In-plane shear force from wind loading (V) = 10 kips
*(e)* Axial loads (Concentrically applied):
Dead load = 300 pounds per foot
Live load = 600 pounds per foot

*(f)* Reinforcement:
#5 bars @ 24” o.c.
$f_y$ = 60,000 psi
$E_s$ = 29,000,000 psi

*(g)* Modular ratio (n) = $E_s/E_m$ = 21.5

*(h)* Equivalent wall thickness = 4.1 inches (table 5-2).

*(i)* Type S mortar is used with:
$f'_m$ = 1350 psi
$E_m$ = l000$f'_m$ 1,350,000 psi
$E_v$ = 0.4$E_m$ = 540,000 psi.

*(j)* There is a door and a window opening as shown in figure 7-7.

(2) *Problem.* Design the given shear wall to withstand the shear and axial forces applied.

(3) *Solution.* The design procedure involves determining the rigidities of each segment (pier) within the shear wall. The method used is based on the deflection charts of figure 7-5. The horizontal loading is then proportioned to each segment based on its rigidity relative to the other segments, with longer and shorter segments receiving the greater load. Each wall segment will then be analyzed by checking the flexural, axial, and shear stresses.

*(a)* The first step in designing the shear wall is to determine the relative rigidities or stiffnesses of the shear wall segments. The method used in determining the relative rigidities is similar to the procedure followed in design example 1.

A solid wall containing ABCDE with no openings is assumed fixed at the bottom only. The deflection and rigidity are determined (use rectangular pier cantilever curve #4 from figure 7-5) as follows:

$$\textbf{Solid Wall: } \frac{h}{d} = \frac{12}{20} = 0.60$$

$$\Delta_{Solid} = 0.165'' \times \frac{12''}{4.1''} = 0.4829''$$

$$k_{Solid} = \frac{1}{\Delta_{Solid}} = \frac{1}{0.4829} = 2.07 \text{ (Stiffness of solid wall)}$$

The deflection of the solid bottom strip, 7.33 feet high, containing BCDE is determined (use rectangular pier cantilever curve #4 from figure 7-5) as follows:

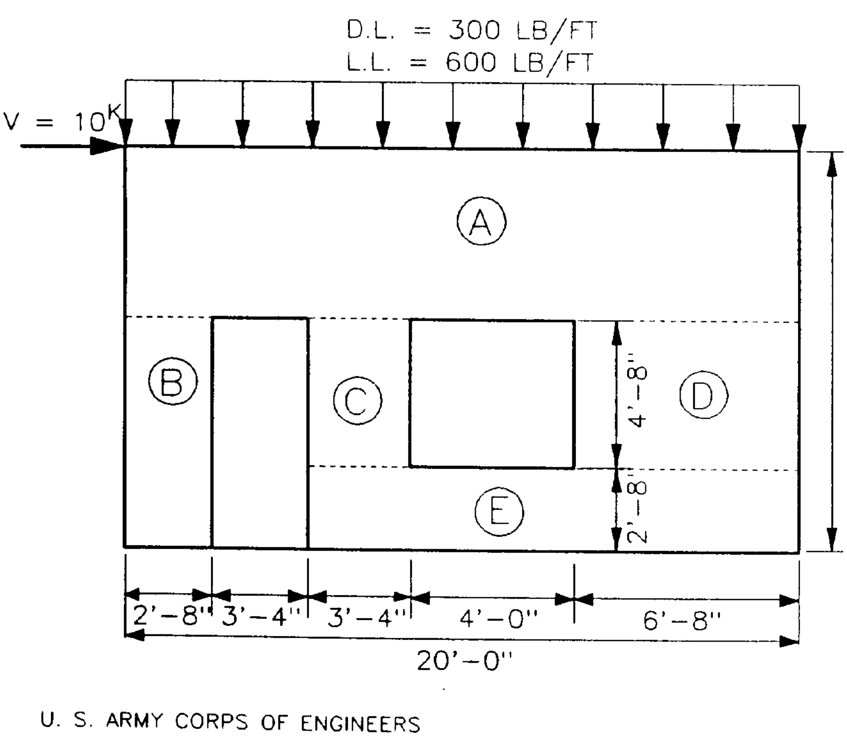



*Figure 7–7. Design example 2 wall elevation.*

$$\frac{h}{d} = \frac{7.33}{20} = 0.367$$

$$\Delta_{Strip} = 0.08'' \times \frac{12''}{4.1''} = 0.2341''$$

The combined deflection of piers B, C, D, and E are determined from the summation of their own individual rigidities (use rectangular pier fixed curve #2 from figure 7-5) as follows:

$$\text{Pier B: } \frac{h}{d} = \frac{7.33}{2.67} = 2.75$$

$$\Delta_B = 1.79'' \times \frac{12''}{4.1''} = 5.239''$$

$$k_B = \frac{1}{5.239} = 0.191$$

$$\text{Pier C: } \frac{h}{d} = \frac{4.67}{3.33} = 1.40$$

$$\Delta_C = 0.43'' \times \frac{12''}{4.1''} = 1.259''$$

$$k_C = \frac{1}{1.259} = 0.794$$

$$\text{Pier D: } \frac{h}{d} = \frac{4.67}{6.67} = 0.70$$

$$\Delta_D = 0.15'' \times \frac{12''}{4.1''} = 0.439''$$

$$k_D = \frac{1}{0.439} = 2.278$$

$$k_{CD} = k_C + k_D = 0.794 + 2.278 = 3.072$$

$$\Delta_{CD} = \frac{1}{k_{CD}} = \frac{1}{3.072} = 0.3255''$$

$$\text{Pier E: } \frac{h}{d} = \frac{2.67}{14.0} = 0.19$$

$$\Delta_E = 0.036'' \times \frac{12''}{4.1''} = 0.1054''$$

$$\Delta_{CDE} = \Delta_{CD} + \Delta_E = 0.3255 + 0.1054 = 0.4309''$$

$$k_{CDE} = \frac{1}{\Delta_{CDE}} = \frac{1}{0.4309} = 2.321$$

$$k_{BCDE} = k_B + k_{CDE} = 0.191 + 2.321 = 2.512$$

$$\Delta_{BCDE} = \frac{1}{k_{BCDE}} = \frac{1}{2.512} = 0.3981''$$

The total shear wall deflection and stiffness can now be found as follows:

$$\Delta_{TOTAL} = \Delta_{Solid} - \Delta_{Strip} + \Delta_{BCDE}$$
$$= 0.4829'' = 0.2341'' + 0.3981'' = 0.6469''$$

$$k_{TOTAL} = \frac{1}{\Delta_{TOTAL}} = \frac{1}{0.6469} = 1.55 < k_{Solid} = 2.07$$

. . . O.K.

*(b)* The next step in the design is to determine the force distribution to the individual piers B, C, and D. This can be done by dividing the stiffness of the individual piers by the summation of stiffnesses of all the piers as follows:

$$\text{Percentage of force to each pier} = \frac{k}{\Sigma k}$$

$$\text{To Pier B: } \frac{k_B}{k_{BCDE}} = \frac{0.191}{2.512} = 0.076 = 7.6\%$$

$$\text{To Piers CDE: } \frac{k_{CDE}}{k_{BCDE}} = \frac{2.321}{2.512} = 0.924 = 92.4\%$$

Thus 7.6% of the total in-plane shear force on the wall will be resisted by Pier B and 92.4% of the force will be resisted by piers C and D. The 92.4% will be distributed to piers C and D in proportion to their relative rigidities as follows:

$$\text{Pier C: } \frac{k_C}{k_{CD}} = \frac{0.794}{3.072} = 0.26 \times .924 = .24 = 24\%$$

$$\text{Pier D: } \frac{k_D}{k_{CD}} = \frac{2.278}{3.072} = 0.74 \times .924 = .684 = 68.4\%$$

Therefore, 24% of the total shear force on the wall will be distributed to Pier C; 68.4% to Pier D; and 7.6% to Pier B.

*(c)* Now that the distribution of in-plane shear forces to each pier is known, the design of the piers can now be accomplished. The design of each pier will begin by checking the shear and flexural stresses due to in-plane wind loads. Axial stresses due to dead and live loads will also be checked. The flexural and axial stresses will then be combined using the unity equation. For loading combinations that include wind loads, the allowable stresses will be increased 33%.

*Pier B design.* The design for out-of-plane wind loadings (not part of this example) require that all cells be reinforced and fully grouted, thus the cross section of the pier is 2'-8" by 7.62" with #5 bars in each cell. When checking in-plane shear stresses, the assumed length of the pier, $d_{Bv}$, will be the actual pier length or 2'-8". When checking in-plane flexural stresses, the assumed effective depth of the beam section, $d_{Bb}$, will be the actual length less the 8-inch distance from the centroid of the two end bars to the end of the pier; therefore $d_{Bb}$ = 2'-0".

*Shear Check*

Lateral force, $V_B$, to Pier B = 7.6% of 10k

$V_B$ = 0.076 x 10k = 760 lbs.

Shear stress in pier B, $f_{vB}$, is determined as follows:

$$f_{vB} = \frac{V_B}{td_{Bv}} = \frac{760 \text{ lbs.}}{7.62'' \text{ X } 32''} = 3.1 \text{ psi}$$

The allowable shear stress assuming no shear reinforcement, $F_{vm}$, will be determined by equation 7-2 as follows (assume pier fixed top and bottom):

$$\frac{M}{Vd_{Bv}} = \frac{h}{2d_{Bv}} = \frac{7.33' \text{ X } 12''/,}{2 \text{ X } 32''} = 1.38 > 1.0$$

Therefore;

$F_{vm} = 1.0(f'_m)^{1/2} = (1350)^{1/2} = 36.7$ psi.

But shall not exceed 35 psi; thus $F_{vm}$ = 35 psi.

$f_{vB}$ = 3.1 psi < $F_{vm}$ = 35 psi x 1.33 = 46.5 psi;

Therefore, no shear reinforcement is required.

*Flexural Check.* Both flexural compression and tension must be considered.

Flexural compressive stress in Pier B, $f_{bB}$, is determined as follows:

$$f_{bB} = \frac{2M}{bd_{Bb}{}^2jk}$$

Where:

$$M = \frac{v_B h}{2} = \frac{760 \text{ lb} \times 7.33' \times 12''/\text{ft}}{2} = 33{,}440 \text{ in-lb}$$

$b = 7.62''$

$A_s = 0.62 \text{ in}^2$ (2 =#5's)

$$p = \frac{A_s}{bd_{Bb}} = \frac{0.62 \text{ in}^2}{7.62'' \text{ X } 24''} = 0.00344$$

np = 21.5 (0.0034) = 0.073; thus k = 0.316 and j = 0.895

$$f_{bB} = \frac{2 \times 33{,}440}{7.62(24)2(0.895)(0.316)} = 53.9 \text{ psi}$$

Allowable flexural compressive stress in Pier B, Fb, is determined as follows:

$F_b = 0.33f'_m$ x 1.33 = 0.33(1350) X 1.33 = 600 psi

$F_{bB}$ = 53.9 psi < $f_b$ = 600 psi

...O.K.

Flexural tensile stress in Pier B, $f_{sB}$, is determined as follows:

$$f_{sB} = \frac{M}{A_s j d_{Bb}} = \frac{33{,}440}{(0.62)(0.895)(24)} = 2511 \text{ psi}$$

Allowable flexural tensile stress in Pier B, $F_s$, is determined as follows:

$F_s$ = 24,000 x 1.33 = 32,000 psi (Grade 60 steel)

$f_{sB}$ =2511 psi < $F_s$ = 32,000 psi

...O.K.

*Axial Load Check.* Since the maximum moment occurs at the top or the bottom of the pier and the axial load is maximum at the bottom of the pier, the axial load will be determined at the bottom of the pier. The fully grouted weight of the wall, $w_2$, is 92 psf.

Axial load at the bottom of Pier B, P, is determined as follows:

$P_{TOTAL} = P_{DL} + P_{LL}$ + Wall Wt. to bottom of Pier B

$P_{TOTAL}$ = [(300 lb/ft) + (600 lb/ft)] (4.33 ft)
+ 92psf [(7.33')(2.67') + (4.67')(4.33')]
= 3900 + 3611 = 7561 lbs.

Axial stress due to axial load in Pier B, $f_{aB}$, is determined as follows:

$$f_{aB} = \frac{P}{A} = \frac{7561 \text{ lbs.}}{7.62'' \times 32''} = 31.0 \text{ psi}$$

Allowable axial stress in Pier B, $F_a$, is determined as follows:

$F_a = (0.2 f'_m)R$

Where:

R = The stress reduction factor.

Since buckling is not a concern at the bottom of the pier, R will be omitted and including wind loading:

$F_a = (0.2 f'_m) \times 1.33$
$= 0.2\ (1350) \times 1.33 = 360$ psi

$f_{aB}$ = 31.0 psi < $F_a$ = 360 psi

...O.K.

Axial stress on Pier B due to the overturning moment of the entire wall panel, $f_{oB}$, is determined as follows:

$f_{oB} = M_o C_B / I_n$,

Where:

$M_o$ = The overturning moment, ft-lbs.
= Vh = 10,000 lb. X 12.0' = 120,000 ft-lb.

$C_B$ = Distance from the center of gravity of the net wall section to the centroid of the pier in question (Pier B). See table 7-1.

$I_n$ = Moment of inertia of the net wall section

$I_n = \Sigma(I_{Cen} + AC^2) = \Sigma AC^2$ (Because $I_{Cen}$, which is equal to $bd^3/12$, is usually negligible compared to $AC^2$. Therefore, use $I_n = \Sigma AC^2$. See table 7-1.)

From table 7-1, $C_B$ and $I_n$ are determined and the axial stress on pier B due to overturning, $f_{oB,}$ is determined as follows:

$$f_{oB} = \frac{120{,}000 \text{ ft-lb. X } 7.94 \text{ ft}}{237.42 \text{ ft}^4 \text{ X } 144 \text{ in}^2/\text{ft}^2} = 27.9 \text{ psi}$$

$$f_{oB} = 27.9 \text{ psi} < F_a = 360 \text{ psi}$$

...O.K.

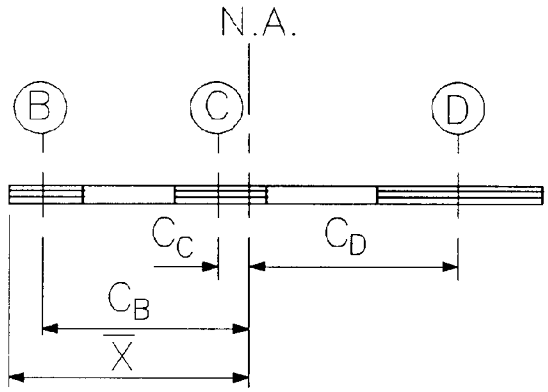
*Combined Load Check.* The combined effects of flexural, axial, and overturning on pier B can be evaluated using the unity equation as follows:

$$\frac{f_{bB}}{f_b} + \frac{f_{aB}}{F_a} + \frac{f_{oB}}{F_a} \leq 1.0$$

$$\frac{53.9}{600} + \frac{31.0}{360} + \frac{27.9}{360} = 0.25 < 1.0$$

...O.K.

*Pier C design.* The design for out-of-plane wind loadings (not part of this example) require that all cells be reinforced and fully grouted, thus the cross section of the pier is 3'-4" by 7.62" with 5 bars in each cell. When checking in-plane shear stresses, the assumed length of the pier, $d_{Cv}$, will be the actual pier length or 3'-4". When checking in-plane flexural stresses, the assumed effective depth of the beam section, $d_{Cb}$, will be the




| PIER | AREA ($ft^2$) | X (ft) | AX ($ft^3$) | C (ft) | $AC^2$ ($ft^4$) |
|---|---|---|---|---|---|
| B | 1.70 | 1.33 | 2.26 | 7.53 | 96.39 |
| C | 2.11 | 7.67 | 16.18 | 1.19 | 2.99 |
| D | 2.05 | 16.33 | 33.48 | 7.47 | 114.39 |
| | Σ 5.86 | | Σ 51.92 | | Σ 213.77 |

$$\bar{X} = \frac{51.92\,ft^3}{5.86\,ft^2} = 8.86\ ft \qquad I_n = \Sigma\ AC^2 = 213.77 ft^4$$

Note: The area of Pier D is based on an equivalent solid thickness of 4.1 inches for a wall section with grouted cells at 24 inches on center. Pier B and C are grouted full and have a thickness of 7.62 inches.

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*Table 7–1. Centroid and moment of inertia of net wall section.*

actual length less the 8-inch distance from the centroid of the two end bars to the end of the pier; so $d_{cB}$ = 2'-8".

*Shear Load*

Lateral force, $V_c$, to Pier C = 24% of 10k

$V_c = 0.24 \times 10k = 2400$ lbs.

Shear stress in pier C, $f_{vC}$, is determined as follows:

$$f_{vC} = \frac{V_C}{td_{Cv}} = \frac{2400\ lbs}{7.62'' \times 40''} = 7.9\ psi$$

The allowable shear stress, $F_{vm}$, will be determined by equation 7-1 as follows (assume pier fixed top and bottom):

$$\frac{M}{Vd_{Cv}} = \frac{h}{2d_{Cv}} = \frac{4.67' \times 12''/,}{2 \times 40''} = 0.70 < 1.0$$

Therefore;

$$F_{vm} = \frac{1}{3}\left[4 - \frac{M}{Vd}\right](f'_m)^{1/2}$$

$$= \frac{1}{3}[4 - 0.70](1350)^{1/2} = 40.4\ psi$$

But shall not exceed: 80 - 45[M/($Vd_{Cv}$)]

$F_{vm}$ = 80 - 45(0.70) = 48.5 psi; thus $F_{vm}$ = 40.4 psi

$f_{vC}$ = 7.9 psi < $F_{vm}$ = 40.4 psi × 1.33 = 53.7 psi

Therefore; no shear reinforcement is required.

*Flexural Check.* Both flexural compression and tension must be considered.

Flexural compressive stress in Pier C, $f_{bC}$, is determined as follows:

$$f_{bC} = \frac{2M}{bd_{Cb}{}^2jk}$$

Where:

$$M = \frac{V_C h}{2} = \frac{2400\text{lb} \times 4.67' \times 12''/\text{ft}}{2} = 67{,}248 \text{ in-lb}$$

$$b = 7.62''$$

$$A_s = 0.62 \text{ in}^2 \ (2 = \#5\text{'s})$$

$$p = \frac{A_s}{bd_{Cb}} = \frac{0.62 \text{ in}^2}{7.62'' \text{ X } 32''} = 0.0025$$

$$np = 21.5(0.0025) = 0.054; \text{ thus } k = 0.28 \text{ and } j = 0.907$$

$$f_{bc} = \frac{2 \times 67{,}248}{7.62(32)2(0.907)(0.28)} = 67.9 \text{ psi}$$

$$f_{bc} = 67.9 \text{ psi} < F_b = 600 \text{ psi}$$

. . . O.K.

Flexural tensile stress in Pier C, $f_{sC}$, is determined as follows:

$$f_{sC} = \frac{M}{A_s jd} = \frac{67{,}248}{(0.62)(0.907)(32)} = 3737 \text{ psi}$$

$$f_{sC} = 3737 \text{ psi} < F_s = 32{,}000 \text{ psi}$$

...O.K.

*Axial Load Check.* Since the maximum moment occurs at the top or the bottom of the pier and the axial load is maximum at the bottom of the pier, the axial load will be determined at the bottom of the pier. The fully grouted weight of the wall, $w_2$, is 92psf.

Axial load at the bottom of Pier C = P (lbs.)

$P_{TOTAL} = P_{DL} + P_{LL}$ + Wall wt. to bottom of Pier C

$P_{TOTAL}$ = [(300 lb/ft) + (600 lb/ft)] (7.0 ft)

+ 92 psf[(4.67)(3.33') + (4.67')(7.0')]

= 6300 + 4438 = 10,738 lbs.

Axial stress due to axial load in Pier C, $f_{aC}$ determined as follows:

$$f_{aC} = \frac{P}{A} = \frac{10{,}738 \text{ lbs.}}{7.62'' \times 40''} = 35.2 \text{ psi}$$

Allowable axial stress in Pier C, $F_a$, is determined as follows:

$F_a = (0.2\ f'_m)R$

Where:

R = The stress reduction factor.

Since buckling is not a concern at the bottom of the pier, R will be omitted and including wind loading:

$F_a = 0.2\ f'_m \times 1.33$

$= 0.2\ (1350) \times 1.33 = 360$ psi

$f_{aC} = 35.2$ psi $< F_a = 360$ psi

...O.K.

Axial stress in Pier C due to the overturning moment of the entire wall panel, $f_{oC}$, is determined as follows:

$$f_{oC} = \frac{M_o C_C}{I_n}$$

Where:

$M_o$ = Overturning Moment = Vh

= 10,000 lbs. × 12.0' = 120,000 ft-lb.

$C_c$ = Distance from the center of gravity of the net wall section to the centroid of the pier in questions (Pier C). See table 7-1.

$I_n$ = Moment of inertia of the net wall section

= $\Sigma(I_{Cen} + AC^2) \approx \Sigma AC^2$ (Because $I_{Cen}$, which is equal to $bd^3/12$, is usually negligible compared to $AC^2$. Therefore, use $I_n$, = $\Sigma AC^2$. See table 7-1.)

$$F_{oC} = \frac{120{,}000 \text{ ft-lb.} \times 1.60 \text{ ft}}{237.42 \text{ ft}^4 \times 144 \text{ in}^2/\text{ft}^2} = 5.6 \text{ psi}$$

$f_{oC}$ = 5.6 psi < Fa = 360 psi

...O.K.

*Combined Load Check.* The combined effects of flexural, axial, and overturning of pier C can be evaluated using the unity equation as follows:

$$\frac{67.9}{600} + \frac{35.2}{360} + \frac{5.6}{360} = 0.23 < 1.0$$

...O.K.

*Pier D design.* The pier is reinforced with #5 bars at 24 inches o.c., so the equivalent solid wall thickness is 4.1". The cross section of the pier is 6'-8" by 4.1" and the area assumed effective in shear parallel to the wall face, $A_{eff}$, is 49.0 $in^2$/ft. The design of pier D will follow the same procedure as previously shown for piers B and C except for the conditions stated herein. The resulting design stress values are as follows:

*Shear Load.*

Lateral force to Pier D, $V_D$, is 68.4% of 10k:

$V_D$ = 0.684 x 10k = 6840 lbs.

Shear stress in pier D, $f_{vD}$, is determined as follows:

$$f_{vD} = \frac{V_D}{A_{eff}} = \frac{6840 \text{ lbs.}}{49 \text{ in}^2/\text{ft} \times 6.67 \text{ ft}} = 20.9 \text{ psi}$$

The allowable shear stress, $F_{vm}$, will be determined by equation 7-1 as follows (assume pier fixed top and bottom):

$$\frac{M}{Vd_{DV}} = \frac{h}{2d_{Dv}} = \frac{4.67' \times 12''/\text{ft}}{2 \times 80''} = 0.35 < 1.0$$

Therefore;

$$F_{vm} = \frac{1}{3}\left[4 - \frac{M}{Vd}\right](f'_m)^{1/2}$$

$$= \frac{1}{3}[4 - 0.35](1350)^{1/2} = 44.7 \text{ psi}$$

But shall not exceed: 80 - 45 [M/($Vd_{Cv}$)]

$F_{vm}$ = 80 - 45(0.35) = 64.3 psi; thus $F_{vm}$ = 44.7 psi

$f_{vD}$ = 20.9 psi < $F_{vm}$ = 44.7psi × 1.33 = 59.5 psi

Therefore, no shear reinforcement is required.

Flexural Check:

Flexural compressive stress in Pier D, $f_{bD}$, is determined as follows:

$$f_{bD} = \frac{2M}{bd_{Db}^2 jk}$$

Where:

$$M = \frac{v_D h}{2} = \frac{6840\text{lb} \times 4.67' \times 12''/\text{ft}}{2} = 191{,}657 \text{ in-lb}$$

$$b = 4.1''; \; d = 80'' - 8'' = 72''$$

$$A_s = 0.62 \text{ in}^2 \text{ (2 - \#5's)}$$

$$p = \frac{A_s}{bd} = \frac{0.62 \text{ in}^2}{4.1'' \times 72''} = 0.0021$$

$$np = 21.5(0.0021) = 0.05; \text{ thus } k = 0.27 \text{ and } j = 0.91$$

$$f_{bD} = \frac{2 \times 191{,}657}{4.1(72)2(0.91)(0.27)} = 73.4 \text{ psi}$$

$f_{bD}$ = 73.4 psi < $f_b$ = 600 psi ...O.K.

Flexural tensile stress in Pier D, $f_{sD,}$ is determined as follows:

$$f_{sD} = \frac{M}{A_s jd} = \frac{191{,}657}{(0.62)(0.91)(72)} = 4718 \text{ psi}$$

...O.K.

Axial Load Check: The weight of the wall, grouted at 24 inches on center, $w_2$, is 69 psf.

Axial load at the bottom of Pier D, P, is determined as follows:

$P_{TOTAL}$ = [(300 lb/ft) + 600lb/ft)] (8.67 ft)
+ 69 psf [(4.67')(6.67') + (4.67')(8.67')]
= 7803 + 4943 = 12,746 lbs.

Axial stress due to axial load in Pier D, $f_{aD}$, is determined as follows:

$$f_{aD} = \frac{P}{A} = \frac{12{,}746 \text{ lbs.}}{4.1'' \times 80''} = 38.9 \text{ psi}$$

$f_{aD}$ = 38.9 psi < $F^a$ = 360 psi (See Pier B for $F_a$)

...O.K.

Axial stress in Pier D stress due to the overturning of the entire wall panel, is determined as follows: (See Pier B design):

$$f_{oD} = \frac{M_o C_D}{I_n} = \frac{120{,}000 \text{ ft-lb.} \times 7.40 \text{ ft}}{237.42 \text{ ft}^4 \times 144 \text{ in}^2/\text{ft}^2} = 26.0 \text{ psi}$$

$f_{oD}$ = 26.0 psi < $F_a$ = 360 psi

...O.K.

Combined Load Check: (Use the unity equation, see Pier B design.)

$$\frac{73.4}{600} + \frac{38.9}{360} + \frac{26.0}{360} = 0.30 < 1.0$$

...O.K.

(4) *Summary*. The design example solution provided above has shown that the assumed wall section is adequate to withstand the applied axial and in-plane shear loads.