CHAPTER 2

RELIABILITY AND ITS MATHEMATICAL FOUNDATIONS

2-1. Reliability as an engineering discipline

Reliability is a measure of a product's performance that affects both mission accomplishment and operating and support (O&S) costs. Too often we think of performance only in terms of speed, capacity, range, and other "normal" measures. However, if a product fails so often (i.e., poor reliability) that it's seldom available, speed, range, and capacity are irrelevant. Reliability is very much like these other performance parameters, however, in a very important way. Reliability results from a conscious effort to design for reliable performance and to ensure that manufacturing processes do not compromise the "designed-in" level of reliability.

a. *Designing for reliability.* Perfect reliability (i.e., no failures, ever, during the life of the product) is difficult if not impossible to achieve. So even when a "good" level of reliability is achieved, some failures are expected. To keep the number of failures, especially those that could result in catastrophic or serious consequences, designers must conduct analyses, use good design practices, and conduct development tests.

(1) The designer has many analytical methods for identifying potential failure modes, determining the probability of a given failure, identifying single-point and multiple failures, identifying weaknesses in the design, and prioritizing redesign efforts to correct weaknesses. More traditional analytical methods are being complemented or, in some cases, replaced by computer simulation methods.

(2) Some designs are more reliable than others. The most reliable designs tend to be simple, be made with parts appropriately applied, be robust (i.e., tolerant to variations in manufacturing process and operating stresses), and be developed for a known operating environment.

(3) Although designers may apply many analytical tools and design techniques to make the product as reliable as necessary, these tools and techniques are not perfect. One way to compensate for the imperfections of analysis and design techniques is to conduct tests. These tests are intended to validate the design, demonstrate functionality, identify weaknesses, and provide information for improving the design. Some tests are conducted specifically for verifying reliability and identifying areas where the reliability can or must be improved. Even tests that are not conducted specifically for reliability purposes can yield information useful in designing for reliability.

b. *Retaining the "designed-in" level of reliability*. Once a design is "fixed," it must be "transformed" to a real product with as much fidelity as possible. The process of transforming a design to a product is manufacturing. Building a product involves processes such as welding and assembly, inspecting materials and parts from suppliers, integrating lower-level assemblies into high-level assemblies, and performing some type of final inspection. Poor workmanship, levels of part quality that are less than specified by the designer, out-of-control processes, and inadequate inspection can degrade the designed-in level of reliability. To ensure that manufacturing can make the product as it was designed, manufacturing/production engineers and managers should be involved during design. In this way, they will know if new equipment or processes are needed, gain insight into the type of training needed for the manufacturing/production personnel, potential problems, and so forth. They can also help the designers by describing current manufacturing/production capabilities and limitations.

2-2. Mathematical foundations: probability and statistics

Reliability engineering is not equivalent to probability and statistics or vice versa. One would never equate mechanical engineering with calculus – mathematics only provides the basis for measurement in engineering. To quote William Thomson, Lord Kelvin, "When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science." Probability and statistics are the mathematical foundation of reliability.

a. *The mathematics of reliability*. Probability and statistics constitute the mathematics of reliability engineering. They allow us to express our discipline in numbers, thereby making a science of what would otherwise be "opinion." But, they do not constitute the whole of reliability engineering. Far from it. One would not expect a mathematician to design an aircraft. Likewise, one should not expect a statistician to design a reliable product.

b. *Probability*. Probability had its beginnings in gambling. Whether playing cards or throwing dice, a player has always wanted to increase his or her chances of winning. In any game of chance, a certain level of uncertainty exists, often indicated by the odds. The higher the odds, the higher the degree of uncertainty.

(1) The odds that a toss of an honest coin will be heads or tails (ignoring the extremely unlikely event of the coin landing on its edge) are 1 in 2, or 50%. In the language of probability, we can say that the probability of tossing a head is 0.5, as is the probability of tossing a tail. Now it is possible to toss 2, 3, or even more heads in a row with an honest coin. In the long run, however, we would expect to toss 50% heads and 50% tails.

(2) The reason that the probability of tossing a head or a tail is 0.5 is that there is no reason that either outcome should be favored. Thus, we say that the outcome of the coin toss is random, and each possible outcome, in the case of a coin there are two, is equally likely to occur.

(3) A coin toss is perhaps the simplest example that can be used to describe probability. Consider another gambling object – the die. Rolling an honest die can result in one of six random events: 1, 2, 3, 4, 5, or 6. The result of any single roll of the die or toss of a coin is called a *random variable*. Since both a coin and a die have a limited number of outcomes, we say that the outcome is a *discrete random variable*. If we call x the value of this discrete random variable for a roll of the die or toss of a coin, then the probability, or likelihood, of x is f(x). That is, the probability is a function. For the coin, f(heads) = f(tails) = 0.5. For the die, f(1) = f(2) = f(3) = f(4) = f(5) = f(6) = 1/6 = 0.167, or 16.7%.

(4) More complicated examples can, of course, be given of calculating probability in gambling. Take, for example, an honest deck of 52 cards. The probability of drawing any given card, the ace of spades, for example, is 1 in 52, or 1.92%. To calculate the probability of drawing another ace, given that we drew an ace of spades the first time requires some thought. If we have drawn an ace of spades, only three aces remain and only 51 cards. Therefore, the probability of drawing another ace of any suit (except for spades, of course) is 3 in 51, or 5.88%. The probability of drawing an ace of spades and one other ace is, therefore, 1.92% x 5.88% = 0.11%.

(5) For discrete random variables, such as the outcome of a coin toss or roll of a die, the random events have an underlying probability function. When there are an infinite number of possible outcomes, such as the height of a person, we say that the random variable is continuous. Continuous random variables have an underlying probability density function (pdf). A pdf familiar to many people is the Normal or Gaussian. It has the familiar bell-shaped curve as shown in figure 2-1. This distribution can be applicable even when some of the possible value can be negative as shown in the figure. The Normal distribution is symmetrical, with half of the possible values above the mean value and half below. For example, the average or mean height of an American male, a continuous random variable, tends to be Normally distributed, with half of the men taller than some mean (e.g., 5 feet-9 inches) and half shorter. Appendix A describes some of the pdfs most used in reliability calculations.

(6) The probability of an event is bounded – it can never be greater than 1 (absolute certainty) or less than 0 (absolute uncertainty). As we have seen, if one rolls a die, the probability of any possible outcome is 1/6. The sum of the probabilities of all possible outcomes (1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6) is 1. This is true for discrete and continuous random variables. For this reason, the area under the pdf for a continuous random variable is 1. One way of calculating the area under any curve is take the integral. So the integral of the pdf over the complete range of possible values is 1.

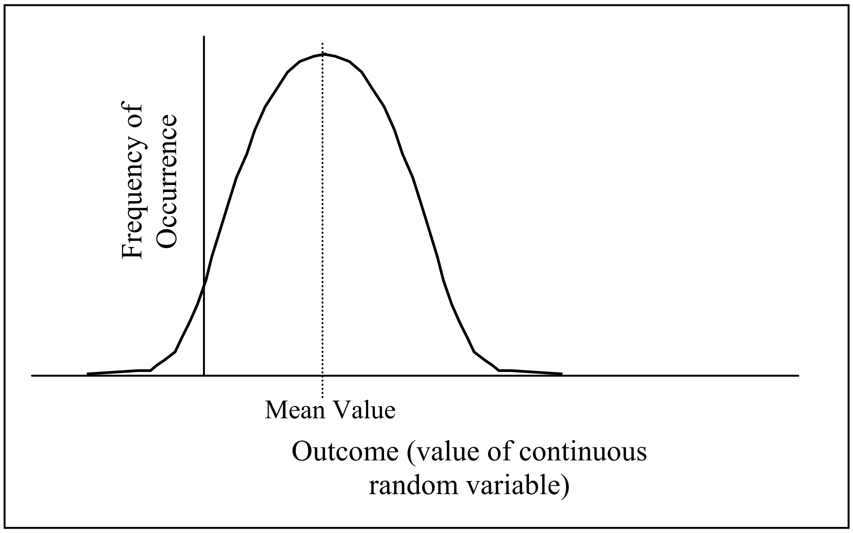

Figure 2-1. Graph of the normal or gaussian probability density function.

c. *Statistics*. One definition of statistics is "a numerical characteristic of a sample population." If the sample population is all males in America, then one statistic, or numerical characteristic of that population, is the average or mean height, assuming that the height is Normally distributed. So the parameters of a population from which we might draw a sample are called statistics. Statistics include means, averages, medians, modes, and standard deviations.

(1) Since we seldom can measure an entire sample population, we can never be absolutely sure of the probability distribution. Hence, we draw a sample from the population. We do this for many purposes, and examples include exit polls during an election and opinion polls. On the basis of the sample, we attempt to determine the most likely probability distribution of the population from which the sample was drawn, and the numerical characteristics of the population. Paragraph 2-4 will discuss sampling in more detail.

(2) Probability and statistics are used to measure reliability. Hence, we can talk about the probability of an item failing over a given time under stated conditions. Or we can talk about mean life or mean time to (or between) failures. Chapter 3 will discuss the various measures of reliability and how they are determined.

2-3. Reliability

Having some background on probability and statistics, we can now discuss reliability in more detail than was given in chapter 1.

a. *Mission success probability.* Reliability is defined as the probability that an item will operate for some period of time within some limits of performance. Reliability is then expressed as a decimal fraction of the number of times that the item will operate for the full mission time. Like the mean for a normally distributed population which states that 0.50 of the population are more than or less than this mean value, this reliability value expresses the decimal fraction of a population of equipment that could be expected to operate for the full mission time. The actual operating time for a single item within a system can be greater or less than the mission time. The reliability value only expresses the probability of completing the mission. To arrive at this figure, however, the basic underlying probability distribution is needed. When the underlying probability distribution is the exponential distribution, reliability is equal to e (the base of natural logarithms) raised to the negative power of the failure rate times the time, or $R(t) = e^{-\lambda t}$, where λ is the failure rate.

b. *MTBF.* Earlier we looked at the probability distribution of the height of a large group of American males. The assumed distribution was the normal distribution and the average height was the mean or expected value. If we had considered the operating times to failure of a population of equipment, instead of the height of men, and if these times were normally distributed, then the expected value of the time to failure of a single equipment would have been the mean of the times to failure, or Mean Time to Failure (MTTF). If the equipment were reparable and we had considered the operating times between failures of a population of equipment, then the expected value of the time between repaired failures would have been this mean, commonly described as Mean Time Between Failure, MTBF. Thus, reliability can be defined in terms of the average or mean time a device or item will operate without failure, or the average time between failures for a reparable item. For the exponential distribution, MTBF or MTTF is equal to the inverse of the failure rate, λ.

(1) Note that, like the average height of males, the MTBF of a particular system is an average and that it is very unlikely that the actual time between any two failures will exactly equal the MTBF. Thus, for example, if a UHF receiver has an MTBF of 100 hours, we can expect that 50% of the time the receiver will fail at or before this time and that 50% of the time it will fail after this time (assuming a Normal distribution).

(2) Over a very long period of time or for a very large number of receivers, the times between failures will average out to the MTBF. It is extremely important to realize that an MTBF is neither a minimum value nor a simple arithmetic average.

2-4. Sampling and estimation

If we could measure the height of every male in America, we would know the exact mean height and the amount of variation in height among males (indicated by the "spread" of the Normal curve). Likewise, if we could observe how long a population of non-repairable valves, for example, operated before failing, we would know the exact mean time to failure, could determine the exact underlying pdf of times to failure, and could calculate the probability of the valves failing before a certain time. We seldom have the luxury of measuring an entire population or waiting until an entire population of parts has failed to make a measurement. Most of the time, we want to estimate a statistic of the population based on a sample.

a. *Unbiased sample.* When taking a sample, it would be possible to skew the results one way or the other, purposely or unintentionally. For example, when taking an opinion poll to determine what percentage of Americans are Republicans, you could take a poll of those leaving the Republican convention. Obviously, such a sample would be biased and not representative of the American population. You must have an unbiased sample. The same principle holds when trying to assess the reliability of a population of valves, for example, based on a sample of the population of valves.

b. *Estimating a statistic*. Once we have an unbiased sample, we can estimate a population statistic based on the sample. For example, we can select a sample of 1,000 valves, test them to failure, determine the underlying distribution of times to failure, and then calculate the reliability as the mean life of the sample. We then use this value of mean life as an estimate of the mean life of the population of valves. Again, we are assuming that the sample is representative of the population. The process of estimating the reliability of an item is usually called prediction and will be addressed in chapter 3.