APPENDIX C

POINT ESTIMATES AND CONFIDENCE BOUNDS

C-1. Introduction to point estimates and confidence bounds

Predicting reliability and maintainability is necessary because both characteristics are very difficult to measure, especially early in a development program. Some experts even deny that they can be measured at all. Without going into the philosophical and mathematical theories on which these experts base this denial, suffice it to say that predictions continue to be used in most every product development program.

a. What is a prediction? Mathematically speaking, predictions are *estimates* of the true values of the parameters of any probability distribution. R&M, at least in part, are probability concepts. Reliability can be stated as the probability that a product will operate successfully under given conditions. Maintainability can be stated as the probability of completing a repair within a certain time under given conditions. If we observed the results of operating an entire population of a product in a given year, we could divide the number of failures by total attempted operations and determine the reliability of the product in that year *after the fact*. This "measurement" is not made to try and tell us what will happen next year or what would have happened in the year in question had only half the products been operated - it is not a prediction. Probability, on the other hand, is that branch of mathematics that allows statements to be made about a population based only on a sample of the population or when we try to predict *before the fact* the outcome of events. These predictions are estimations of the "truth."

b. Types of estimates. Two kinds of estimation can be made: point estimates and interval estimates.

(1) *Point estimates*. A point estimate is, as implied by the title, a single number that is an estimate of the distribution parameter in which we are interested. For example, on the basis of analysis or test, we might estimate the reliability to be 95%. How much confidence do we have in the estimate? It depends on the information used to make the estimate. If we used only analytical methods of arriving at the estimate (i.e., no actual testing performed), it would depend on how much the new product resembled the old product(s) from which the data used in the analysis were derived. If we had one test that was the basis for the estimate, we could be 50% sure that the true reliability is higher or lower than our estimate. Finally, if we test 100 products (out a larger total population), it seems intuitive that we could be more confident in our estimate than when we tested only one product.

(a) As our sample size increases, we can make estimates in which we can have a higher confidence. Unfortunately, the subject of confidence is a bit more complex than that. For example, take the case where we have a sample of 100 products to test. Suppose that 10 failures occur. We could estimate the reliability to be 10/100 or 90%. We could also estimate it as 85% or 75%. How can that be? Well, the 90% figure is a point estimate in which we can be 50% confident. If we want to be more confident, say 95% confident that the true value is equal to or higher than the estimate, *our estimate must be more conservative*.

(b) The reader might wonder what is wrong with just using a point estimate. Nothing is "wrong" with using a point estimate. But a point estimate isn't discriminating; it tells us nothing about the risk involved with the estimate. And there is always the risk that our estimate of reliability is optimistic, i.e., too high (customers don't care, at least in theory, if it's too low, i.e., conservative). Consider the example estimates in table C-1. From the table, one can understand why a point estimate is not discriminating! Most people would more readily accept the point estimate made using 1000 products than that made with only 10.

Table C-1. Point estimates for different sample sizes

Size of Sample (or number of tests)	Number of Failures	Point Estimate of Reliability
10	1	90%
100	10	90%
1000	100	90%

(2) *Interval estimates*. An interval estimate is one in which we calculate a range of values and can state the probability of the true value of the parameter being estimated being contained in the interval. The lower and upper values of the interval are called lower and upper confidence limits, respectively. The confidence level is the probability that the range or interval of values actually includes the true value of reliability. A confidence bound can be two-sided or one-sided. A two-sided bound can be compared to a tolerance. Most of us are familiar with a measurement that is stated, for example, as 12 feet ± 0.01 feet. A two-sided bound on a reliability estimate of 95% might be ±2.1%, at 95 confidence. In other words, we are 95% confident that the interval of 92.9% to 97.1% includes the true reliability. We may, however, only be interested in the lower limit. A one-sided confidence bound would be stated as, for example, "we are 95% confident that the true reliability is greater than 90%." In this case, we are not worried about how much higher it may be. If we are trying to determine if a contractor has met (or exceeded) the reliability requirement, the one-sided confidence bound is sufficient. If we want to plan a spares buy based on the reliability of the product, the two-sided bound should be used.

c. Estimation and confidence are topics filling entire chapters of textbooks. The discussion herein is necessarily simplified and abbreviated. For a more rigorous and mathematically accurate treatment of estimation and confidence, the reader is directed to "Practical Statistical Analysis for the Reliability Engineer (SOAR-2)," The Reliability Analysis Center, Kieron A. Dey, 1983. (Available from the Reliability Analysis Center: 1-800-526-4802), or "Methods for Statistical Analysis of Reliability and Life Test Data," Nancy R. Mann, Ray E. Schafer, and Nozer D. Singpurwalla, John Wiley and Sons, Inc., Somerset, NJ, 1974, or any good text on probability and statistics.