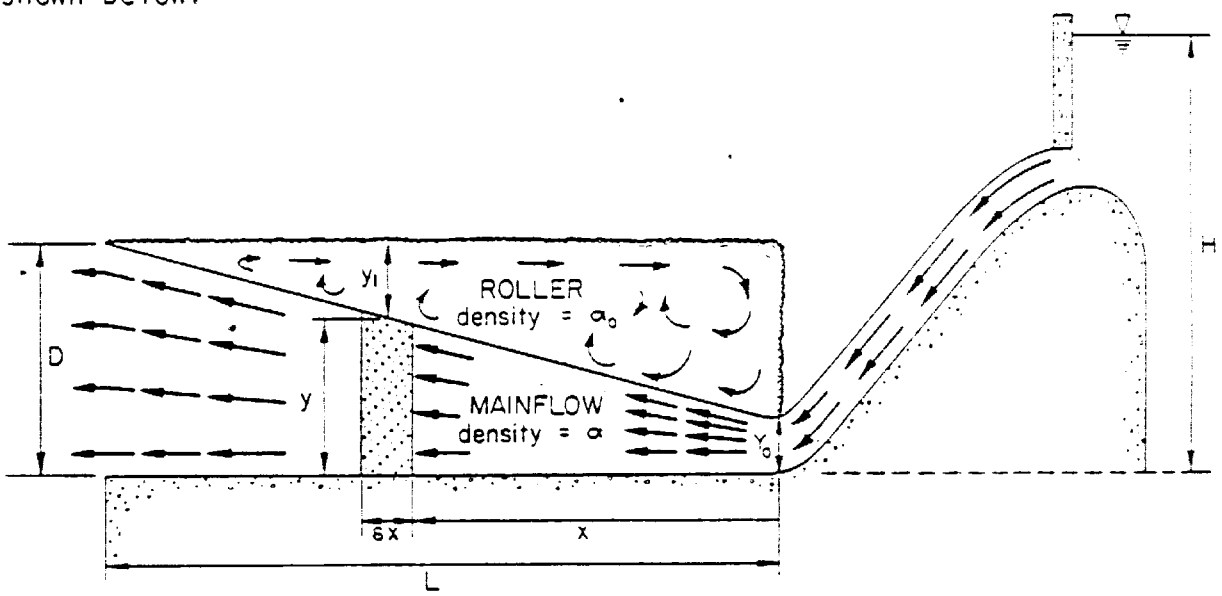
DERIVATION OF THE SPILLWAY-STILLING BASIN MODEL*

Consider the conceptual representation of the stilling basin shown below.



CONCEPTUAL REPRESENTATION OF SPILLWAY-STILLING BASIN COMBINATION

The water parcel indicated in cross-section by the shaded area moves through the stilling basin, decelerating and increasing in height. It extends laterally the full effective width, ω of the stilling basin as illustrated in Figure 3 of the main report.

We now make the following assumptions for the water parcel and stilling basin:

1. For that length of spillway that is in operation at a given time, the discharge is uniform along the

*Taken from: "A Nitrogen Gas (N_2) Model for the Lower Columbia River, "Final Report, Water Resources Engineers, Inc., under contract to US Army Corps of Engineers, North Pacific Division, January 1971

crest (this is equivalent to assuming that the properties of the water parcel are constant along any line parallel to the spillway crest).

2. The value Y_o is the initial depth of the spill before the jump. It is computed as:

$$Y_o = \frac{q}{V_o} = \frac{q}{\sqrt{2gH}} \qquad \text{(A-1)}$$

where

q = discharge per foot along the crest

H = total reservoir head above the stilling basin floor.

3. The only effect of the roller which overlies the main flow is to increase the static pressure within the water parcel by an amount $\alpha_o y_1$.

4. A given mass of air M_1 is entrained as discrete bubbles into the water parcel at the point $x = 0$ and remains uniformly distributed within the water parcel as it passes through the stilling basin.

5. The distribution of the mass of air among the various bubble sizes remains unchanged during the water parcel's journey through the stilling basin.

6. The dissolved nitrogen within the water parcel is uniformly distributed.

7. Rate of nitrogen dissolution $\frac{dM}{dt}$ in the water parcel is governed by Fickian diffusion as:

$$\frac{dM}{dt} = K_L A (C_E - C) \qquad \text{(A-2)}$$

where

M = the mass of dissolved nitrogen in the water parcel,

K_L = rate coefficient,

A = total surface area of the air bubbles contained in the water parcel,

C_E = effective saturation concentration of dissolved nitrogen in the water parcel, and

C = actual concentration of dissolved nitrogen in the water parcel.

With these assumptions, we can now define the parameters M, A, and C_E in equation A-2 as functions of the location of the water parcel in the stilling basin.

Assumption 6 allows us to write the mass M as the product of the concentration C and the volume of the water parcel,

$$M = (wy\delta x)C \tag{A-3}$$

where w is the effective width of the stilling basin, i.e., w = (number of gates open) x (width per gate).

The saturation concentration of a gas such as N_2 or O_2 that is only slightly soluble in water is governed by Henry's Law which states that the equilibrium or saturation concentration of the gas in solution is directly proportional to the pressure existing at the gas-liquid interface. In the water parcel the pressure P at an elevation z above the stilling basin floor is

$$P = P_o + \alpha_o y_1 + \alpha(y-z) \tag{A-4}$$

where P_o is the atmospheric (or barometric) pressure, and the α parameters are the densities of the roller and main flow as shown in Figure A-1. Hence, the saturation concentration at any elevation z in the parcel is given as:

$$C_{sat} = [P_o + \alpha_o y_1 + \alpha(y-z)]C^* \qquad \text{(A-5)}$$

where C^* is the saturation concentration under one atmosphere of pressure. In equation A-5, the pressure term has units of atmospheres of pressure. From equation A-5, it is seen that C_{sat} varies linearly with z. It follows that the average or *effective* saturation concentration, C_E in the water parcel is the value of C_{sat} at mid-depth, or at $z = y/2$: Thus,

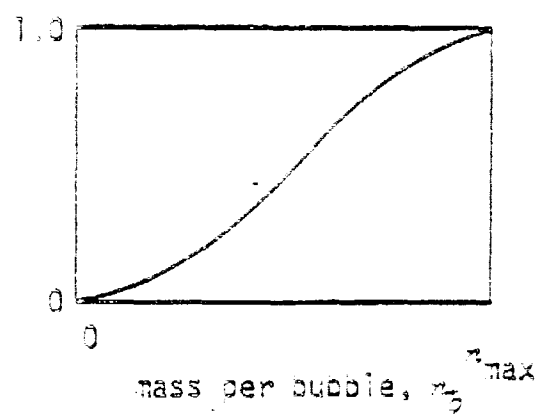
$$C_E = [P_o + \alpha_o y_1 + \alpha(y/2)]C^* \qquad \text{(A-6)}$$

Noting that $y_1 = D-y$ gives the final form of C_E as

$$C_E = [P_o + \alpha_o D - (\alpha_o - \frac{\alpha}{2})y]C^* \qquad \text{(A-7)}$$

The total surface area A of the air bubbles in the water parcel depends upon the total mass of air entrained and, upon the bubble size distribution. It is not unreasonable to expect that the entrained mass of air will be distributed among the various bubble sizes in a manner similar to that shown below.

β = fraction of total air mass in the water parcel with bubbles having a mass less than or equal to m_b



The volume V_b of an air bubble with mass m_b can be found from the ideal gas law:

$$V_b = \frac{mRT}{P} \tag{A-8}$$

where

m = number of moles of air in the bubble,

R = universal gas constant,

T = absolute temperature, and

P = the total pressure in the bubble.

In equation A-8, m can be replaced by $n_b/28.9$ where 28.9 is the molecular weight of air. The diameter d_b and the area A_b of a sphere are given by:

$$d_b = \left(\frac{6}{\pi} V_b\right)^{1/3} \tag{A-9a}$$

$$A_b = \pi d_b^{\,2} \tag{A-9b}$$

Now, combining equations A-8 and A-9, the following expression results for the surface area A_b of an air bubble with mass n_b:

$$A_b = \left(\frac{6\sqrt{\pi}RT}{28.9}\right)^{2/3} \left(\frac{n_b}{P}\right)^{2/3} \tag{A-10}$$

Thus, if the total air mass entrained per unit volume of water at Y_o is M_A, the total air bubble surface areas A', per unit volume of water is found from the bubble size distribution and equation A-10 as

$$A' = \int_0^{n_{max}} A_b \frac{M_A \frac{dB}{dn_b} dn_b}{n_b} \tag{A-11}$$

or

$$A' = \left(\frac{6\sqrt{\pi}RT}{28.9P}\right)^{2/3} M_A \int_0^1 n_b^{-1/3} dB \tag{A-12}$$

Finally, to get the total bubble surface area in the water parcel it is necessary to integrate equation A-12 over the volume of the parcel $wy\delta x$, i.e.,

$$A = \int_{\delta x}\int_{w}\int_{z} A' \, dx \, dw \, dz \qquad \text{(A-13)}$$

Applying assumptions 4 and 5 and substituting for A' from equation A-12 gives

$$A = w\delta x \left(\frac{6\sqrt{\pi R m}}{28.9}\right)^{2/3} M_A \int_0^1 n_b^{-1/3} dB \int_{z=0}^{y} \frac{dz}{P^{2/3}} \qquad \text{(A-14)}$$

Replacing P with equation A-4 and integrating,

$$A = 3\left(\frac{6\sqrt{\pi R m}}{28.9}\right)^{2/3} M_A \int_0^1 n_b^{-1/3} dB \; (w\delta x) \; \frac{\left(P_o + \alpha_o y_1 + \alpha y\right)^{1/3} - \left(P_o + \alpha_o y_1\right)^{1/3}}{\alpha} \qquad \text{(A-15)}$$

Substituting $(D-y)$ for y_1 gives the final form as

$$A = K_A \; (w\delta x) \left\{ \left[P_o + \alpha_o D + (\alpha - \alpha_o) y\right]^{1/3} - \left[P_o + \alpha_o D - \alpha_o y\right]^{1/3} \right\} \qquad \text{(A-16)}$$

where

$$K_A = \frac{3}{\alpha} \left(\frac{6\sqrt{\pi R m}}{28.9}\right)^{2/3} M_A \int_0^1 n_b^{-1/3} dB$$

If the expressions for M, C_y, and A from equation A-3, A-7 and A-15 respectively are substituted into the rate expressions given in equation A-2, there results

$$(y w \delta x)\frac{dC}{dt} = (w \delta x)\ K_L K_A \left\{ [P_o + \alpha_o D + (\alpha - \alpha_o) y]^{1/3} - [P_o + \alpha_o D - \alpha_o y]^{1/3} \right\}$$
$$\left\{ [P_o + \alpha_o D - (\alpha_o - \frac{\alpha}{2}) y] C^* - C \right\} \qquad \text{(A-17)}$$

We can now write rate expression $\frac{dC}{dt}$ in terms of the location in the stilling basin by using the relationship

$$\frac{dC}{dt} = \frac{dx}{dt}\ \frac{dC}{dx} = v\ \frac{dC}{dx} = \frac{q}{y}\ \frac{dC}{dx} \qquad \text{(A-18)}$$

where v is the velocity of the parcel and q is the discharge per unit width of the stilling basin. In addition, we define a system parameter K, which we will call the *entrainment coefficient*, as

$$K = K_L K_A = \frac{3}{\alpha}\left(\frac{5\sqrt{\pi}R}{28.9}\right)^{2/3} \left[T^{2/3} K_L\ M_A \int_0^1 n_b^{-1/3} dB\right] \qquad \text{(A-19)}$$

Substituting equation A-18 and A-19 into A-17 gives the expression for the concentration change in the water parcel as

$$\frac{dC}{dx} = \frac{K}{q} \left\{ [P_o + \alpha_o D + (\alpha - \alpha_o) y]^{1/3} - [P_o + \alpha_o D - \alpha_o y]^{1/3} \right\}$$
$$\left\{ [P_o + \alpha_o D - (\alpha_o - \frac{\alpha}{2} y)] C^* - C \right\} \qquad \text{(A-20)}$$

The solution is obtained as follows. Evaluate the pressure terms at the midpoint of the stilling basin $y = \frac{D+Y_o}{2}$ to obtain

$$\frac{dC}{dx} = \frac{K}{q} \left\{ [\overline{P} + \frac{\alpha}{4}(D + Y_o)]^{1/3} - [\overline{P} - \frac{\alpha}{4}(D + Y_o)]^{1/3} \right\} \left\{ \overline{P} C^* - C \right\} \qquad \text{(A-21)}$$

where $$\overline{P} = P_o + \frac{\alpha_o}{2}(D - Y_o) + \frac{\alpha}{4}(D + Y_o)$$

Now let

$$\overline{\Delta P^{1/3}} = [\overline{P} + \frac{\alpha}{4}(D + Y_o)]^{1/3} - [\overline{P} - \frac{\alpha}{4}(D + Y_o)]^{1/3} \qquad \text{(A-22)}$$

Rewriting equation A-21, with these substitutions gives

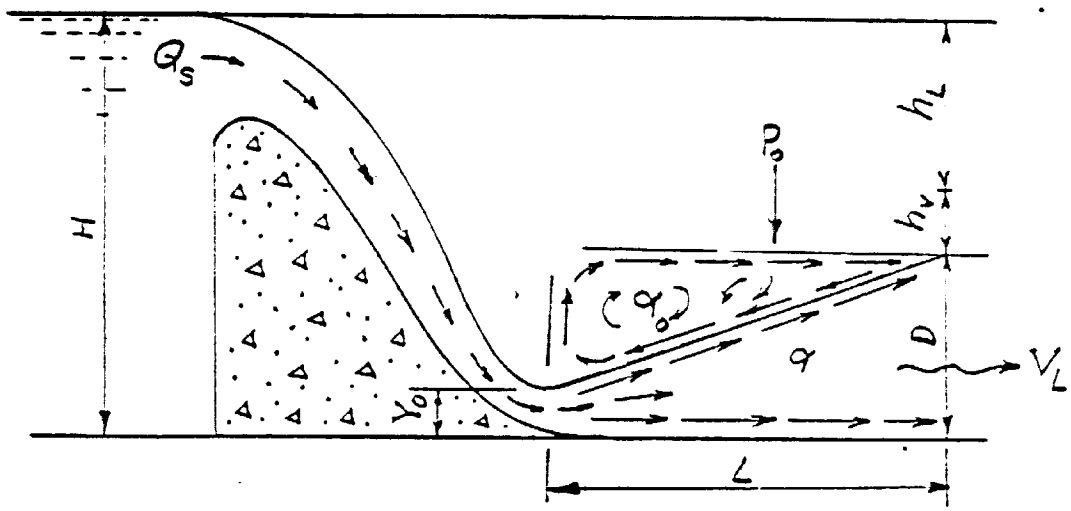
$$dC + C\frac{K}{q}\overline{\Delta P^{1/3}}\,dx = \frac{K}{q}\overline{\Delta P^{1/3}}\,\overline{P}C^*\,dx \qquad \text{(A-23)}$$

which has the solution

$$C = \overline{P}C^* + ke^{-\frac{K}{q}\overline{\Delta P^{1/3}}\,x} \qquad \text{(A-24)}$$

Evaluating equation A-24 at $x = 0$, where C equals the forebay concentration C_F, and at $x = L$ where C equals the stilling basin concentration C_S, yields the spillway-stilling basin model as

$$C_S = \overline{P}C^* - (\overline{P}C^* - C_F)\,e^{-\frac{K}{q}L\overline{\Delta P^{1/3}}} \qquad \text{(A-25)}$$



DEFINITION SKETCH

RESIDENCE TIME = $t_R = WDL/Q_s = DL/q \approx L/V_L$

TOTAL HEAD LOSS = $h_L = H-D-h_v = H-(D+v_L^2/2g)$

ENERGY LOSS RATE = $E = h_L/t_R$

AVE. PRESSURE = $\overline{P} = P_o + \frac{\alpha_o}{2}(D-Y_o) + \frac{\alpha}{4}(D+Y_o)$. $\alpha = 0.0295$ atm./ft. $\alpha_o = \alpha c$

N_2 CONCENTRATION AT END OF STILLING BASIN, L

$$C_s = \overline{P}C^* - (\overline{P}C^* - C_F)\exp\left(-\frac{K}{q} L \overline{\Delta P}^{1/3}\right)$$

$$\overline{\Delta P}^{1/3} = \left[\overline{P} + \frac{\alpha}{4}(D+Y_o)\right]^{1/3} - \left[\overline{P} - \frac{\alpha}{4}(D+Y_o)\right]^{1/3}$$

$$K = \frac{q}{L\,\overline{\Delta P}^{1/3}} \ln\left(\frac{\overline{P}C^* - C_f}{\overline{P}C^* - C_s}\right) = K_{20}(1.028)^{(T-20)}$$ T = Water Temperature.

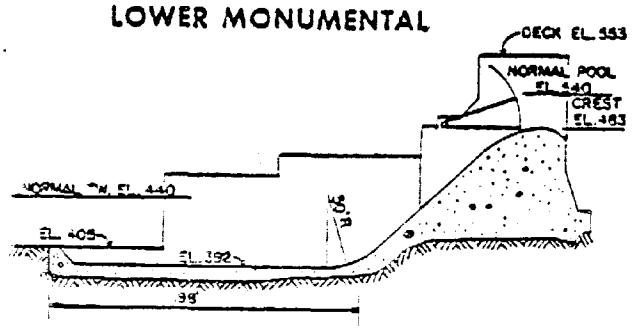
$K_{20} = aE^b$. a & b are empirically determined from observed data. They are shown below:

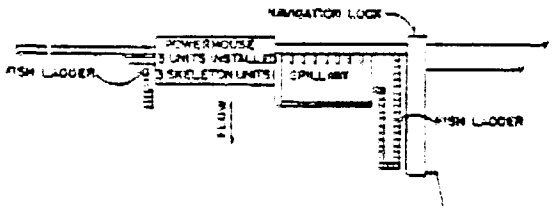
MODEL COEFFICIENTS

PROJECT	c	a	b
Little Goose	1.00	0.09	2.45
Lower Monumental	1.00	0.09	2.45
Ice Harbor	1.00	0.30	1.00
McNary	1.00	1.00	2.00
John Day	1.00	0.20	2.10
The Dalles	0.50	0.80	2.50
Bonneville	1.00	1.90	1.00

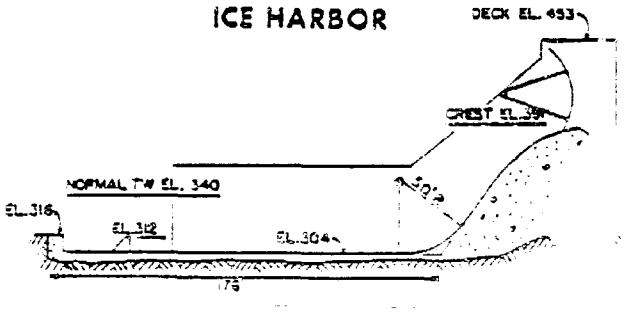
1/ Developed by Water Resources Engineers, Inc. for the Corps in 1971.

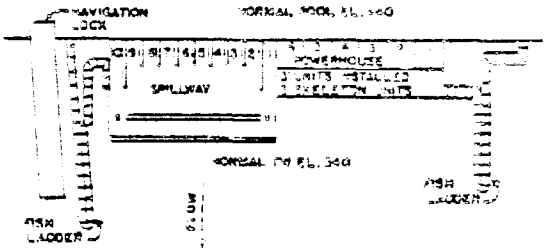
LOWER MONUMENTAL



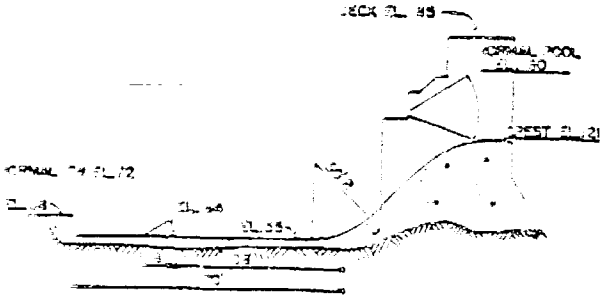


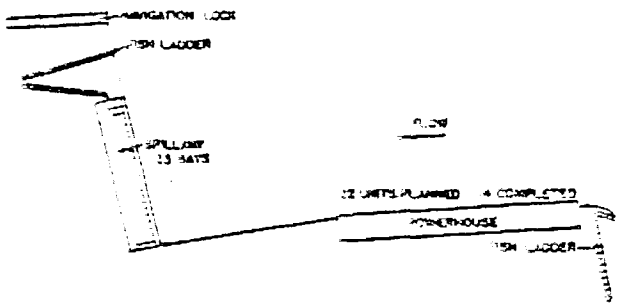
ICE HARBOR



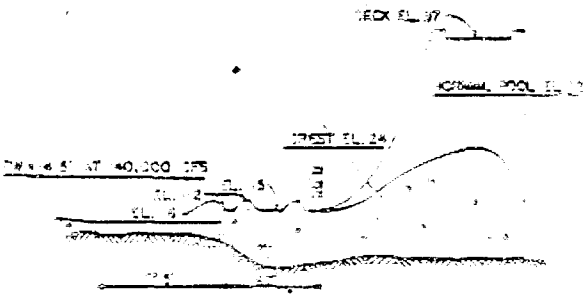


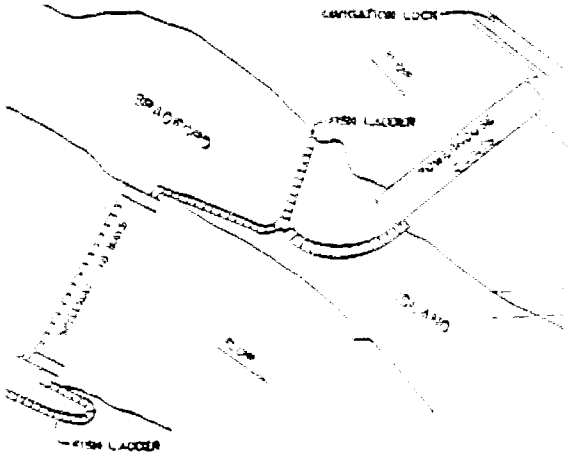
THE DALLES



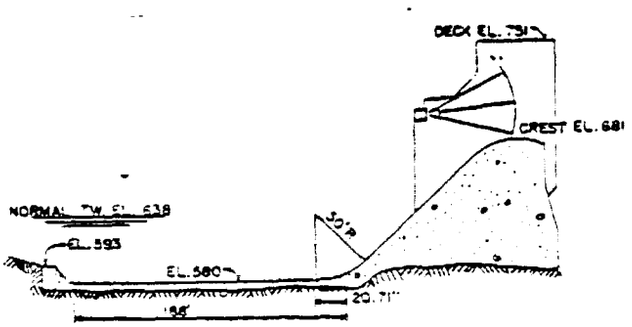


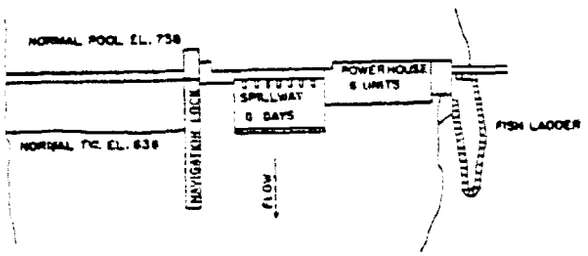
BONNEVILLE



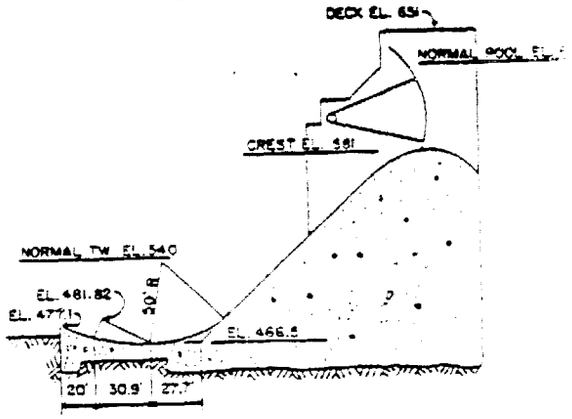


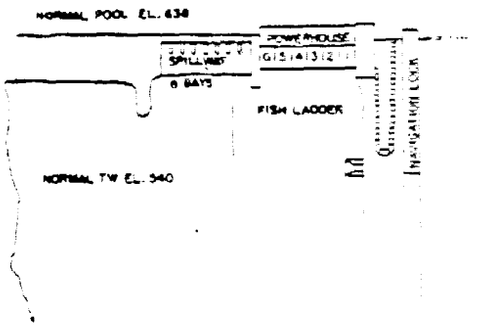
LOWER GRANITE



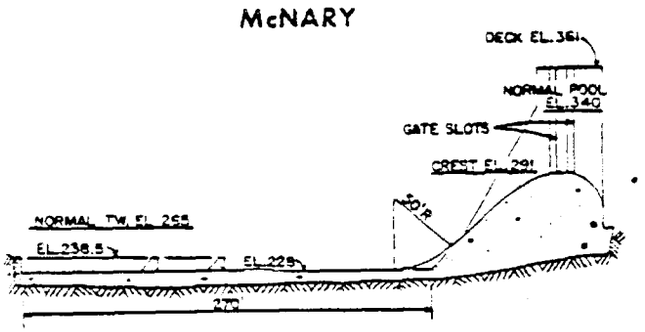


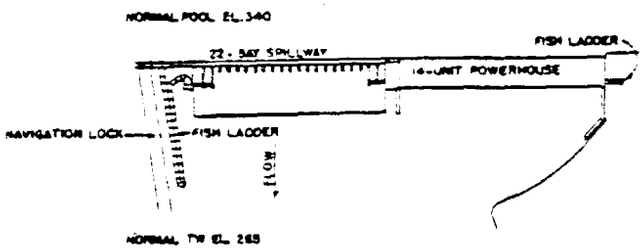
LITTLE GOOSE



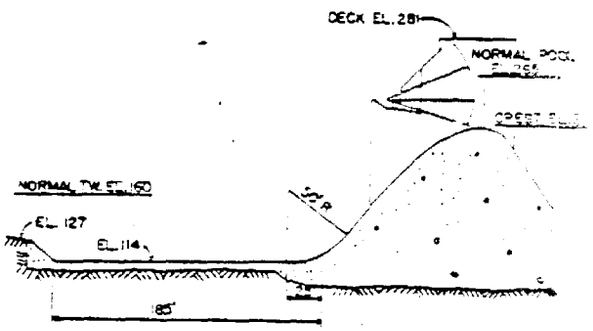


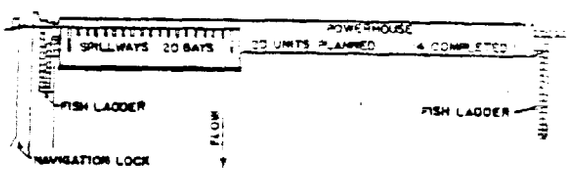
McNARY





JOHN DAY





PREDICTION OF DISSOLVED GAS AT HYDRAULIC STRUCTURES[1/]

by Perry L. Johnson[2/] and
Danny L. King[3/]

Introduction

With the increased interest in the effects of hydraulic structures on the dissolved gas concentration of the flow, it becomes desirable to be able to predict how particular structures operating under specific conditions will change the dissolved gas concentration.

At existing structures a predictive ability would enable the facility operator to select the method of release that would have the most desirable effect on the dissolved gas concentration of the flow. Prototype data indicate that the change in the dissolved gas concentration is dependent on the type of structure through which the flow passes, the magnitude of the discharge, the barometric pressure, and the water temperature. To establish an operating criteria for each structure based on actual measurement of resulting dissolved gas concentrations would be a difficult task. A predictive ability could yield an understanding of a structure's potential and allow preparation for the possible consequences, even if the structure had never operated.

Also, with a predictive ability designers would have an additional factor which could be considered in structure selection. Depending on the situation, it is conceivable that the dissolved gas potential might even control the design. Planners could also use a predictive ability to evaluate the potential effects of a single hydraulic structure, or a series of hydraulic structures, on a river.

Initially, the dissolved gas concentration above the structure (both oxygen and nitrogen) is equal to the concentration established by the inflowing stream. The nitrogen, being relatively inert, will maintain this concentration for quite some time. The oxygen, however, especially in the lower depths of a resevoir, may be depleted from the decaying of organit material. Thus, if water is released it may be low in dissolved oxygen and yet may conceivably be high in dissolved nitrogen. Furthermore, the water may be high in biochemical oxygen demand (BOD) which would reduce the dissolved oxygen concentration in the stream below the dam. Therefore, the analysis should be able to evaluate how effectively structures increase depleted gas concentrations as well as evaluate whether supersaturated conditions might be created.

Such predictive methods have been developed for the spillways of the U.S. Army Corps of Engineers dams on the Columbia River (1). Most of these structures are geometrically similar. They are low head, run-of-the-river structures, with gate-controlled ogee spillways. The stilling basins are also of similar design. This similarity enabled the development of a predictive analysis that is quite satisfactory for the structures considered. The Bureau of Reclamation has few structures that correspond to these Columbia River dams. In general, Bureau structures vary widely in type and size. Thus, a much more generalized predictive analysis is required for significant application.

1/ Reprinted with the permission of the authors
2/ Hydraulic Engineer, Bureau of Reclamation, Denver, Colorado
3/ Chief, Hydraulics Branch, Bureau of Reclamation, Denver, Colorado

As a basis for development of the analysis, the following data were collected:

1. Reservoir water temperature, dissolved oxygen concentration, and dissolved nitrogen concentration at the elevation from which the water is withdrawn
2. Discharge and a record of which gates or valves are operating if releases are being controlled
3. Tailwater elevation, temperature, and dissolved oxygen and nitrogen concentrations in the tailrace
4. Local barometric pressure
5. Photographs of the structure operating and dimensioned drawings of the structure's configuration

By fall of 1973 the monitoring program of the Bureau's Engineering and Research Center had reached 16 sites and had observed 24 structures in operation. Forty-nine different operating conditions had been studied. In addition the Pacific Northwest Region of the Bureau of Reclamation has closely studied Grand Coulee Dam and made observations at 36 other sites. The Upper Missouri Region of the Bureau has performed monitoring at Yellowtail Afterbay Dam. Combined, these data provided an adequate base from which the predictive analysis could be developed.

Analysis

The process of gas transfer is described by the equation:

$$C(t) = C_s - (C_s - C_i)\, e^{-Kt} \qquad (1)$$

where $C(t)$ = final dissolved gas concentration
C_s = saturation concentration
C_i = initial concentration
K = a constant of proportionality
t = time

$C(t)$, C_s, and C_i are concentrations in mg/L of water.

Equation 1 shows that the final dissolved gas concentration, $C(t)$, below a hydraulic structure is dependent on the initial concentration, C_i, in the reservoir, the saturation concentration, C_s, in the stilling basin, the length of time, t, that gas is being dissolved into the flow, and a constant that would be expected to vary with the specific hydraulic structure and operating condition. C_i will be either set at a known level or assumed. The other three parameters (C_s, t, and K) are dependent on the type of stucture, operating condition, temperature, and barometric pressure. Efforts were directed at evaluating C_s, t, and K computationally.

The saturation concentration level, C_s, in the stilling basin, is dependent on the pressure that can be developed in the basin and the water temperature. The pressure obtained in a stilling basin is dependent on the depth of water over the flow in which the bubbles are entrained and the barometric pressure. Thus, surface water at sea level will hold 33 percent more gas than surface water at an elevation of 8000 ft (2438 m). Also, water at the surface of a pool will hold 60 percent less gas than water at a depth of 34 ft (10.4 m). Barometric pressure is basically controlled by the elevation at which the

structure is located, with daily fluctuations that result from atmospheric conditions. The effects caused by daily fluctuations in atmospheric pressure are not large but they may be significant and should be considered in the evaluation of C_s. In this analysis measured barometric pressures were used when available. If measured values were not available a standard atmosphere was assumed and barometric pressures were computed according to elevation.

The depth of water over the flow in which gas is being dissolved is generally dependent on the depth of water in the stilling basin. Thus, variations in the tailwater elevation will have some effect. Throughout this analysis a water depth equal to two-thirds of the basin depth was used to compute saturation concentrations. It was thought that initially the fairly compact jet from a spillway or outlet would penetrate to the floor of the stilling basin. The flow would then be deflected downstream and out of the basin. As the flow moved through the basin it would be diffused and its velocity reduced. This diffusion would be linear and result in a triangular pattern with the average depth through the diffusion being two-thirds of the total basin depth. Bubbles rising from the flow and incomplete flow penetration would tend to reduce this average depth, but the two-thirds depth was considered representative and therefore used in the analysis. A major point of support for the two-thirds depth assumption is the fact that later applications proved the assumption reasonable. If the flow being studied does not penetrate to the bottom of the pool the maximum depth of flow penetration may be used in this calculation in place of the basin depth.

Evaluation of C_s is achieved by summing the barometric pressure and two-thirds of the basin depth (expressed in mm of Hg) and dividing this total pressure by standard atmospheric pressure (760 mm of Hg) to obtain the average absolute pressure on the dissolving bubbles in terms of atmospheres. This average absolute pressure is then multiplied by the dissolved gas saturation concentration at sea level, for the desired water temperature, to obtain C_s.

The next parameter from equation 1 to be considered is the time, t. It is representative of the length of time that the inflowing jet with entrained air is under pressure in the stilling basin and, thus, the length of time that gas is being dissolved in the flow. Consideration of time revealed two possible limitations that could control its value. First, it would seem that given sufficient time the entrained air bubbles would rise out of the flow and end the dissolving of gas. In some cases it would seem that an evaluation of this bubble rise time could be used to represent time. On the other hand, situations might occur where the flow with entrained air would pass through the basin and be deflected to a shallow depth in a fairly short time. Therefore, the actual length of time required for the flow to pass through the basin could represent t. During this analysis the assumption was made that either of these time periods might be critical in specific situations. For each flow condition and structure studied, t was evaluated for both limitations. The smaller of the two computed values was considered applicable to the particular situation and was used in the remainder of the analysis.

Bubble rise time. - Evaluation of t based on the bubble rise time, t_1, would be, if strictly pursued, a very complex computation which would probably produce questionable results. The vertical dimension of the jet (thickness of jet that the bubble would rise through) is never constant. The time, t, based on bubble rise time, t_1, was evaluated by dividing the calculated vertical

thickness of the jet at the tailwater surface by the terminal rise velocity the bubble. By trial and error, it was determined that an assumed 0.028-i (0.7-mm) diameter bubble with a theoretical terminal velocity of 0.696 ft/ (0.2 m/s) yielded the most consistent results with respect to observed pro. yo conditions. Also, when an analysis was developed that predicted K (equation 1 from two dimensionless parameters, it was found that the 0.028-inch-diameter bubble yielded predicted values of K that were consistent with the predicted values of K based on the basin retention time.

Basin retention time. - Computation of the flow retention time, t_2, in the basin is accomplished by dividing the path length of the flow by the average flow velocity along the path. The path length is generally controlled by the basin shape. The path length is the distance from the point at which the jet enters the tailwater pool to the point at which the majority of the flow is directed toward the surface and, therefore, into a lower pressure zone. If a large portion of the flow is deflected upward at a point by baffle piers, for example, this point would be considered the end of the path.

To compute the average flow velocity over the path length, the first step is to obtain the jet velocity at the tailwater surface (or at the start of the flow path) from the previous analysis of bubble rise time. To determine the average flow velocity, the velocity at the end of the path must be found. This is done through the use of figure 1 which is a summary of information from studies of jet diffusion by Yevdjevich (2) and Henry (3). Observation of velocity distributions in jet diffusions indicates that half of the maximum velocity would be an approximation of the jet's average velocity at the en the flow path. This average velocity might also be evaluated by dividing discharge, Q, by the channel cross sectional area, A, which would assume complete diffusion of the jet. The larger of the computed velocities should be used, since the average jet velocity at the end of the path could be higher, but not lower than the average velocity through the full cross secti The velocities at the beginning and end of the flow path are then averaged, then this average is divided into the flow path length to obtain the basin flow retention time (t_2). As previously stated, the value of t to be used in equation 1 is the smaller of the two computed values (t_1 or t_2).

The final term in equation 1 to be evaluated is K. K is unlike the other terms evaluated in that it is not directly representative of any specific physical parameter. K is a measure of the ability of a particular structure operating under a particular condition, to dissolve gas. It is representati of the degree of air entrainment and the rate at which the water at the gas-liquid interface is replenished.

It appears that K is dependent only on the hydraulic performance of the basin. Attempts to find a predictive procedure that could be used to evaluate K resulted in the curves shown in figure 2. To obtain these curves the prototype data were manipulated into various parameters until useable results were found. Only dissolved nitrogen data were used in the development due to the stability of nitrogen. At a few of the reservoirs data were collected at several depths. These data indicated that dissolved oxygen concentration vary widely through the depth of a reservoir but that dissolved nitrogen concentrations are fairly constant. At some other reservoirs dissolved gas data were collected only near the surface and not at the withdrawal elevation. Therefore, if dissolved nitrogen and oxygen concentrations are measured at the

reservoir surface and the withdrawals are made from deep in the reservoir, the measured values of the initial dissolved gas concentrations, C_I, are probably more accurate for nitrogen than oxygen. Even though dissolved nitrogen data were used as a base for the analysis, application of the analysis for observed prototype conditions indicates that resulting dissolved oxygen levels may also be predicted.

Figure 2 shows that the value of K is dependent on two parameters. The first is H_v/X, the velocity head, H_v, at the tailwater surface divided by the flow path length X. H_v/X is an energy gradient parameter for the flow; it relates the amount of energy in the flow to the path length in the basin over which the energy is dissipated. The greater the value of H_v/X the more turbulent the basin flow and the larger the resulting K value. The path length used corresponds to the value of t selected. If t_2 is applicable, then the value used for X would be the path length used to evaluate t_2. But if t_1 is applicable, the path length is adjusted to determine the effective path length for the time interval, that is, the length of time the bubbles remain in the jet. Flow deceleration is assumed linear and the ratio of t_1/t_2 is multiplied by the total velocity drop to determine the velocity drop along the adjusted path length. The average velocity along the adjusted path is then computed (initial velocity minus one-half the velocity drop) and multiplied by t_1 to determine the adjusted path length.

The other parameter on which the value of K is based is a ratio of the shear perimeter of the jet to the jet's cross-sectional area at the tailwater surface. This term is a measure of the jet compactness and shape. The shear perimeter for a jet is defined as the length of the jet's perimeter over which a shearing action is occurring between the jet and the water of the stilling basin pool. For a free jet plunging into a pool the shear perimeter would equal the total perimeter of the jet, while for a flow passing down a chute spillway and into a basin the shear perimeter would be the chute width at the tailwater surface. Situations exist where the walls of the stilling basin are offset from the jet entering the basin. If this offset is small, questions may arise as to whether the sides of the jet should be included in the shear perimeter. This is a judgment factor and is probably best handled by individual consideration. Another common structure that might raise a similar question would be a hollow jet valve discharging into a pool. Although the flow would have a ring-shaped cross-section, only the outside perimeter should be included in the evaluation. In general, if it appears that significant shear will occur along the section of perimeter in question then those lengths should be included in the analysis.

With the evaluation of K from figure 2, equation 1 may be applied and the final dissolved gas concentration, C(t), determined. The prototype data were used extensively to evaluate the coefficients that are applied throughout the analysis. This empirical approach is mandatory because of the complexity of the flows being considered. Very few of the situations studied have clearly defined flow conditions that are well suited for direct analysis. Not only are the jets that leave the spillway chutes, the valves, and the gates often quite complex, but the stilling basin pools are equally complex. Any analysis of these flow conditions would be quite involved and the accuracy would be questionable. However, the coefficients resulting from this analysis do have a rational basis and are representative of the various physical parameters. The coefficients can be interpreted to yield additional insight into the significance of the various factors.

Although some entrainment of air is needed for the dissolved gas uptake to occur, the amount of entrained air required seems to be quite small. At so of the prototype structures releases were exposed only briefly to the air. some of these cases the water surfaces of the releases were also relativel smooth. Thus, it is assumed that little air was entrained. This assumption was verified by the small quantities of air that were observed returning to the tailwater surface. However, in some instances, the structures with little apparent air entrainment were among the worst in creating supersaturated conditions.

Example Application

Included with the example is a drawing of the structure (figure 3) and photographs (figure 4) of operation. The computations are described step by step. All critical points and all judgments or approximations are discussed and the results of the analysis are compared to actual field findings. Results are also included for examples for which the calculations are not shown. Variations between the observed and calculated dissolved gas concentrations may be attributed to several factors. First, and probably one of the most important, is that the entire analysis was based on average prototype data. Therefore, some structures will fit the analysis better than others and some structures will yield more accurate predicted results. A second significant source of variation would be errors in measuring the prototype dissolved gas concentrations. The chemical analyses used are not completely accurate, but even more important, samples may be collected from regions that are not representative of the total flow. Extreme errors of this sort may or may not be obvious. several cases, two or more readings were available which gave some additio assurance. Variations due to errors in data collection may be small or they may be quite large. Application of the analysis and use of the graphs may also result in some error, but this error should be small. All factors considered, the results are very encouraging.

Example. - Sluiceway. - The following information is known:

Reservoir water surface elevation = 3136 ft (971 m)
Tailwater surface elevation = 3168 ft (966 m)
Barometric pressure = 677 mm Hg
Water temperature = 4.1 °C
Discharge = 3550 ft^3/s (100 m^3/s)
Reservoir dissolved nitrogen concentration = 104 percent of saturation
Reservoir dissolved oxygen concentration = 85 percent

The structural dimensions in figure 3 and the photograph in figure 4 are also available. From these sources the following terms are deduced:

H_v = 3136 - 3168 = 29 ft (8.5 m)
Angle of jet penetration ≃ 25°
Basin depth = 3168 - 3146 = 22 ft (6.7 m)
Basin flow path length, X ≃ 95 ft (29 m)

It should be observed that no head loss was included in the evaluation of jet velocity head, H_v. For this particular structure, this assumption should be reasonably valid in that the flow path between the control gate and

the stilling basin pool is short and unobstructed. Because of the changing slope of the flow surface as it enters the stilling basin, the angle of penetration was approximated to be 25° below horizontal. The basin depth of 22 ft (6.7 m) was computed for the deepest portion of the pool. Finally, the flow path length, X, of 95 ft (29 m) is approximately the distance from the point where the jet would attain significant penetration to the end sill of the basin. It was reasoned that at the end sill a large portion of the flow will be deflected upward, the flow will no longer be under the higher pressure and dissolving of gases in the basin will be complete. These approximations are quite rough, but attempts to refine the evaluations would yield only slight improvements and would call for and indicate unwarranted accuracy.

The absolute dissolved nitrogen concentration in the reservoir is evaluated as the first step in the analysis. This is accomplished by referring to appropriate standard tables and obtaining the nitrogen saturation concentration for the specific water temperature (4.4 °C) and multiplying it by the relative reservoir dissolved nitrogen concentration (104 percent).

$$C_I = (1.04)\ (20.7) = 21.5 \text{ mg/L}$$

Next the potential absolute dissolved nitrogen concentration for the stilling basin is computed. As stated before, it is dependent on the barometric pressure, water temperature, and basin depth. Two-thirds of the basin depth is assumed as the average depth over the flow while the gas is being dissolved. Using this approximation an average pressure on the flow (in atmospheres) is computed and multiplied by the absolute dissolved nitrogen concentration obtained earlier.

$$C_S = \frac{677 + 2/3(22)(304.8/13.55)}{760}\ (20.7) = 27.4 \text{ mg/L}$$

This term has been adjusted to reflect the barometric pressure and, thus, the structure's elevation. If the barometric pressure is unknown, a standard atmosphere may be used.

Two of the terms (C_S and C_I) of equation 1:

$$C(t) = C_S - (C_S - C_I)\ e^{-Kt}$$

have now been evaluated. The time, t, that gas is being dissolved, is the next term of interest. The bubble rise time, t_I, is evaluated first. To do this, the vertical dimension of the jet at the tailwater surface is found. The 28-foot velocity head yields a velocity of 42.5 ft/s (13.0 m/s). The discharge is then divided by the velocity to obtain a total flow cross sectional area for three gates.

$$3550/42.5 = 83.5 \text{ ft}^2\ (7.8 \text{ m}^2)$$

Assuming equal flow through each results in a flow cross sectional area of 27.8 ft^2 (2.6 m^2) for a single gate. When equal flow conditions are assumed for the gates, the analysis of each individual gate is identical and, thus, the analysis of the flow for only one gate will predict the performance of the entire structure. If the flow cross sectional area is then divided by the gate width (8 ft) the flow depth is determined.

$$27.8/8 = 3.5 \text{ ft}\ (1.1 \text{ m})$$

Since the flow is not horizontal the flow depth must be divided by the cosine of the angle of penetration to obtain the vertical dimension of the jet.

$$3.5/\cos 25^\circ = 3.5/0.9063 = 3.9 \text{ ft } (1.2 \text{ m})$$

If this distance is then divided by the terminal bubble velocity, a bubble rise time, t, is obtained.

$$t_1 = 3.9/0.696 = 5.6 \text{ seconds}$$

The length of time, t, is also evaluated by considering the length of time that the flow is at an effective depth in the basin. To do this the curves in figure 1 are used. First, the flow path length, X, is divided by the flow depth, B_o.

$$X/B_o = 95/3.5 = 27.1$$

The flow width (L_o) is then divided by the flow depth.

$$L_o/B_o = 8/3.5 = 2.3$$

Figure 1 is then referred to and the ratio of the maximum velocity, V_m, within the velocity distribution at the end of the flow path to the initial flow velocity, V_o, is obtained.

$$V_m/V_o = 0.36$$

or

$$V_m = (0.36)(42.6) = 15.3 \text{ ft/s } (4.7 \text{ m/s})$$

If the average flow velocity at the end of the path is then assumed to be one-half of V_m, an average velocity through the basin can be determined.

$$V = ((15.3)/2 + 42.6)/2 = 25.1 \text{ ft/s } (7.7 \text{ m/s})$$

An average velocity at the end of the path based on cross sectional area and discharge would be:

$$3550/((22)(28)) = 5.8 \text{ ft/s } (1.8 \text{ m/s})$$

This is less than (15.3/2) or 7.7 ft/s (2.3 m/s), so 7.7 ft/s should be used.

The path length divided by this average velocity gives the basin retention time:

$$t_2 = 95/25.1 = 3.8 \text{ seconds}$$

The smaller of the two computed times is the one that is applicable to the problem. For this particular case, the shorter time is 3.8 seconds, the t interval based on the flow velocity.

The final term to be evaluated is K, which is found through the use of figure 2. To apply figure 2, two parameters must be computed. The ratio of the velocity

head, H_v, to the appropriate flow path length, X, is H_v/X. If the time interval used is based on basin retention time, the basin flow path length (evaluated from the basin geometry) is used. If the smaller time results from the consideration of the bubble rise time then the flow path length to be used is less then the basin flow path length. For the sample problem the time based on the basin retention time is the smaller so the initially determined path length of 95 ft (29 m) is used. Therefore,

$$H_v/X = 28/95 = 0.295$$

For application of figure 2, the second parameter that must be evaluated is the ratio of the shear perimeter length of the jet to the cross sectional area of the jet. For this problem the shear perimeter is the jet width plus the jet height for each side or

$$8 + 3.5 + 3.5 = 15.0 \text{ ft } (4.6 \text{ m})$$

The cross sectional area has already been found to be 27.8 ft^2 (2.6 m^2). Thus the ratio is

$$15.0/27.8 = 0.54$$

The value of K is 0.1 from figure 2. The user will note the possibility of interpolation error. All the terms may now be substituted into equation 1 and a dissolved nitrogen concentration that is not corrected for barometric pressure is obtained.

$$C(t) = 27.4 - (27.4 - 21.5)\, e^{-(0.1)(3.8)} = 23.4 \text{ mg/L}$$

If this is then divided by the saturation concentration, the percent nitrogen saturation is obtained.

$$23.4/20.7 \times 100 = 113 \text{ percent}$$

The observed value for nitrogen, N_2 was also 113 percent. To obtain a predicted absolute concentration, multiply the predicted percentage by the absolute concentration adjusted for barometric pressure.

$$(1.13)(677/760)(20.7) = 20.8 \text{ mg/L of } N_2$$

Considering dissolved oxygen, we compute:

$$C_I = (0.85)(12.9) = 11.0 \text{ mg/L}$$

where 12.9 mg/L is the saturation concentration of oxygen at 4.4 °C.

Also:

$$C_S = \frac{677 + 2/3(22)(304.8/13.55)}{760}(12.9) = 17.1 \text{ mg/L}$$

t = 3.8 seconds

K = 0.1

all of which follow from the nitrogen calculations above. Applying equation 1:

$$C(t) = 17.1 - (17.1 - 11.0)\, e^{-(0.1)(3.8)} = 12.9 \text{ mg/L}$$

The percent oxygen saturation calculated is:

$$12.9/12.9 \times 100 = 100 \text{ percent}$$

The actual observed value for oxygen, O_2 was also 100 percent.

An approximation of the percent total dissolved gas would be:

$$(100)\ (23.4 + 12.0)/(20.7 + 12.9) = 105 \text{ percent}$$

This considers nitrogen and oxygen, which together comprise over 99 percent of the total dissolved gas.

Several other examples were calculated with the following results:

Structure	Calculated N_2	Calculated O_2	Observed N_2	Observed O_2
Spillway with roller bucket, three gates operating	201%	197%	199% 1/	2/
Chute spillway into hydraulic jump basin	116	112	116	108
Auxiliary outlet works (four discharges) through spillway face into hydraulic jump basin	148 152 153 154	146 152 153 153	147 156 153 125 3/	130 4/ 132 4/ 134 4/ 130 4/
Chute spillway with flip bucket and shallow plunge pool	109	2/	100	2/

1/ Considerably less after dilution by powerplant discharge.
2/ Data not available.
3/ Believe that gas escaped from sample.
4/ Possibly lower because of heavy organic loading.

Conclusions

1. Given the velocity head of the inflow jet at the tailwater surface, the angle of penetration of the jet into the tailwater, the shape of the jet, the basin length and depth, the water temperature, the barometric pressure, and the initial dissolved gas levels in the reservoir, the dissolved gas levels that will result from the passage of flow through a hydraulic structure can be predicted with reasonable accuracy. Model studies can be used to great advantage in defining the hydraulic characteristics to be used in the analysis.

2. The basic equation developed to predict the resulting dissolved gas concentrations is:

$$C(t) = C_S - (C_S - C_I)\, e^{-Kt}$$

where $C(t)$ is the dissolved gas concentration created by the hydraulic structure, C_I is the dissolved gas concentration in the reservoir, C_S is the saturated dissolved gas concentration at a depth which is two-thirds of the maximum basin depth, t is representative of the length of time during which gas is being dissolved, and K is a constant that varies with structure and operating condition. A method is developed for prediction of the K value.