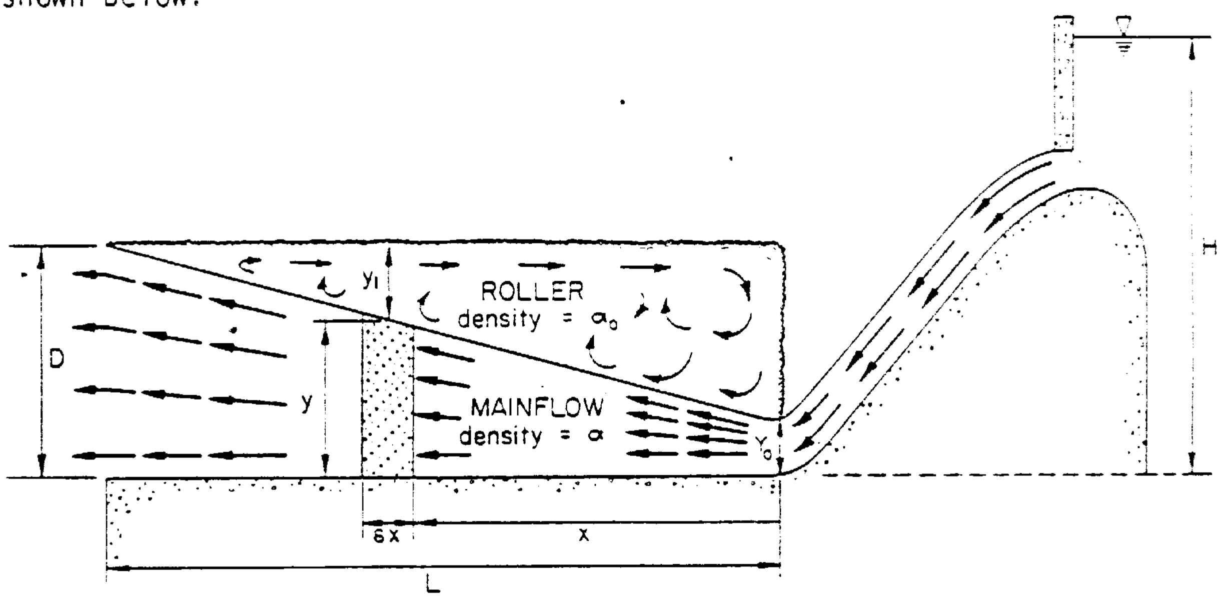
DERIVATION OF THE SPILLWAY-STILLING BASIN MODEL*

Consider the conceptual representation of the stilling basin shown below.



CONCEPTUAL REPRESENTATION OF SPILLWAY-STILLING BASIN COMBINATION

The water parcel indicated in cross-section by the shaded area moves through the stilling basin, decelerating and increasing in height. It extends laterally the full effective width, ω of the stilling basin as illustrated in Figure 3 of the main report.

We now make the following assumptions for the water parcel and stilling basin:

1. For that length of spillway that is in operation at a given time, the discharge is uniform along the

*Taken from: "A Nitrogen Gas (N_2) Model for the Lower Columbia River, "Final Report, Water Resources Engineers, Inc., under contract to US Army Corps of Engineers, North Pacific Division, January 1971 crest (this is equivalent to assuming that the properties of the water parcel are constant along any line parallel to the spillway crest).

2. The value Y_o is the initial depth of the spill before the jump. It is computed as:

$$Y_o = \frac{q}{V_o} = \frac{q}{\sqrt{2gH}} \qquad \text{(A-1)}$$

where

q = discharge per foot along the crest

H = total reservoir head above the stilling basin floor.

3. The only effect of the roller which overlies the main flow is to increase the static pressure within the water parcel by an amount $\alpha_g y_1$.

4. A given mass of air M_a is entrained as discrete bubbles into the water parcel at the point $x = 0$ and remains uniformly distributed within the water parcel as it passes through the stilling basin.

5. The distribution of the mass of air among the various bubble sizes remains unchanged during the water parcel's journey through the stilling basin.

6. The dissolved nitrogen within the water parcel is uniformly distributed.

7. Rate of nitrogen dissolution $\frac{dM}{dt}$ in the water parcel is governed by Fickian diffusion as:

$$\frac{dM}{dt} = K_L A \, (C_s - C) \qquad \text{(A-2)}$$

where

M = the mass of dissolved nitrogen in the water parcel,

K_L = rate coefficient,

A = total surface area of the air bubbles contained in the water parcel,

C_E = effective saturation concentration of dissolved nitrogen in the water parcel, and

C = actual concentration of dissolved nitrogen in the water parcel.

With these assumptions, we can now define the parameters M, A, and C_E in equation A-2 as functions of the location of the water parcel in the stilling basin.

Assumption 6 allows us to write the mass M as the product of the concentration C and the volume of the water parcel,

$$M = (wy\delta x)C \tag{A-3}$$

where w is the effective width of the stilling basin, i.e., w = (number of gates open) x (width per gate).

The saturation concentration of a gas such as N_2 or O_2 that is only slightly soluble in water is governed by Henry's Law which states that the equilibrium or saturation concentration of the gas in solution is directly proportional to the pressure existing at the gas-liquid interface. In the water parcel the pressure P at an elevation z above the stilling basin floor is

$$P = P_o + \alpha_o y_1 + \alpha(y-z) \tag{A-4}$$

where P_o is the atmospheric (or barometric) pressure, and the α parameters are the densities of the roller and main flow as shown in Figure A-1. Hence, the saturation concentration at any elevation z in the parcel is given as:

$$C_{sat} = [P_o + \alpha_o y_1 + \alpha(y-z)]C^* \tag{A-5}$$

where C^* is the saturation concentration under one atmosphere of pressure. In equation A-5, the pressure term has units of atmospheres of pressure. From equation A-5, it is seen that C_E varies linearly with z. It follows that the average or *effective* saturation concentration, C_E in the water parcel is the value of C_{sat} at mid-depth, or at $z = y/2$: Thus,

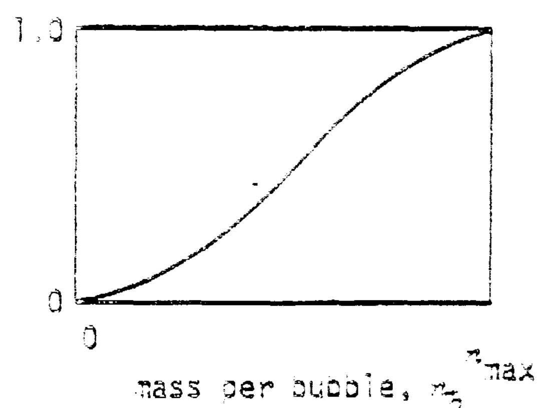
$$C_E = [P_o + \alpha_o y_1 + \alpha(y/2)]C^* \tag{A-6}$$

Noting that $y_1 = D-y$ gives the final form of C_E as

$$C_E = [P_o + \alpha_o D - (\alpha_o - \frac{\alpha}{2})y]C^* \tag{A-7}$$

The total surface area A of the air bubbles in the water parcel depends upon the total mass of air entrained and, upon the bubble size distribution. It is not unreasonable to expect that the entrained mass of air will be distributed among the various bubble sizes in a manner similar to that shown below.

β = fraction of total air mass in the water parcel with bubbles having a mass less than or equal to m_b



The volume V_b of an air bubble with mass m_b can be found from the ideal gas law:

$$V_b = \frac{mRT}{P} \tag{A-8}$$

where

m = number of moles of air in the bubble,

R = universal gas constant,

T = absolute temperature, and

P = the total pressure in the bubble.

In equation A-8, m can be replaced by $n_b/28.9$ where 28.9 is the molecular weight of air. The diameter d_b and the area A_b of a sphere are given by:

$$d_b = \left(\frac{6}{\pi} V_b\right)^{1/3} \tag{A-9a}$$

$$A_b = \pi d_b^{\,2} \tag{A-9b}$$

Now, combining equations A-8 and A-9, the following expression results for the surface area A_b of an air bubble with mass n_b:

$$A_b = \left(\frac{6\sqrt{\pi}RT}{28.9}\right)^{2/3} \left(\frac{n_b}{P}\right)^{2/3} \tag{A-10}$$

Thus, if the total air mass entrained per unit volume of water at Y_o is M_A, the total air bubble surface areas A', per unit volume of water is found from the bubble size distribution and equation A-10 as

$$A' = \int_o^{n_{max}} A_b \frac{M_A \dfrac{dB}{dn_b} dn_b}{n_b} \tag{A-11}$$

or

$$A' = \left(\frac{6\sqrt{\pi}RT}{28.9P}\right)^{2/3} M_A \int_0^1 n_b^{-1/3} dB \tag{A-12}$$

Finally, to get the total bubble surface area in the water parcel it is necessary to integrate equation A-12 over the volume of the parcel $wy\delta x$, i.e.,

$$A = \int_{\delta x}\int_{w}\int_{z} A' \, dx \, dw \, dz \qquad \text{(A-13)}$$

Applying assumptions 4 and 5 and substituting for A' from equation A-12 gives

$$A = w\delta x \left(\frac{6\sqrt{\pi}RT}{28.9}\right)^{2/3} M_{A_j}\int^{1} n_D^{-1/3}\,dB \int_{z=0}^{y} \frac{dz}{P^{2/3}} \qquad \text{(A-14)}$$

Replacing P with equation A-4 and integrating,

$$A = 3\left(\frac{6\sqrt{\pi}RT}{28.9}\right)^{2/3} M_{A_j}\int^{1} n_D^{-1/3}\,dB \quad (w\delta x) \quad \frac{\left(P_o + a_o y_1 + ay\right)^{1/3} - \left(P_o + a_o y_1\right)^{1/3}}{a} \qquad \text{(A-15)}$$

Substituting $(D-y)$ for y_1 gives the final form as

$$A = K_A \, (w\delta x) \left\{\left[P_o + a_o D + (a-a_o)y\right]^{1/3} - \left[P_o + a_o D - a_o y\right]^{1/3}\right\} \qquad \text{(A-16)}$$

where
$$K_A = \frac{3}{a}\left(\frac{6\sqrt{\pi}RT}{28.9}\right)^{2/3} M_{A_j}\int^{1} n_D^{-1/3}\,dB$$

If the expressions for M, C_E, and A from equation A-3, A-7 and A-15 respectively are substituted into the rate expressions given in equation A-2, there results

$$(yw\delta x)\frac{dC}{dt} = (w\delta x)\ K_L K_A \left\{[P_o + \alpha_o D + (\alpha-\alpha_o)y]^{1/3} - [P_o + \alpha_o D - \alpha_o y]^{1/3}\right\}$$
$$\left\{[P_o + \alpha_o D - (\alpha_o - \frac{\alpha}{2})y]C^* - C\right\} \quad \text{(A-17)}$$

We can now write rate expression $\frac{dC}{dt}$ in terms of the location in the stilling basin by using the relationship

$$\frac{dC}{dt} = \frac{dx}{dt}\ \frac{dC}{dx} = v\ \frac{dC}{dx} = \frac{q}{y}\ \frac{dC}{dx} \quad \text{(A-18)}$$

where v is the velocity of the parcel and q is the discharge per unit width of the stilling basin. In addition, we define a system parameter K, which we will call the *entrainment coefficient*, as

$$K = K_L K_A = \frac{3}{\alpha}\left(\frac{5\sqrt{\pi}R}{28.9}\right)^{2/3} \left[T^{2/3} K_L\ M_A \int_0^1 n_b^{-1/3} dB\right] \quad \text{(A-19)}$$

Substituting equation A-18 and A-19 into A-17 gives the expression for the concentration change in the water parcel as

$$\frac{dC}{dx} = \frac{K}{q}\left\{[P_o + \alpha_o D + (\alpha - \alpha_o)y]^{1/3} - [P_o + \alpha_o D - \alpha_o y]^{1/3}\right\}$$
$$\left\{[P_o + \alpha_o D - (\alpha_o - \frac{\alpha}{2}y)]C^* - C\right\} \quad \text{(A-20)}$$

The solution is obtained as follows. Evaluate the pressure terms at the midpoint of the stilling basin $y = \frac{D+Y_o}{2}$ to obtain

$$\frac{dC}{dx} = \frac{K}{q}\left\{[\overline{P} + \frac{\alpha}{4}(D + Y_o)]^{1/3} - [\overline{P} - \frac{\alpha}{4}(D + Y_o)]^{1/3}\right\}\left\{\overline{P}C^* - C\right\} \quad \text{(A-21)}$$

where $\quad \bar{P} = P_o + \frac{\alpha_o}{2}(D - Y_o) + \frac{\alpha}{4}(D + Y_o)$

Now let

$$\overline{\Delta P^{1/3}} = [\bar{P} + \frac{\alpha}{4}(D + Y_o)]^{1/3} - [\bar{P} - \frac{\alpha}{4}(D + Y_o)]^{1/3} \tag{A-22}$$

Rewriting equation A-21, with these substitutions gives

$$dC + C\frac{K}{q}\overline{\Delta P^{1/3}}\,dx = \frac{K}{q}\overline{\Delta P^{1/3}}\,\bar{P}C^*\,dx \tag{A-23}$$

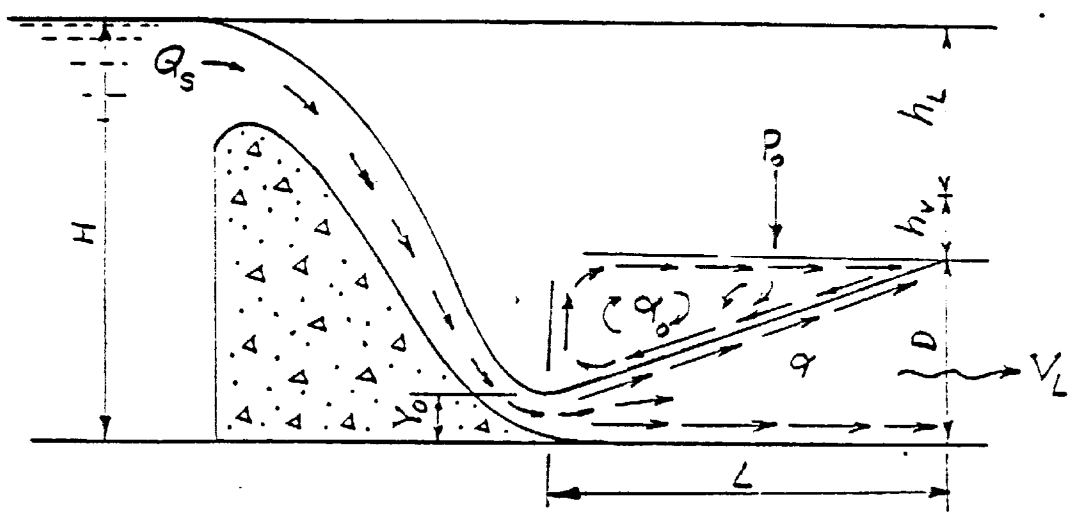
which has the solution

$$C = \bar{P}C^* + ke^{-\frac{K}{q}\overline{\Delta P^{1/3}}\,x} \tag{A-24}$$

Evaluating equation A-24 at $x = 0$, where C equals the forebay concentration C_F, and at $x = L$ where C equals the stilling basin concentration C_S, yields the spillway-stilling basin model as

$$C_S = \bar{P}C^* - (\bar{P}C^* - C_F)\,e^{-\frac{K}{q}L\,\overline{\Delta P^{1/3}}} \tag{A-25}$$

ESSENTIALS OF N_2 MODEL 1/
LOWER COLUMBIA & LOWER SNAKE RIVERS



DEFINITION SKETCH

RESIDENCE TIME = $t_R = WDL/Q_s = DL/q \approx L/V_L$

TOTAL HEAD LOSS = $h_L = H-D-h_v = H-(D+v_L^2/2g)$

ENERGY LOSS RATE = $E = h_L/t_R$

AVE. PRESSURE = $\bar{P} = P_o + \frac{\alpha_o}{2}(D-Y_o) + \frac{\alpha}{4}(D+Y_o)$. $\alpha = 0.0295$ atm./ft.
$\alpha_o = \alpha c$

N_2 CONCENTRATION AT END OF STILLING BASIN, L

$$C_s = \bar{P}C^* - (\bar{P}C^* - C_F)\exp\left(-\frac{K}{q} L\, \overline{\Delta P}^{1/3}\right)$$

$$\overline{\Delta P}^{1/3} = \left[\bar{P} + \frac{\alpha}{4}(D+Y_o)\right]^{1/3} - \left[\bar{P} - \frac{\alpha}{4}(D+Y_o)\right]^{1/3}$$

$$K = \frac{q}{L\,\overline{\Delta P}^{1/3}} \ln\left(\frac{\bar{P}C^* - C_f}{\bar{P}C^* - C_s}\right) = K_{20}(1.028)^{(T-20)}$$ T = Water Temperature.

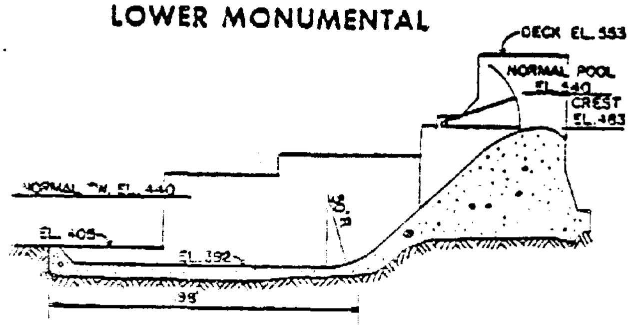
$K_{20} = aE^b$. a & b are empirically determined from observed data. They are shown below:

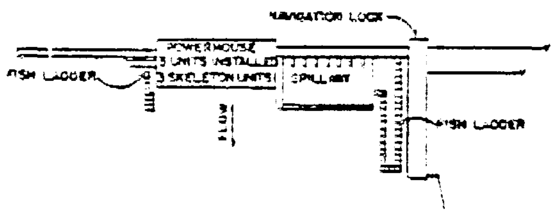
MODEL COEFFICIENTS

PROJECT	c	a	b
Little Goose	1.00	0.09	2.45
Lower Monumental	1.00	0.09	2.45
Ice Harbor	1.00	0.30	1.00
McNary	1.00	1.00	2.00
John Day	1.00	0.20	2.10
The Dalles	0.50	0.80	2.50
Bonneville	1.00	1.90	1.00

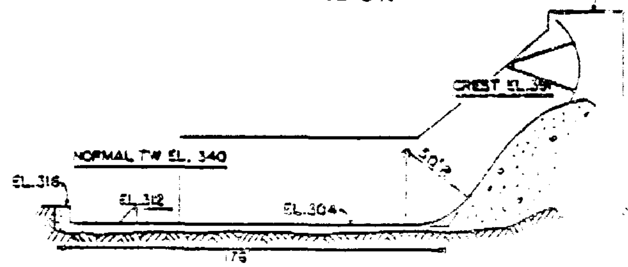
1/ Developed by Water Resources Engineers, Inc. for the Corps in 1971.

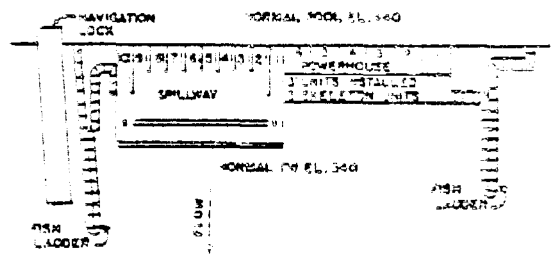
LOWER MONUMENTAL



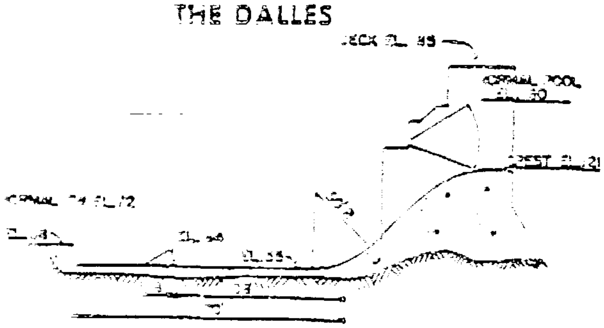


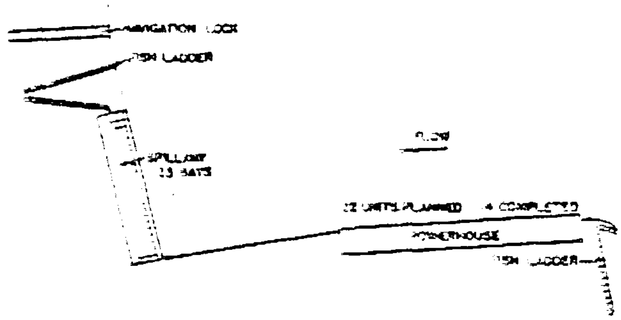
ICE HARBOR



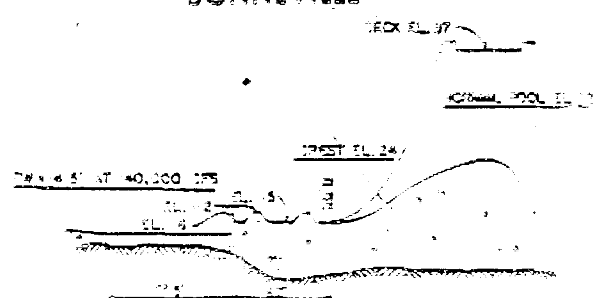


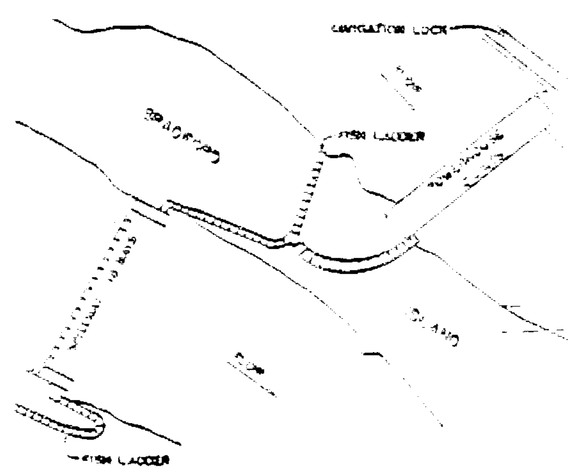
THE DALLES



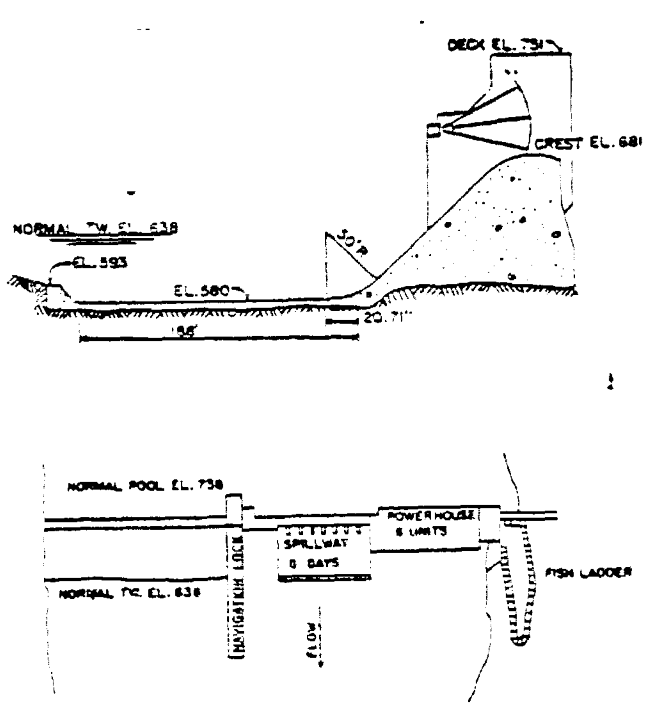


BONNEVILLE

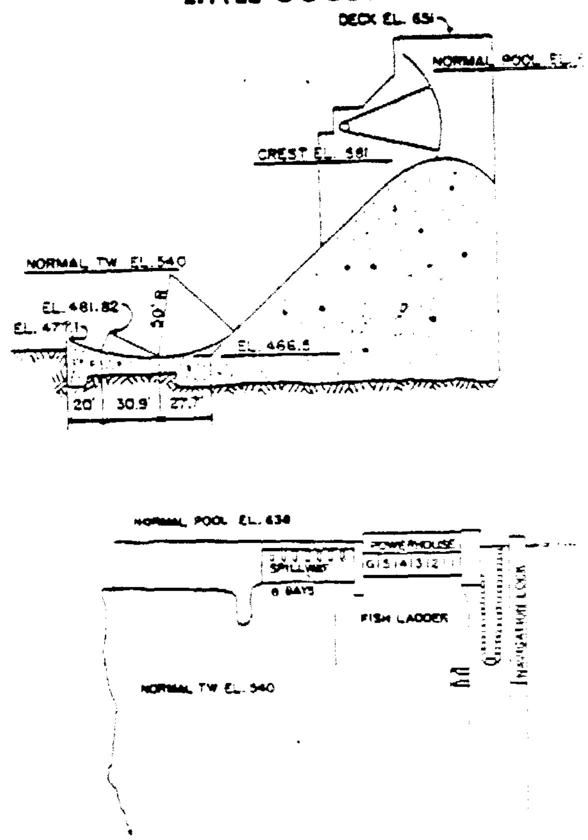




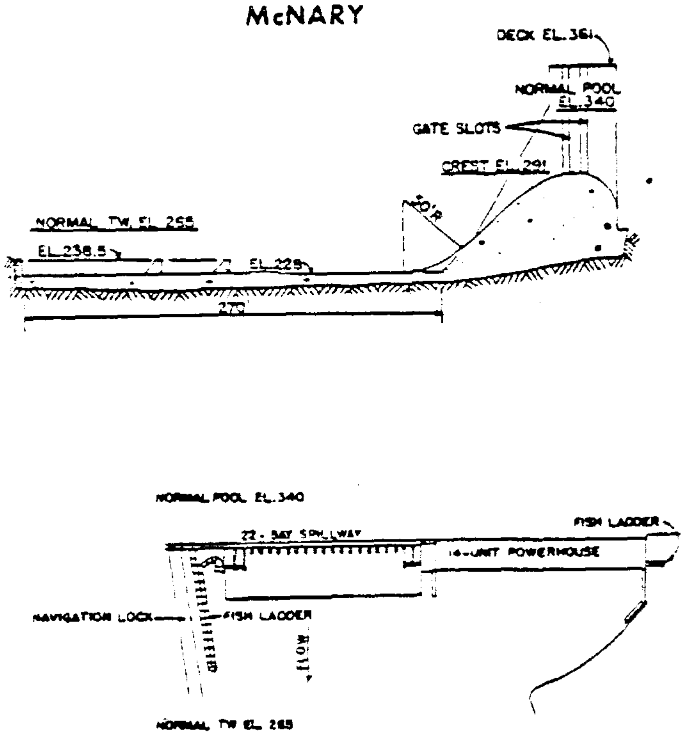
LOWER GRANITE



LITTLE GOOSE



McNARY



JOHN DAY