* # Chapter 9
# Sediment Transport Mechanics

Section I
Introduction

9-1. Definition

Sedimentation embodies the processes of erosion, entrainment, transportation, deposition, and compaction of sediment. These are natural processes that have been active throughout geological times and have shaped the present landscape of our world. The principal external dynamic agents of sedimentation are water, wind, gravity, and ice. Although each may be important locally, only hydrodynamic forces are considered herein. Transport functions, as typified by Einstein (1950), treat only the "transportation" process.

9-2. Topics Beyond the Material Presented in This Chapter

a. Local scour/deposition. Local scour, as compared to general erosion/deposition, refers to the scour hole that forms around a bridge pier or downstream from a hydraulic structure or along the outside of a bend, etc. It involves fluid forces from multidimensional flow accelerations, pressure fluctuations, and gravity forces on the sediment particles. The complexity of local scour processes relegates analysis to empirical equations or physical model studies. This chapter does not address local scour.

b. Cohesive sedimentation theory. The concept of the equilibrium condition does not apply to cohesive sediment transport as it does to noncohesive sediment transport. That is, in noncohesive sediment transport, there is a continual exchange of sediment particles between the water column and the bed surface. The equilibrium condition exists when the same number of a given type and size of particles are deposited on the bed as are entrained from it. That exchange process does not exist in cohesive sediment movement. Particle inertia due to its mass is insignificant in cohesive sedimentation problems in rivers. The dominant forces preventing cohesive particles from being eroded are electrochemical forces. That is, when cohesive particles come in contact with the bed, they are likely to adhere to it and resist re-entrainment. Deposition rates depend on flocculation of cohesive particles in suspension. There are analytical techniques for calculating the erosion, entrainment, transportation, deposition, and consolidation of cohesive sediments. However, it is a basic requirement to develop site-specific sediment properties from testing samples. Two fundamental properties are: (1) the shear stress for the initiation of erosion and deposition, and (2) the erosion rate. The erosion/deposition shear stresses are called erosion and deposition thresholds. Erosion rate is expressed as a function of bed shear stress. These relationships are needed for the full range of hydraulic conditions expected at the site. Finally, settling velocities are needed.

Section II
Initiation of Motion

9-3. General

Thresholds for particle erosion can be calculated, using average values for hydraulic parameters, if the fluid and sediment properties are known. The significant fluid properties are specific weight and viscosity. Significant sediment properties are particle size, shape, specific gravity, and position in the matrix of surrounding particles. In the case of cohesive particles the electrochemical bonds, related primarily to mineralogy, are the most significant sediment properties. Significant hydraulic forces are bed shear stress, lift, pressure fluctuations related to turbulence, and impact from other particles.

9-4. Shields Parameter

Although velocity has been used historically for predicting whether or not a particle will erode, Shields relationship between dimensionless shear stress (or Shields parameter), τ_*, and grain Reynolds number, R_*, is now recognized as a more reliable predictor. Shields parameter and grain Reynolds number are dimensionless, so that any consistent units of measurement may be used in their calculation. Although the experimental work and analysis were performed by Shields, the curve termed the Shields Curve, which is shown in Figure 9-1, was actually proposed by Rouse (ASCE 1975). Shields curve may be expressed as an equation, which is useful for computer programming.

$$\tau_* = 0.22\ \beta + 0.06 \times 10^{-7.7\beta} \tag{9-1}$$

$$\beta = \left(\frac{1}{\nu} \sqrt{ \left(\frac{\gamma_s - \gamma}{\gamma} \right) g d^3 } \right)^{-0.6} \tag{9-2}$$

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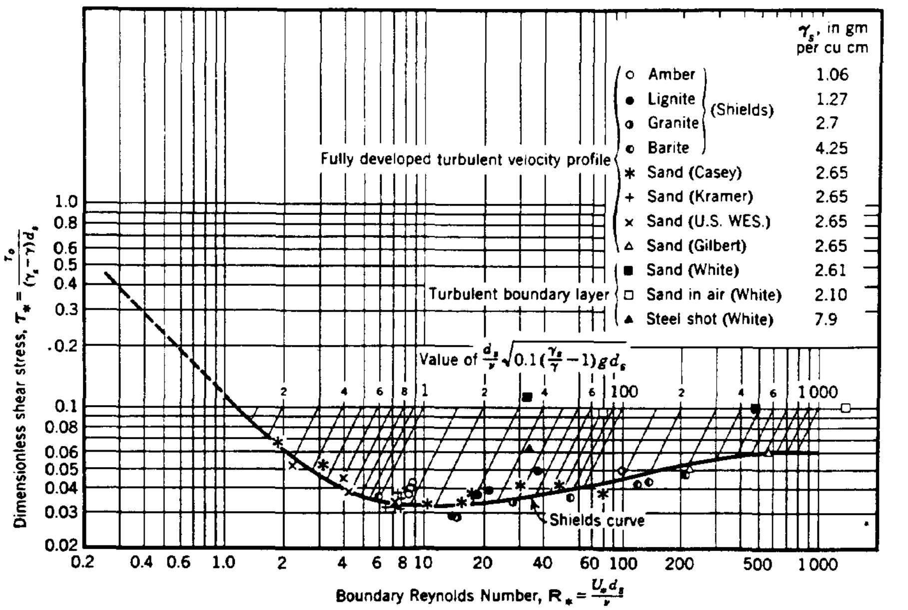

Figure 9-1. Shields curve (ASCE 1975)

where

τ_o = bed shear stress

γ_s = particle specific weight

γ = fluid specific weight

ν = kinematic viscosity of the fluid

g = acceleration of gravity

d = particle diameter

u_* = shear velocity = $(gRS)^{0.5}$

R = hydraulic radius

S = slope

The critical shear stress, τ_c, for stability of a particle having a diameter, d is then calculated from the following equation:

$$\tau_c = \tau_* (\gamma_s - \gamma)d \qquad (9\text{-}3)$$

9-5. Adjusted Shields Parameter

Shields obtained his critical values for τ_* experimentally, using uniform bed material, and measuring sediment transport at decreasing levels of bed shear stress and then extrapolating to zero transport. There are three problems associated with the critical dimensionless shear stress as determined by Shields. First, the procedure did not account for the bed forms that developed with sediment transport. A portion of the total shear is required to overcome the bed form roughness; therefore the calculated dimensionless shear stress was too high. Gessler (1971) reanalyzed Shields' data so that the critical Shields parameter represented only the grain shear stress which determines sediment transport and entrainment (Figure 9-2). Secondly, the critical dimensionless shear stress is based on the average sediment transport of numerous particles and does not account for the sporadic entrainment of individual particles at very low shear stresses. This becomes very important when transport of gravels and cobbles is of interest in low energy environments, and in the design of armor protection. This phenomenon was demonstrated by Paintal (1971) and is shown in Figure 9-3. Note that the extrapolated critical dimensionless shear

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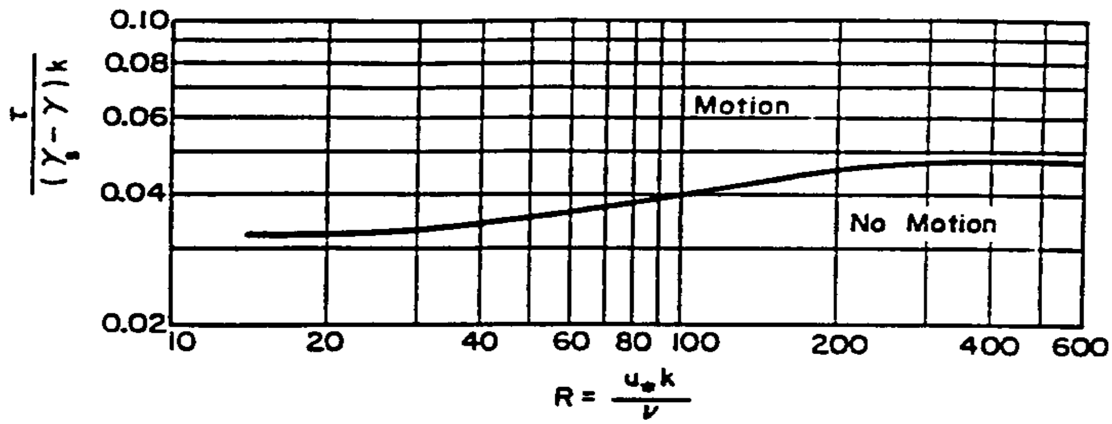

Figure 9-2. Shields diagram (Gessler 1971)

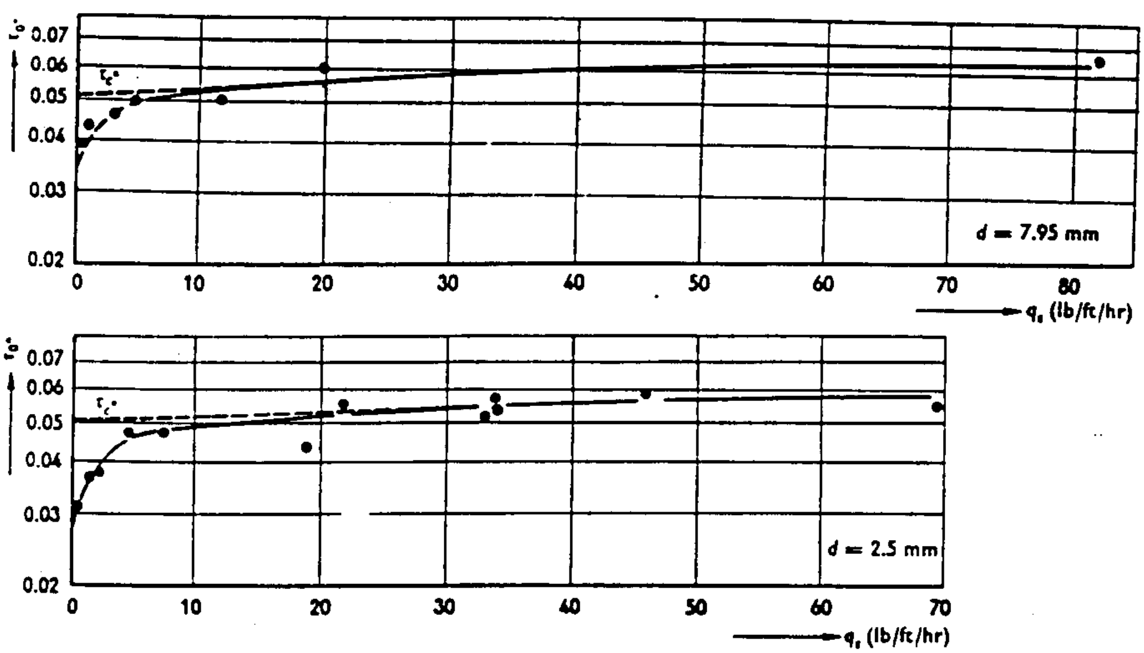

Figure 9-3. Determination of critical shear stress (Paintal 1971)

stress was about 0.05, but the actual critical dimensionless shear stress was 0.03. Thirdly, critical dimensionless shear stress for particles in a sediment mixture may be different from that for the same size particle in a uniform bed material. Meyer-Peter and Muller (1948) and Gessler (1971) determined from their data sets that the critical Shields parameter for sediment mixtures was about 0.047. Neill (1968) determined, from his data, that in gravel mixtures, most of the particles become mobile when τ_* for the median grain size was 0.030. Andrews (1983) found a slight difference in τ_*, for different grain sizes in a mixture, and presented the following equation:

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$$\tau_{*_i} = 0.0834 \left(\frac{d_i}{d_{50}} \right)^{-0.872} \qquad (9\text{-}4)$$

where the subscript, i, indicates the Shields parameter value for size class i, and d_{50} is the median diameter of the subsurface material. The minimum value for τ_{*i} was found to be 0.020. According to Andrews, the critical shear stress for individual particles has a very small range; therefore, the entire bed becomes mobilized at nearly the same shear stress.

9-6. Gessler's Concept for Particle Stability

a. Critical shear stress is difficult to define because at low shear stresses entrainment is sporadic, caused by bursts of turbulence. It is even more difficult to define for particles in a coarse surface layer because the critical shear stress of one size class is affected by the presences of other size classes. Gessler (1971) developed a probabilistic approach to the initiation of motion for sediment mixtures. He reasoned that due to the random orientation of grains on the bed and the random strength of turbulence on the bed, for a given set of hydraulic conditions, part of the grains of a given size will move while others of the same size may remain in place. Gessler assumed that the critical Shields parameter represents an average condition, where about half the grains of a uniform material remain stable and half move. It follows then that when the critical shear stress was equal to the bed shear stress there was a 50 percent chance for a given particle to move. Using experimental flume data, he developed a probability function, **p**, dependent on τ_c/τ where τ_c varied with bed size class (Figure 9-4). He determined that the probability function had a normal distribution and that the standard deviation (slope of the probability curve) was a function primarily of turbulence intensity and equal to 0.057. Gessler found the effect of grain-size orientation to be negligible. The standard deviation also accounts for hiding effects, i.e. no attempt was made to separate hiding from the overall process. Gessler's analysis demonstrates that there can be entrainment of particles even when the applied shear stress is less than the critical shear stress, and that not all the particles of a given size class on the bed will necessarily be entrained until the applied shear stress exceeds the critical shear stress by a factor of 2.

b. Gessler suggested that the mean value of the probabilities for the bed surface to stay should be a good indicator of stability:

$$\bar{P} = \frac{\int_{i_{min}}^{i_{max}} P^2 f_i \; di}{\int_{i_{min}}^{i_{max}} P \; f_i \; di} \qquad (9\text{-}5)$$

Where $\bar{p}$ is the probability function for the mixture and depends on the frequency of all grain sizes in the underlying material, and f_i is the fraction of grain size i. Gessler suggested that when $\bar{p} > 0.65$ that the surface layer of the bed would be unstable.

9-7. Grain Shear Stress

a. The total bed shear stress may be divided into that acting on the grains and that acting on the bed forms. Entrainment and sediment transport are a function only of the grain shear stress. Grain shear stress thus must be determined in order to make sediment transport calculations. Einstein (1950) determined that the grain shear stress could best be determined by separating total bed shear stress into a grain component and a form component which are additive. The equation for total bed shear stress is:

$$\tau_o = \tau' + \tau'' = \gamma R S \qquad (9\text{-}6)$$

where

τ_o = total bed shear stress

τ' = grain shear stress

τ'' = form shear stress

b. Einstein (1950) also suggested that the hydraulic radius could be divided into grain and form components that are additive. The equations for grain and form shear stress then become

$$\begin{aligned} \tau' &= \gamma R' S \\ \tau'' &= \gamma R'' S \end{aligned} \qquad (9\text{-}7)$$

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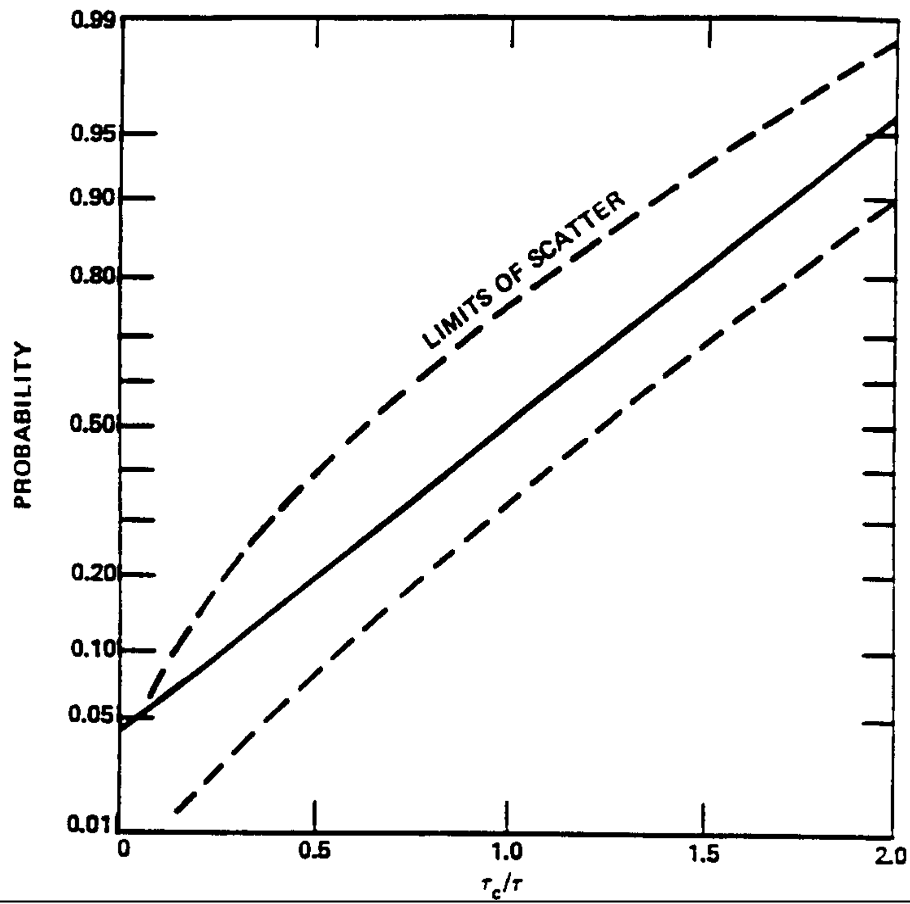

Figure 9-4. Probability of grains to stay (Gessler 1971)

where R' and R'' are hydraulic radii associated with the grain and form roughness, respectively. The total bed shear stress can be expressed as

$$\tau_o = \gamma R' S + \gamma R'' S \qquad (9\text{-}8)$$

Slope and the specific weight of water are constant, so that the solution becomes one of solving for one of the R components. The Limerinos (1970) equation can be used to calculate the grain roughness component.

$$\frac{V}{U_*'} = 3.28 + 5.66 Log_{10} \frac{R'}{d_{84}} \qquad (9\text{-}9)$$

$$U_*' = \sqrt{g R' S}$$

where V is the average velocity and d_{84} is the particle size for which 84 percent of the sediment mixture is finer. Limerinos developed his equation using data from gravel-bed streams. Limerinos' hydraulics radii ranged between 1 and 6 ft; d_{84} ranged between 1.5 and 250 mm. This equation was confirmed for sand-bed streams without bed forms by Burkham and Dawdy (1976). The equation can be solved iteratively when average velocity, slope, and d_{84} are known.

9-8. Bed-Form Shear Stress

Einstein and Barbarossa (1952) used data from several sand-bed streams to develop an empirical relationship between bed form shear velocity and a dimensionlesssediment mobility parameter, Ψ'. The relationship is shown in Figure 9-5.

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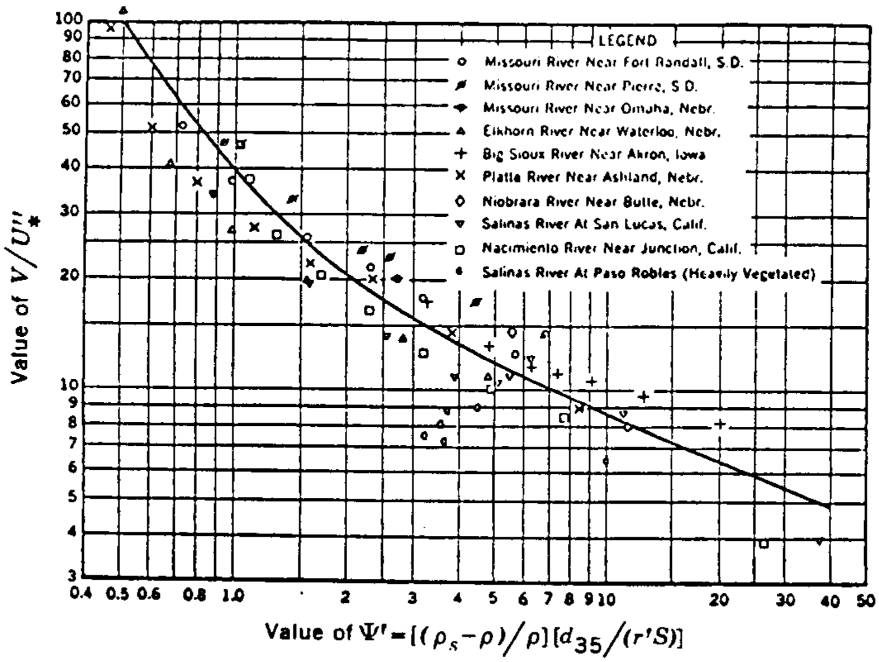

Figure 9-5. Bar resistance curve (Einstein and Barbarossa 1952)

$$\Psi' = \left(\frac{\gamma_s - \gamma}{\gamma}\right)\frac{d_{35}}{R'S} \tag{9-10}$$

where d_{35} is the particle size for which 35 percent of the sediment mixture is finer. R'' can be solved for directly using the following equation:

$$R'' = \frac{(U''_*)^2}{g\ S} \tag{9-11}$$

Typically, either the grain or form hydraulic radius is calculated directly, and the other hydraulic radius component is determined to be the difference between the total hydraulic radius and the calculated component.

9-9. Bank or Wall Shear Stress

Whenever the streambanks contribute significantly to the total roughness of the stream, the shear stress contributing to sediment transport must be further reduced. This is accomplished using the side-wall correction procedure which separates total roughness into bed and bank roughness and conceptually divides the cross-sectional area into additive components. The procedure is based on the assumption that the average velocity and energy gradient are the same in all segments of the cross section.

$$\begin{aligned} A_{total} &= A_b + A_w \\ A_{total} &= P_b R_b + P_w R_w \end{aligned} \tag{9-12}$$

where A is cross-sectional area, P is perimeter, and subscripts b and w are associated with the bed and wall (or banks), respectively. Note that the hydraulic radius is not additive with this formulation as it was with R' and R''. Using the Manning equation, with a known average velocity, slope, and roughness coefficient, the hydraulic radius associated with the banks can be calculated:

$$\frac{V}{1.486\ S^{1/2}} = \frac{R^{2/3}}{n} = \frac{R_w^{2/3}}{n_w} \tag{9-13}$$

$$R_w = \left(n_w \frac{V}{1.486\ S^{1/2}}\right)^{3/2} \tag{9-14}$$

where velocity is in feet per second and R is in feet. The side-wall correction procedure is outlined using the *

* Darcy-Weisbach equation in ***Sedimentation Engineering*** (ASCE 1975, pp 152-154). Total hydraulic radius and shear stress considering grain, form, and bank roughness can be expressed by the following:

$$R_{total} = \frac{P_b(R^{/} + R^{//}) + P_w R_w}{P_{total}} \tag{9-15}$$

$$\tau_{total} = \gamma S \left(\frac{P_b(R^{/} + R^{//}) + P_w R_w}{P_{total}} \right) \tag{9-16}$$

Section III
Stage-Discharge Predictors

9-10. General

There are several stage-discharge predictors that have been developed for alluvial channels and these are presented in ***Sedimentation Engineering*** (ASCE 1975, pp 126-152). The Limerinos (1970) equation is suggested as a stage-discharge predictor for gravel-bed streams. The Einstein-Barbarossa (1952) method was the first stage-discharge predictor to account for variability in stage due to bed-form roughness by calculating separate hydraulic radii for grain and form contributions. More recently, Brownlie (1981) developed regression equations to calculate a hydraulic radius that accounts for both grain and form roughness in sand-bed streams.

9-11. Brownlie Approach

a. Database. Brownlie's resistance equations are based on about 1000 records from 31 flume and field data sets. The data were carefully analyzed for accuracy and consistency by Brownlie. The resistance equations account for both grain and form roughness, but not bank roughness. The data covered a wide range of conditions: grain size varied between 0.088 and 2.8 mm, and depth ranged between 0.025 and 17 m. All of the data had width-to-depth ratios greater than 4, and the gradation coefficients of the bed material were equal to or less than 5.

b. Regression equations. Brownlie developed separate resistance equations for upper and lower regime flow. The equations are dimensionless, and can be used with any consistent set of units.

Upper Regime:

$$R_b = 0.2836\ d_{50}\ q_*^{0.6248}\ S^{-0.2877}\ \sigma^{0.0813} \tag{9-17}$$

Lower Regime:

$$R_b = 0.3742\ d_{50}\ q_*^{0.6539}\ S^{-0.2542}\ \sigma^{O.1050} \tag{9-18}$$

where

$$q_* = \frac{V\ D}{\sqrt{g\ d_{50}^3}} \tag{9-19}$$

R_b = hydraulic radius associated with the bed

d_{50} = median grain size

S = slope

σ = geometric bed material gradation coefficient

V = average velocity

D = water depth

g = acceleration of gravity

To determine if upper or lower regime flow exists for a given set of hydraulic conditions, a grain Froude number, F_g, and a variable, F_g', were defined by Brownlie:

$$F_g = \frac{V}{\sqrt{g\ d_{50} \left(\frac{\gamma_s - \gamma}{\gamma} \right)}} \tag{9-20}$$

$$F_g^{/} = \frac{1.74}{S^{\,0.3333}} \tag{9-21}$$

According to Brownlie, upper regime flow occurs if $S > 0.006$ or if $F_g > 1.25\ F_g'$, and lower regime flow occurs if $F_g < 0.8\ F_g'$. Between these limits is the transition zone.

Section IV
Bed-Load Transport *

* **9-12. General**

Bedload is defined as sediment moving on or near the bed by sliding, rolling, or jumping. Any particle size can move as bed load, depending on hydraulic forces.

9-13. DuBoys' Concept of Bed Load

Between 1879 and 1942 much of the work in sediment transport was influenced by DuBoys. He proposed the idea of a bed shear stress and visualized a process by which the bed material moved in layers. The significant assumptions in the DuBoys approach were that sediment transport could be calculated using average cross-section hydraulic parameters and that transport was primarily a function of the excess shear stress; i.e., the difference between hydraulically applied shear stress and the critical shear stress of the bed material. The general form of the DuBoys equation is

$$q_B = K\tau_o(\tau_o - \tau_c)^m \qquad (9\text{-}22)$$

where

q_B = bed-load transport rate in weight per unit time per unit width

τ_o = hydraulically applied shear stress

τ_c = critical, or threshold shear stress, for the initiation of movement

K and m = constants

The functional relationship between K, τ_c, and grain size was determined experimentally and is presented in ***Sedimentation Engineering*** (ASCE 1975, p 191). In DuBoys' equation m = 1.0. No movement occurs until the bed shear stress exceeds the critical value.

9-14. Einstein's Concept of Particle Movement

A major change in the approach to predicting sediment transport was proposed by Einstein (1950) when he presented a bed-load formula based on probability concepts in which the grains were assumed to move in steps of average length proportional to the sediment size. He describes bed-material transportation as follows:

> "The least complicated case of bed-load movement occurs when a bed consists only of uniform sediment. Here, the transport is fully defined by a rate. Whenever the bed consists of a mixture the transport must be given by a rate and a mechanical analysis or by an entire curve of transport against sediment size. For many years this fact was neglected and the assumption was made that the mechanical analysis of transport is identical with that of the bed. This assumption was based on observation of cases where actually the entire bed mixture moved as a unit. With a larger range of grain diameters in the bed, however, and especially when part of the material composing the bed is of a size that goes into suspension, this assumption becomes untenable."

> "The mechanical analysis of the material in transport is basically different from that of the bed. This variation of the mechanical analysis will be described by simply expressing in mathematical form the fact that the motion of a bed particle depends only on the flow and its own ability to move, and not on the motion of any other particles." (Einstein 1950).

a. Equilibrium condition. Einstein's hypothesis that motion of a bed particle depends only on the flow and its own ability to move and not on the motion of any other particles allowed him to describe the equilibrium condition for bed-material transportation mathematically as two independent processes: deposition and erosion. He proposed an "equilibrium" condition and defined it as the condition existing when the same number of a given type and size of particles must be deposited in the bed as are scoured from it.

b. Bed-load equation. In Einstein's formulation for bed-load transport, he determined the probability of a particle being eroded from the bed, p, to be

$$\frac{p}{1-p} = A_* \ \Phi_{i*}$$
$$\Phi_{i*} = \frac{i_B}{i_b}\frac{q_B}{\gamma_s}\left(\frac{\gamma}{\gamma_s - \gamma}\right)^{1/2}\left(\frac{1}{g d_i^3}\right)^{1/2} \qquad (9\text{-}23)$$

where

$A*$ = constant

Φ_i* = bed-load parameter for size class i *

* i_B = fraction of size class i in the bed-load

i_b = fraction of size class i in the bed material

q_B = bed-load transport in weight per unit time and width

d_i = grain diameter of size class i

He then reasoned that the dynamic lift forces on a particle are greater than particle weight when the probability to go into motion is greater than unity. Assuming a normal distribution for the probability of motion yields

$$p = 1 - \frac{1}{\sqrt{\pi}} \int_{\eta_o}^{\eta} e^{-t^2} dt$$
$$\eta_o = -B_* \Psi_{i*} - 2.0 \tag{9-24}$$
$$\eta = B_* \Psi_{i*} - 2.0$$

where

$B*$ = a constant

Ψ_i* = dimensionless flow intensity parameter

t = variable of integration

Ψ_i* is a function of grain size, hydraulic radius, slope, specific weight, and viscosity. Correction factors are applied to account for hiding and pressure variations due to the composition of the bed-material mixture. Setting the probability of erosion equal to the probability of motion yields the Einstein bed-load function

$$1 - \frac{1}{\sqrt{\pi}} \int_{\eta_o}^{\eta} e^{-t^2} \, dt = \frac{A_* \Phi_*}{1 + A_* \Phi_*} \tag{9-25}$$

The equation can be transformed into the following and solved for sediment transport rate, q_B

$$i_B q_B = i_b \Phi_* \gamma_s d_i \sqrt{g d_i \left(\frac{\gamma_s - \gamma}{\gamma}\right)} \tag{9-26}$$

where Φ* is a function of Ψ* which is determined using empirically derived graphs provided by Einstein (1950) or ASCE (1975, pp 195-200).

c. Limitations. The dependence of the Einstein method on these empirical graphs, which were derived from limited data, limits the applicability of the method. The important contributions of this work were the introduction of the probability concept for bed-load movement, the identification of processes influencing entrainment and transport of sediment mixtures, and a formulation of the interactions. Einstein was aware of the limitations of his method and did not intend that it should be considered as a universal one.

Section V
Suspended Sediment Transport

9-15. Concentration Equation

The most important process in maintaining sediment in suspension is flow turbulence. In steady turbulent flow, velocity at any given point will fluctuate in both magnitude and direction. Turbulence is greatest near the boundary where velocity changes are the greatest. When dye is injected instantaneously at a point in a turbulent flow field, the cloud will expand as it is carried downstream at the mean velocity. This process is called diffusion and is the basis for the analytical description of sediment suspension. The one-dimensional sediment diffusion equation balances the upward flow of sediment due to diffusion with the settling of the sediment due to its weight

$$C\,\omega + \varepsilon_s \frac{\partial C}{\partial y} = 0 \tag{9-27}$$

where

C = sediment concentration

ω = settling velocity

ε_s = sediment diffusion coefficient

y = depth

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* For boundary roughness dominated flows, it is common practice to assume that the sediment diffusion coefficient is equal to the momentum diffusion coefficient, ε_m, which can be described by

$$\varepsilon_s = \varepsilon_m = \kappa U_* \frac{y}{D} (D - y) \tag{9-28}$$

where

κ = Von Karman constant

$U*$ = shear velocity

D = total water depth

Integration yields the Rouse equation:

$$\frac{C_y}{C_a} = \left(\frac{D - y}{y} \quad \frac{a}{D - a} \right)^z \tag{9-29}$$

$$z = \frac{\omega}{\kappa U_*} \tag{9-30}$$

where

a = reference elevation

C_a = concentration at reference elevation

C_y = concentration at depth y

The equation gives the concentration in terms of C_a , which is the concentration at some arbitrary level $y = a$. This requires foreknowledge of the concentration at some point in the vertical. Typically, this point is assumed to be close to the bed and C_a is assumed to be equal to the bed-load concentration. One problem with this equation is that concentration approaches infinity as y approaches zero. Therefore, the equation cannot be used to calculate the total sediment load from the bed to the surface. A graph of the Rouse suspended load distribution equation is shown in Figure 9-6.

9-16. Suspended Sediment Discharge

Suspended sediment discharge is calculated from the concentration profile using the following equation:

$$q_s = \int_{y=y_o}^{D} C_y u \; dy \tag{9-31}$$

where u is the local velocity. Solution of this equation requires an analytical description of the vertical velocity distribution.

a. Einstein's approach. Einstein (1950) assigned the lower limit of integration, $y_o = 2d_i$, and called this the thickness of the bed layer. He assumed that C_a was equal to the bed-load concentration. He used Keulegan's logarithmic velocity distribution equations to determine velocity. Since this work was done prior to the common usage of computer, Einstein prepared tables for the solution of the integral. These are found in Einstein (1950) and ASCE (1975) as well as other sediment transport texts. Total sediment transport can be calculated as a function of the bed-load concentration. The equation for total bed-material transport for particle size i is

$$q_i = q_{Bi} + q_{si} \tag{9-32}$$

$$q_{Bi} = i_b \, \Phi_{*i} \gamma_s d_i \sqrt{g d_i \left(\frac{\gamma_s \gamma}{\gamma} \right)} \tag{9-33}$$

$$q_{si} = i_b C_{ai} \int_{y=y_o}^{D} \left(\frac{D-y}{y} \quad \frac{a}{D-a} \right)^z u_* \, 5.75 \log \left(\frac{30.2y}{\Delta} \right) dy \tag{9-34}$$

where

a = thickness of the bed-load layer (Einstein considered $a = 2d_i$)

C_a = concentration in bed-load layer

d_i = geometric mean of particle diameters in each size class i

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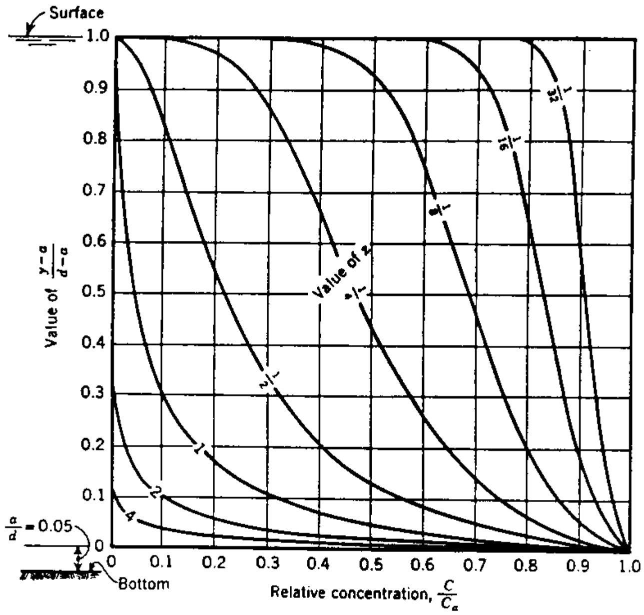

Figure 9-6. Rouse's suspended sediment concentration distribution for a/D = 0.5 and several values of z (ASCE 1975, p 77)

D = flow depth, bed to water surface

i = size class interval number

i_b = fraction of size class i in the bed

κ = von Karman constant = 0.4 in clear water

q_i = unit total bed material load in size class i

q_{si} = unit suspended bed material load in size class i

q_{Bi} = unit bed-load in size class i

y = any point in the flow depth measured above the bed

z = slope of the concentration distribution ($\omega_i/\kappa u_*$)

u_* = bed shear velocity

ω_i = settling velocity for grains of sediment in class interval i

Δ = apparent grain roughness diameter of bed surface

The total unit sediment discharge of the bed-material load is the sum of discharges for all particle sizes in the bed.

$$q_s = \Sigma_1^N q_{si} \qquad (9\text{-}35)$$

where n = number of size classes

b. Brooks approach. Brooks (1965) developed a graph that can be used to calculate suspended sediment transport if the sediment concentration at middepth is known. Using the Rouse equation, Brooks assigned a = 0.5 D. The lower limit of integration, y_o, was determined to be the depth where u = 0. Brooks used a power law velocity distribution equation and numerical integration to develop the curve shown in Figure 9-7. This figure can be used to determine total suspended sediment concentration when the concentration at middepth, the average velocity V, and the shear velocity U^* are known.

Section VI
Selecting a Sediment Transport Function

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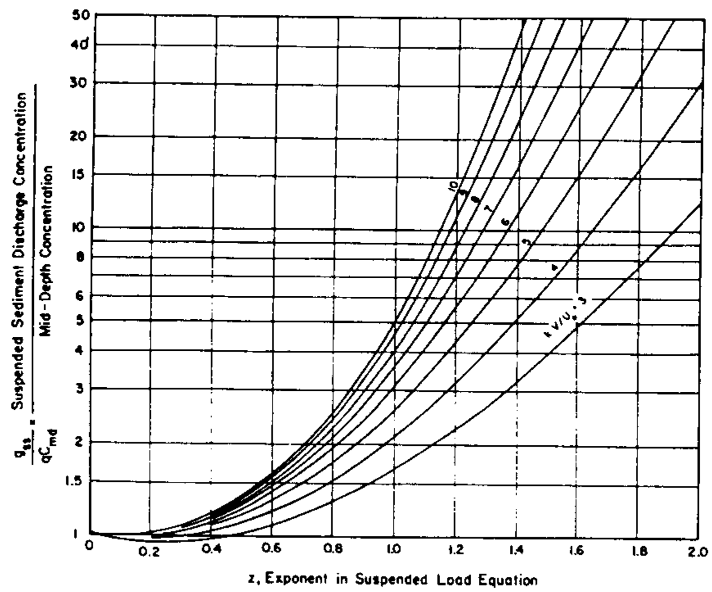

Figure 9-7. Brooks curve for suspended sediment concentration (ASCE 1975)

9-17. General

Most sediment transport functions predict a rate of sediment transport for a given set of steady-state hydraulic and bed-material conditions. Typically, hydraulic variables are laterally averaged. Some sediment transport equations were developed for calculation of bed load only, and others were developed for calculation of total bed material load. This distinction can be critical in sand-bed streams, where the suspended bed-material load may be orders of magnitude greater than the bed load. Another important difference in sediment transport functions is the manner in which grain size is treated. Most sediment transport functions were developed as single-grain-size functions, usually using the median bed-material size to represent the total bed. Single-grain-size functions are most appropriate in cases where equilibrium sediment transport can be assumed, i.e. when the project will not significantly change the existing hydraulic or sediment conditions. When the purpose of the sediment study is to evaluate the effect of a project on sediment transport characteristics (i.e., the project, or a flood, will introduce nonequilibrium conditions), then a multiple-grain-size sediment transport equation should be used. Multiple-grain-size functions are very sensitive to the grain-size distribution of the bed material. Extreme care must be exercised in order to ensure that the fine component of the bed-material gradation is representative of the bed surface for the specified discharge. This is very difficult without measured data. For this reason Einstein (1950) recommended ignoring the finest 10 percent of the bed material sample for computation of bed-material load with a multiple-grain-size function. Frequently, single-grain-size functions are converted to multiple-grain-size functions simply by calculating sediment transport using geometric mean diameters for each size class in the bed (sediment transport potential) and then assuming that transport of that size class (sediment transport capacity) can be obtained by multiplying the sediment transport potential by the bed fraction. This assumes that each size class fraction in the bed acts independent of other size classes on the bed, thus ignoring the effects of hiding, which can produce unreliable results.

9-18. Testing

It is important to test the predictive capability of a sediment transport equation against measured data in the project stream or in a similar stream before its adoption for use in a sediment study. Different functions were developed from different sets of field and laboratory data and are better suited to some applications than others. Different functions may give widely differing results for a

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* specified channel. Experience with sediment discharge formulas can be summed up in Figure 9-8.

9-19. Sediment Transport Equations

A generalized sediment transport equation can be presented in a functional form:

$$Q_s = f(V, D, S_e, B, d_e, \rho_s, G_{sf}, d_s, i_b, \rho, T) \tag{9-36}$$

where

B = effective width of flow

D = effective depth of flow

d_e = effective particle diameter of the mixture

d_s = geometric mean of particle diameters in each size class i

Q_s = total bed material discharge rate in units of weight divided by time

G_{sf} = grain shape factor

i_b = percentage of particles of the ith size class that are found in the bed expressed as a fraction

S_e = slope of energy line

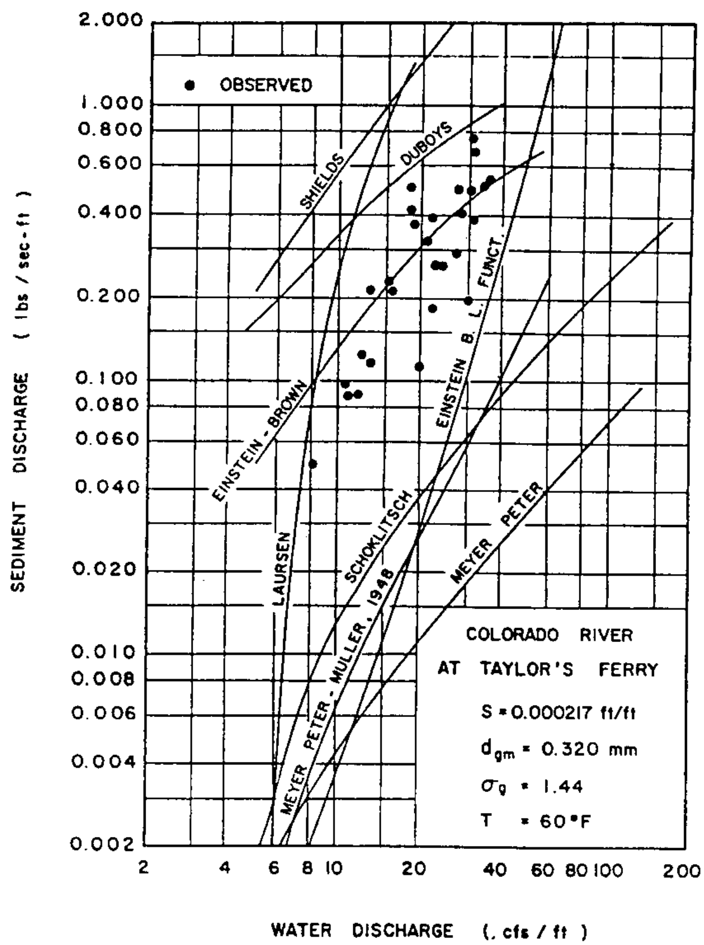

Figure 9-8. Sediment discharge rating curve, Colorado River (ASCE 1975)

*

* ρ = density of fluid for other than temperature effect

ρ_s = density of sediment particles

T = water temperature

V = average flow velocity

Of particular interest are the groupings of terms: hydraulic parameters (V,D,S_e,B), sediment particle parameters (d_e,ρ_s,G_{sf}), sediment mixture parameters (d_s, i_b), and fluid properties (ρ, T).

a. Processes. Although Einstein's (1950) work is classic and presents a complete view of the processes of equilibrium sediment transportation, it is more useful for understanding those processes than for application. Many other researchers have contributed sediment transport functions - always attempting to arrive at one which is always dependable when compared against field data. The choices are too numerous to name, and yet no single function has been proved superior to the others for the general case. The following general guidelines are given to aid in the selection of a transport function. However, it is important to confirm the selection using data from the project site. In the absence of such confirmation, the scatter between calculated values, similar to that shown in Figure 9-8, may be used in establishing a sensitivity range or a risk and uncertainty factor.

b. Colby (1964). The Colby equation has been used successfully on a limited class of shallow sand-bed streams with high sediment transport. The Colby function was developed as a single-grain-size function for both bed load and suspended bed-material load. Its unique feature is a correction factor for very high fine sediment concentrations. This correction factor may be used with other sediment transport equations and has been incorporated into the HEC-6 numerical model where it is used with all sediment-transport equations.

c. Einstein (1950). The Einstein equation has application for both sand and gravel bed streams. It is a multiple-grain-size sediment transport function that calculates both bed-load and suspended bed-material load. The hiding factor in the original equation has been modified by several investigators (Einstein and Chien 1953; Pemberton 1972; and Shen and Lu 1983) to improve performance on specific studies.

d. Laursen-Madden (Madden 1993). The Laursen (1958) sediment transport equation, which was based on flume data, was modified by Madden in 1963 based on data from the Arkansas River and again in 1985 using additional data from other sand-bed rivers. The equation calculates both bed-load and suspended bed-material load. It is a multiple-grain-size function, but it does not have a hiding factor. This feature makes its application in streams with a wide range of grain sizes questionable. The 1963 equation has been used successfully on large and intermediate size sand-bed rivers. The newer equation should be applicable in stream channels having sizes from sand to medium gravels.

e. Meyer-Peter and Muller (1948). This equation was developed from flume data and was developed as a multi-grain-size function, although it is frequently applied as a single-grain-size function. Sediment was transported as bed load in the Meyer-Peter and Muller flume. Its applicability is for bed-load transport in gravel-bed streams. It has been found to significantly underestimate transport of larger gravel sizes in several studies.

f. Toffaleti (1968). This multiple-grain-size function has been successfully used on many large sand-bed rivers. It calculates both bed load and bed-material suspended load and is based on extensive sand-bed river and flume data. Its formulation follows that of Einstein; however, there are significant differences. The Toffaleti equation generally underestimates the transport of gravel size classes. However, it has been combined with the Meyer-Peter and Muller equation in HEC-6 and SAM to provide an equation with more potential to transport a wider range of size classes.

g. Yang (1973, 1984). Yang developed two regression equations, one for sand and one for gravel, from extensive measured data on a wide variety of streams. This is a single-grain-size equation, and when applied as a multiple-grain-size function in HEC-6 or SAM it is done so without a hiding factor. The function is not as sensitive to grain size as other functions and, therefore, is less likely to produce wide variations in calculated sediment transport. It is most applicable to intermediate to small sand bed streams with primarily medium to coarse sand beds. It would not be appropriate if significant armoring or hydraulic sorting of the bed is expected.

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9-20. Guidance Program in SAM

A guidance module was included in the SAM hydraulic design package to aid in the selection of a sediment transport function. The significant hydraulic and sediment variables of slope, velocity, width, depth, and median grain size applicable to a given stream are provided to the computer program. The program then checks the given data against 17 sets of field data collected by Brownlie (1983) and looks for a river with similar characteristics. Ten sediment transport equations were tested with each of the 17 data sets and the best three were determined. The program then reports to the user which are the three best sediment transport equations for each of the data sets with hydraulic characteristics that matched the given stream.

9-21. Procedure for Calculating Sediment-Discharge Rating Curve

The steps in calculating a sediment-discharge rating curve from the bed-material gradation are:

a. Assemble field data (cross sections and bed gradations).

b. Develop representative values for hydraulic variables and for bed gradation from the field measurements.

c. Calculate the stage-discharge rating curve accounting for possible regime shifts due to bed-form change.

d. Calculate the bed-material sediment-discharge rating curve using hydraulic parameters from the stage-discharge calculation.

Section VII
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