Appendix E
Rotational Stability of Intake Towers

E-1. Rocking Potential

For a rigid tower, or a flexible tower idealized by a single-degree-of-freedom (SDOF) system, rocking will occur when the disturbing moment exceeds the restoring moment due to the weight of the tower. This criterion is based on satisfying the moment equilibrium of the rigid block model shown in Figure E-1:

$$MS_A\,(H/2) > Mg\,(B/2) \tag{E-1}$$

where

M = the total mass of the tower including the added hydrodynamic mass of the surrounding and internal water

S_A = spectral acceleration

H = height of the block

g = acceleration due to gravity

B = base of the block

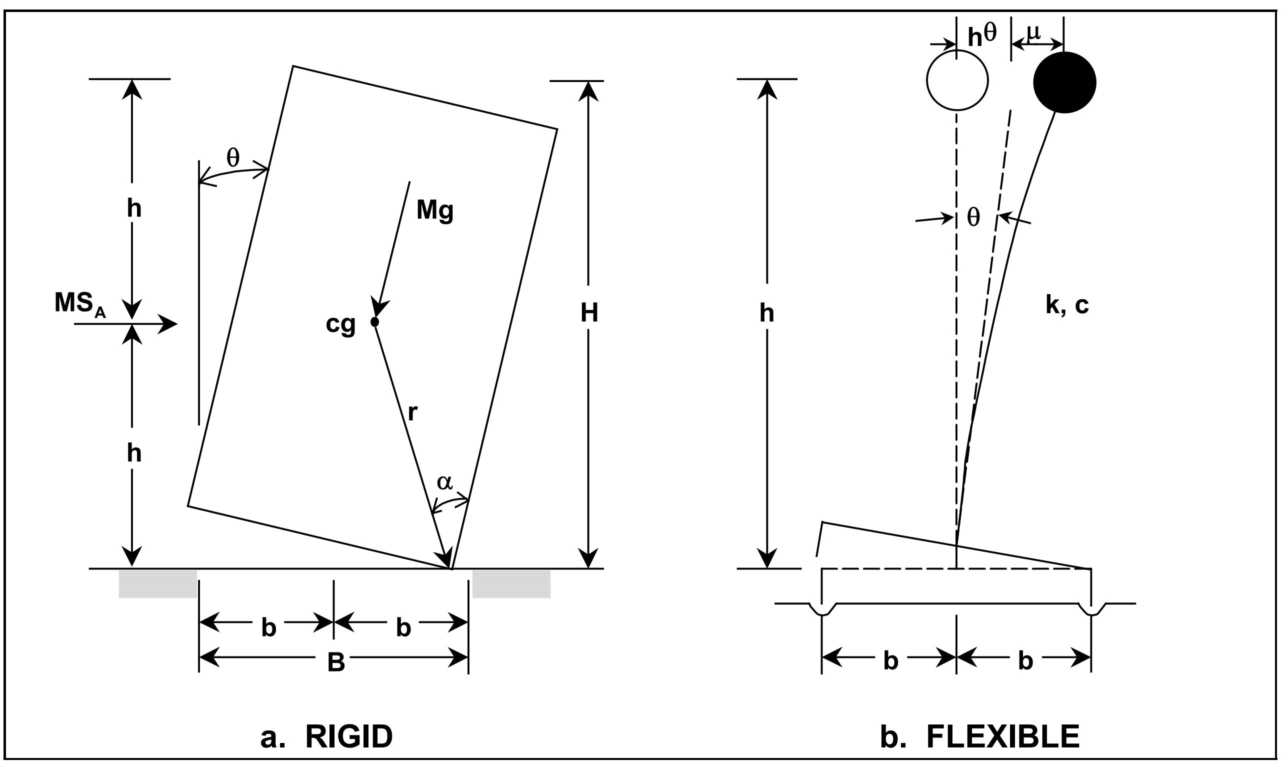

Figure E-1. Rigid block and SDOF models for dynamic rotational stability (where *c* = damping coefficient for the structure; θ = rigid body rotational degree of freedom; μ = flexural deflection of the top of the structure; and *k* = stiffness of the structure

Rocking occurs when the spectral acceleration S_A is greater than the product of the acceleration of gravity g and the ratio of the base width to the tower height:

$$S_A > g\,(b/h) \tag{E-2}$$

where

$b = B/2$ (one-half the base of the block)

$h = H/2$ (one-half the height of the block)

E-2. Rocking Potential Example

The tower in Appendix C (Figure C-1) will be investigated for rotational stability using the response spectrum shown in Figure C-3. From Table C-11, the frequency of the first mode including hydrodynamic effects is 2.22 Hz or a period of 0.45 sec. From the response spectra (for 5 percent damping) shown in Figure C-3, the first mode spectral acceleration S_A is 0.62 g. The tower has a *b/h* ratio of 0.24; therefore, rocking can occur during the maximum design earthquake (MDE) represented by the response spectrum.

E-3. Housner's Rigid Block Model

a. Although rocking occurs, the tower will not necessarily be rotationally unstable during the operational basis earthquake (OBE) or MDE. Using spectral acceleration as the only indicator of earthquake demand is not a sufficient basis for evaluating overturning or tipping during earthquakes. Therefore two other dynamic motion parameters, i.e., spectral velocity and spectral displacement, must be included for evaluating overturning, and an energy-based characterization of the ground motion should be used. Housner (1963) developed a rigid block model for structures with a uniform mass distribution on a rigid foundation that relates overturning or tipping to the spectral velocity S_V. This criterion is based on satisfying conservation of kinetic and potential energies for the slender rigid blocks shown in Figure E-2. If a slender block is given an initial velocity that is just sufficient to rock the block to the point of impending static instability, then equating the initial kinetic energy of the block to the increased potential energy at the point of overturning results in the following equation:

$$\frac{WR\alpha 2}{2} = \frac{1}{2}\frac{W}{g}\frac{MR^2}{I}S_V^2 \tag{E-3}$$

where

W = weight of the structure

R = distance from the center of gravity to the corner about which rotation occurs

I = mass moment of inertia about corner

The spectral velocity S_V approximates the maximum initial velocity of the block, and the critical angle of rotation α is a small angle (less than 20 degrees). For small angles $\alpha \approx b/h$ and $r \approx h$. For slender structures MR^2/I is almost equal to unity. For tall slender blocks Equation E-3 simplifies to

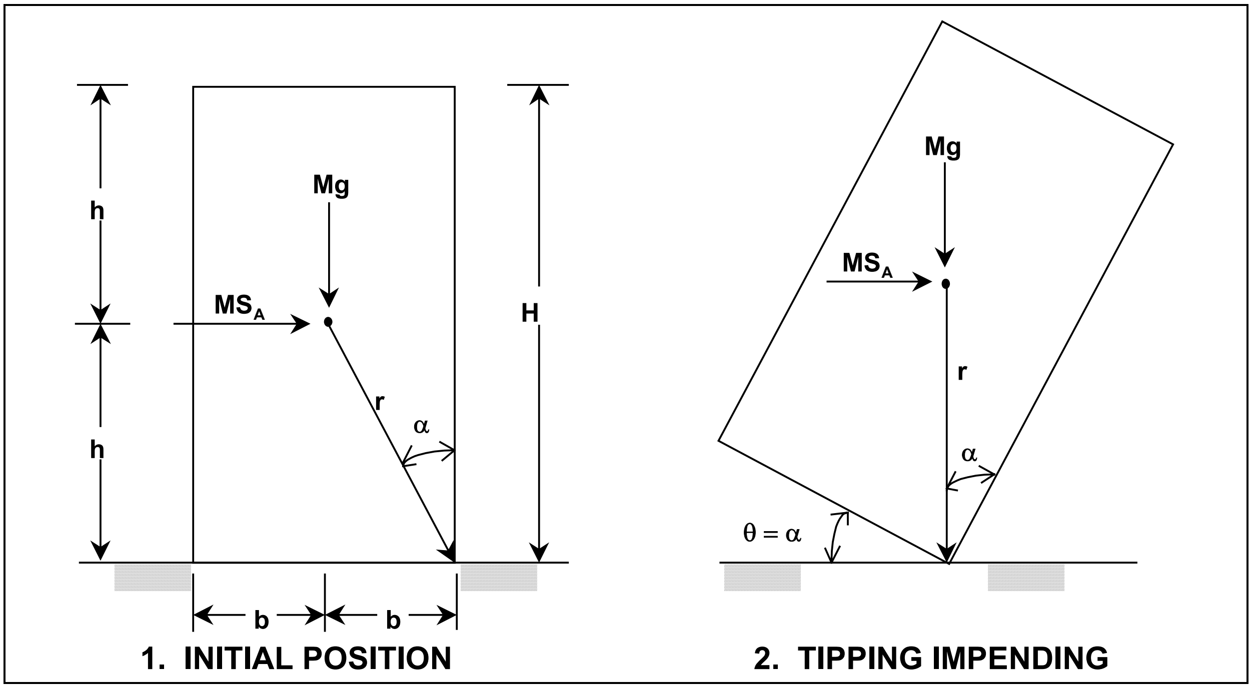

Figure E-2. Housner's model for slender rigid blocks

$$\alpha_{cr} = \frac{S_V}{\sqrt{gr}} \tag{E-4}$$

where

α_{cr} = critical value of the angle of rotation from the initial upright position of static equilibrium to the final tipping position of impending rotational instability. The instantaneous center of rotation is at the toe of the slender rigid block (radians).

S_V = the response spectral-pseudo velocity for the OBE or MDE ground motion (cm per sec)

g = acceleration of gravity (981.4 cm per sec squared)

r = the radial distance from the toe to the center of mass of the tower (cm)

b. According to Housner, this equation may be interpreted as stating that for a given spectral value S_V, a block that rocks through an angle α will have approximately a 50 percent probability of being overturned. However, a 50 percent probability of failure or survival is not always an acceptable criterion for stability because there is an unexpected scale effect that makes the larger of two geometrically similar blocks more stable than the smaller block.

c. The scaling effect can be estimated for a damped structure by using the relationships for the spectral-pseudo velocity S_V, spectral-pseudo acceleration S_a, and the spectral-pseudo displacement S_d. These relationships are appropriate because accelerograms recorded during earthquakes show the maximum relative displacements occur when the relative velocities are zero; the maximum values of the relative displacements are proportional to the maximum absolute accelerations. The shapes of the maximum peaks are approximately sinusoidal with a natural frequency of ω (rad/sec) = $T/2\pi$ where T is the period of the structure.

So

$$S_V = \omega\, S_d = S_a / \omega \qquad \text{(E-5)}$$

and

$$S_V{}^2 = S_a\, S_d \qquad \text{(E-6)}$$

d. This relationship clearly shows that the earthquake demand acceleration and displacement are critical parameters for evaluating overturning, and that considering either S_a or S_d alone as the earthquake demand is not sufficient. Substituting Equation E-6 into Equation E-4 yields:

$$\alpha = \sqrt{\frac{S_a}{g}\frac{S_d}{r}} = \frac{b}{h} \qquad \text{(E-7)}$$

If S_a is sufficient to cause overturning of the rocking block, then combining Equation E-7 and the inequality E-2 yields:

$$\left[\frac{gr}{S_d}\right]\left(\frac{b}{h}\right)^2 \rangle\, g\frac{b}{h} \qquad \text{(E-8)}$$

For slender rigid blocks, r is approximately equal to h, so

$$S_d < b \qquad \text{(E-9)}$$

The scaling effect shows that if S_a is just sufficient to cause static overturning, the block will start to rotate; but, if the block is to overturn, its displacement as approximated by S_d must be equal to half the base width of the block. In general, S_d is never large enough to cause a typical tower to overturn. Conversely, these equations must be extended to evaluate the seismic stability of other structures subjected to significant net lateral loads.

E-4. Seismic Rotational Stability Example

a. For the tower in Appendix C, the geometry in the transverse direction is characterized by the following:

(1) Distance from the bottom of pedestal to c.g. h, of 23.25 m (75.729 ft) and a half base width b of 7.36 m (24 ft)

(2) An angle α = 0.3068 radian (17.58 deg)

(3) A radial distance r of 24.2 m (78.83 ft)

b. The earthquake demand for the site can be estimated using the response spectrum shown in Figure C-3.

(1) The spectral pseudo-acceleration is 0.62g for a period of vibration of 0.45 sec.

(2) The spectral pseudo-velocity and displacement are computed to be 0.436 m per sec (1.42 ft per sec) and 0.031 m (0.1 ft), respectively.

$S_V = S_a T / 2\pi = 0.62 \times 981.4 \times 0.45 / 2 \times 3.14 = 0.436$ m per sec (1.42 ft per sec)

and

$S_d = S_V\, T / 2\pi = 0.436 \times 0.45 / 2 \times 3.14$
$= 0.031$ m (0.1 ft)
$<< b = 7.36$ m (24 ft)

c. The critical angle of rotation is computed from Equation E-4 to be 0.015 rad (0.859 deg).

$$\alpha_{cr} = \frac{S_v}{\sqrt{gr}}$$

$$\alpha_{cr} = \frac{0.435}{\sqrt{981.4 \times 24.2}} = 0.015 \text{ rad (0.859 deg)}$$

The actual angle α is less than the critical value from Equation E-4, and the spectral displacement is less than half of the base width. Therefore, rotational instability due to seismic rocking as represented by the response spectrum is highly unlikely.