Appendix B
Seismic Analysis for Preliminary Design or Screening Evaluation of Free-Standing Intake Towers

B-1. Introduction

a. General. Guidance for the preliminary design or screening evaluation of intake towers is based on linear elastic response spectra modal techniques, and a brief discussion of nonlinear dynamic analyses is also presented. The procedures presented here are adequate for the seismic analysis of intake towers that are square, rectangular, or circular in plan and bend about a elastic center line as a cantilever beam.

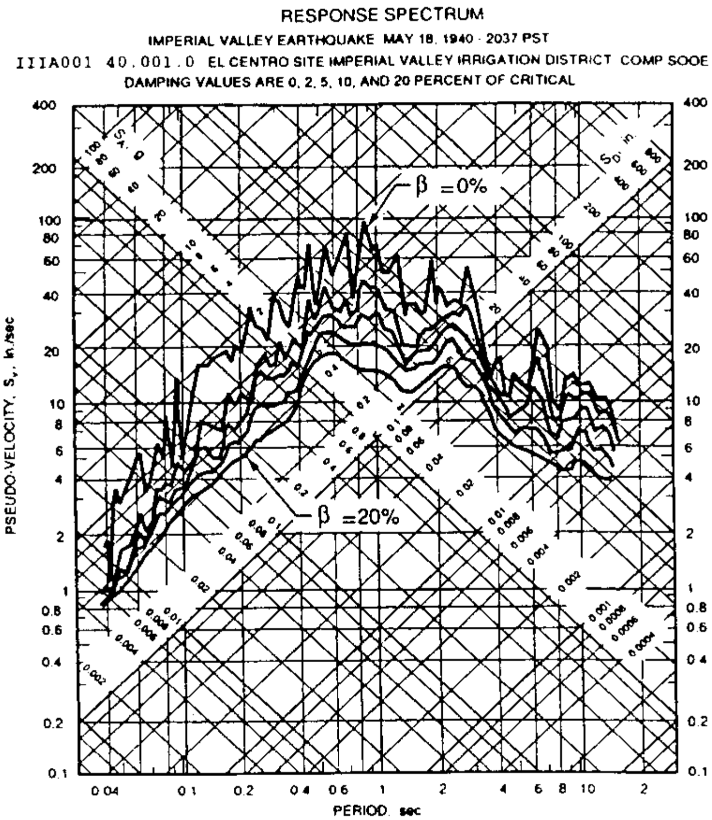
b. Standard spectra. Standard spectra are commonly used for preliminary design when site-specific response spectra are not available. A Newmark and Hall Standard Response Spectrum is illustrated in Figure B-1. ER 1110-2-1806 and EM 1110-2-6050 also provide guidance on the use of standard response spectra, and an example of the standard acceleration response spectra is shown in Figure B-2.

c. Smoothed response spectra. Actual site response spectra generated from specific earthquake accelerograms tend to be very jagged, as indicated in Figure B-2a, and are not necessarily good design tools. Therefore a modified form of the response spectrum, developed by averaging the response values from many response spectra for several different earthquakes and then smoothing the resulting peaks and valleys, is used for analysis. Equal hazard spectra developed by probabilistic methods are also used for analysis. More information about response spectra used for the design or evaluation of hydraulic structures is in EM 1110-2-6050.

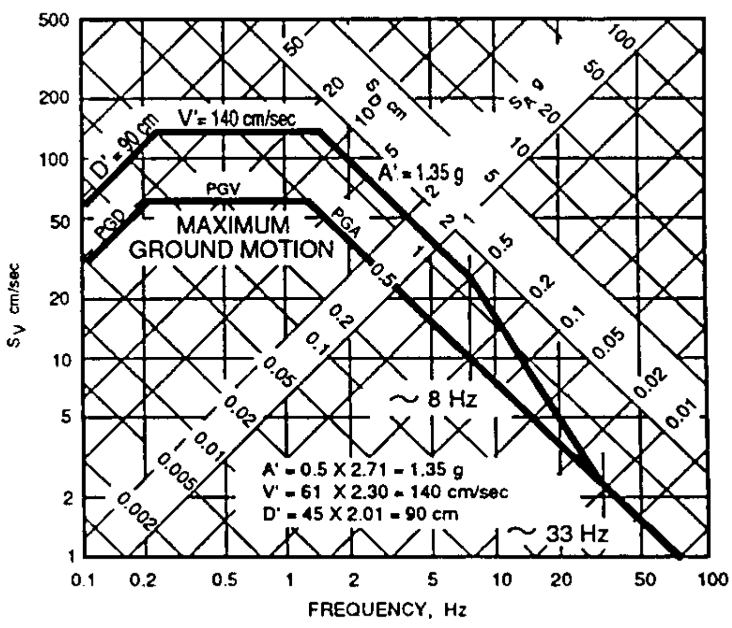
d. Linear elastic response spectrum modal analyses (RSMA). Response spectra may be used for the analysis of multiple-degree-of-freedom (MDOF) systems by entering the spectrum with any one of the natural periods of the MDOF system. In such circumstances, the spectrum will yield only the response for the single vibration mode having that natural period. It is necessary in such an approach to find the individual responses for several modes (and their natural periods), then combine them to find the total response. The response spectrum provide the maximum displacement (and pseudo-quantities) for each natural period, even though each maximum displacement occurs at a different time. Using direct superposition to combine the displacements would accumulate all maximum displacements as if they were occurring at the same instant. Such a solution would grossly distort the structural response. In addition, the displacements bear no algebraic signs. Only the absolute value of the displacement is known. The direct sum, without algebraic signs, would further distort the response. Two rational means of combining several responses from the response spectrum are the square-root-of-the-sum-of-the-squares (SRSS) method and the complete-quadratic combination (CQC) method. Both methods attempt to find the least upper bound for the behavioral response.

(1) SRSS method.

(a) SRSS is an approximate method for combining modal responses. In the SRSS method, the squares of a specific response are summed (e.g., displacement, drift, story/base shear, story/base overturning moments, elemental forces, etc.). The square root of this sum is taken to be the combined effect. It is important to note that the quantities being combined (e.g., story drift, base shear, etc.) are those for each individual mode.



a. Actual Response Spectra for a Recorded Ground Motion



b. Newmark and Hall Smooth-Shaped Response Spectra

Figure B-1. Typical tripartite plot and smooth-shaped response spectrum (Newmark and Hall 1982)

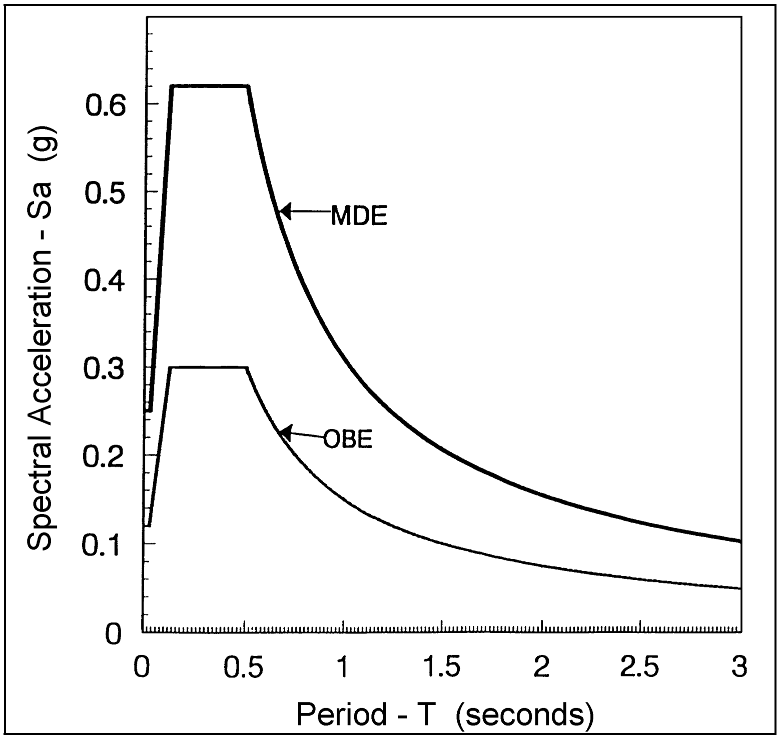


Figure B-2. Standard acceleration response spectra

(b) The governing equation of the SRSS method can be written mathematically as

$$u_i = \sqrt{\sum_{j=1}^{n} u_{ij}^2} \tag{B-1}$$

where

u_i = approximate maximum response for the i^{th} component of the behavioral response

n = number of modes to be used in the analysis

u_{ij} = i^{th} component of the j^{th} modal behavioral response vector

The SRSS method is conservative most of the time. It tends to be unconservative when the modal frequencies are closely spaced.

(2) CQC method. CQC is a modal combination method based on the use of cross-modal coefficients. The cross-modal coefficients reflect the duration and frequency content of the seismic event as well as the modal frequencies and damping ratios of the structure.

(a) Mathematically, the governing equation of the CQC method can be written as:

$$u_i = \sqrt{\sum_{j=1}^{n} \sum_{k=1}^{n} u_{ij} \rho_{jk} u_{ik}} \tag{B-2}$$

where

u_{ik} = i^{th} component of the k^{th} modal response vector

ρ_{jk} = cross-modal coefficient for the j^{th} and k^{th} modes

As indicated in *d* above, the behavioral responses are calculated for each mode prior to their superposition in CQC. If the duration of the earthquake is long compared with the periods of the structure and if the earthquake spectrum is smooth over a wide range of frequencies, the cross-modal coefficient can be approximated as:

$$\rho_{jk} = \frac{8\xi^2(1+r)r^{1.5}}{(1+r^2)^2 + 4\xi^2 r(1+r)^2} \tag{B-3}$$

where *r* is the reference level and ξ is the modal damping ratio and

$$r = \frac{T_k}{T_j} \tag{B-4}$$

where

T_k = period of the system for mode *k*

T_j = period of the system for mode *j*

Note that if the frequencies are well separated, the off-diagonal terms approach zero and the CQC method reverts to the SRSS method.

B-2. RSMA of Intake Tower

A response spectrum modal analysis of an intake tower can be performed the using an approximate two-mode model, a beam model, or a finite element model.

a. Approximate two-mode model. The approximate two-mode model is a manual procedure for the preliminary analysis of intake towers. The two-mode method includes bending deformations only, which provides sufficient accuracy for preliminary analysis of intake towers that are regular in plan and elevation and are supported on rigid foundation. If a sufficient number of lumped masses are used to represent the distributed mass system, any error introduced by the lumped mass approximation will be negligible. A minimum of six lumped masses should be used for modeling an intake tower. A two-mode, added-mass analysis can be performed on a spreadsheet. The following six-step procedure applies to the analysis of a free-standing stepped tower, such as that shown in Figure B-3. An example of the two-mode approximate method is presented in Appendix C:

(1) Determine the mass per unit length.

(a) An added mass concept is used to simulate the hydrodynamic effects of the mass of water in the tower and the water surrounding the tower. Establish all discontinuities in size, stiffness, mass, and added mass. For each piecewise continuous segment, determine the flexural moment of inertia *I* of the cross section about both the xx- and yy-axes, the mass of the structure per unit length m_d, the added mass of water per unit length outside the tower m_a^o, the added mass of water per unit length inside the tower m_a^i

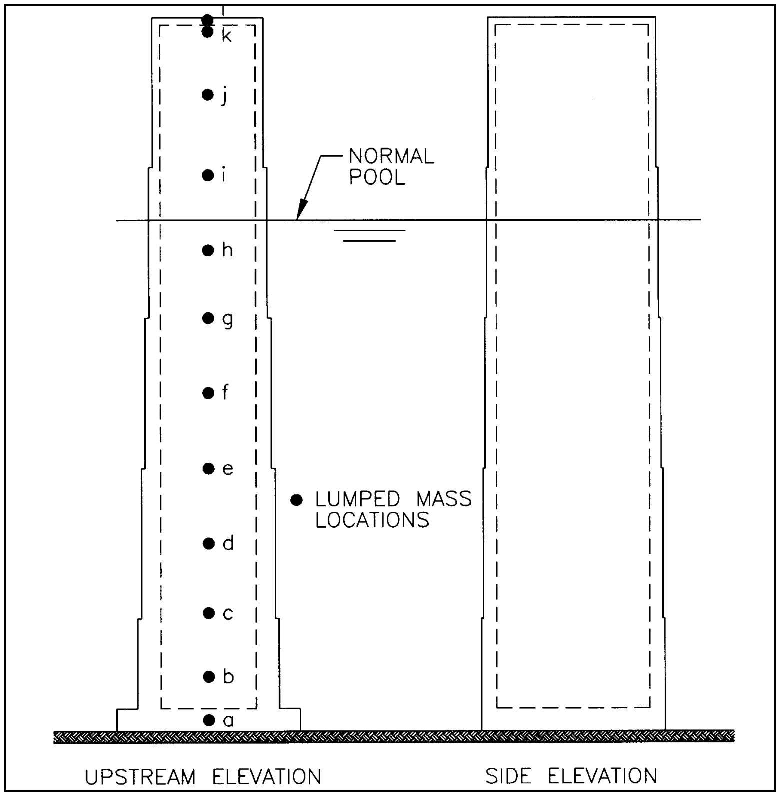


Figure B-3. Tower geometry

and the total mass per unit length $m(z)$ of the tower at any height z/L, where L is the total height of the tower. The total mass per unit length of the tower is the sum of the individual masses:

$$m(z) = m_d(z) + m_a^o(z) + m_a^i(z) \tag{B-5}$$

where

z = coordinate measured along the height of the tower

$m_d(z)$ = actual mass of the tower

$m_a^o(z)$ = hydrodynamic added mass due to the surrounding water

$m_a^i(z)$ = hydrodynamic added mass due to theinternal water

(b) The hydrodynamic added mass $m_a^o(z)$ can be found for the two-mode case from Figure B-4 and the mass $m_a^i(z)$ from Figure B-5. The use of these figures to obtain the added hydrodynamic mass is illustrated in Appendix C. A refined procedure for determining more accurately the added hydrodynamic mass is described in Appendix D. The total mass M_n of any segment n is then computed as

$$M_n = m(z)(\Delta L) \tag{B-6}$$

where ΔL is the length of the segment n.

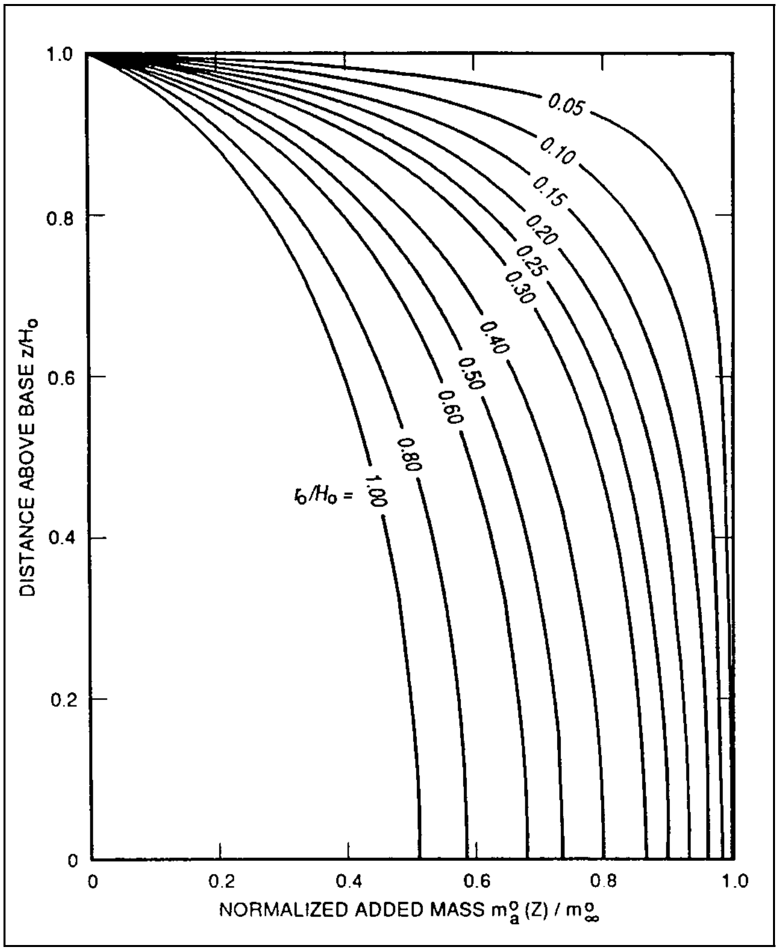


Figure B-4. Hydrodynamic added mass m_a^o from the two-mode approximate analysis procedure

(2) Determine the stiffness per unit length. The shape function φ in a distributed-mass analysis corresponds to the mode shape in a lumped-mass analysis. The shape functions for distributed-mass towers are given in Tables B-1 and B-2 for various amounts of taper. The displacement of the *normalized* shape functions at the tenth points are given directly in the tables. (The normalized shape function has a displacement of 1 at the top of the tower.) If the structural mass is the only mass (no added masses), the period can also be computed from the coefficients given in the tables. Note that if Tables B-1 and B-2 are used, the ratio of moments of inertia for bending about the xx-axis will not likely be the same as that for bending about the yy-axis. There will, therefore, be two values of the natural period T (one for each shape function) in each of the x- and y-directions. The generalized stiffness k^* may be computed from the coefficients given in Tables B-1 and B-2. The value of k^* will be needed in subsequent calculations and should be determined at this point.

(3) Determine the normalization ratio (L_n/m *). The normalization ratio L_n/m^* is computed directly and recorded. Physically, it is the ratio of the displacement (or acceleration) of the actual shape function to the displacement (or acceleration) of the normalized shape function. The normalization factor L_n is given by

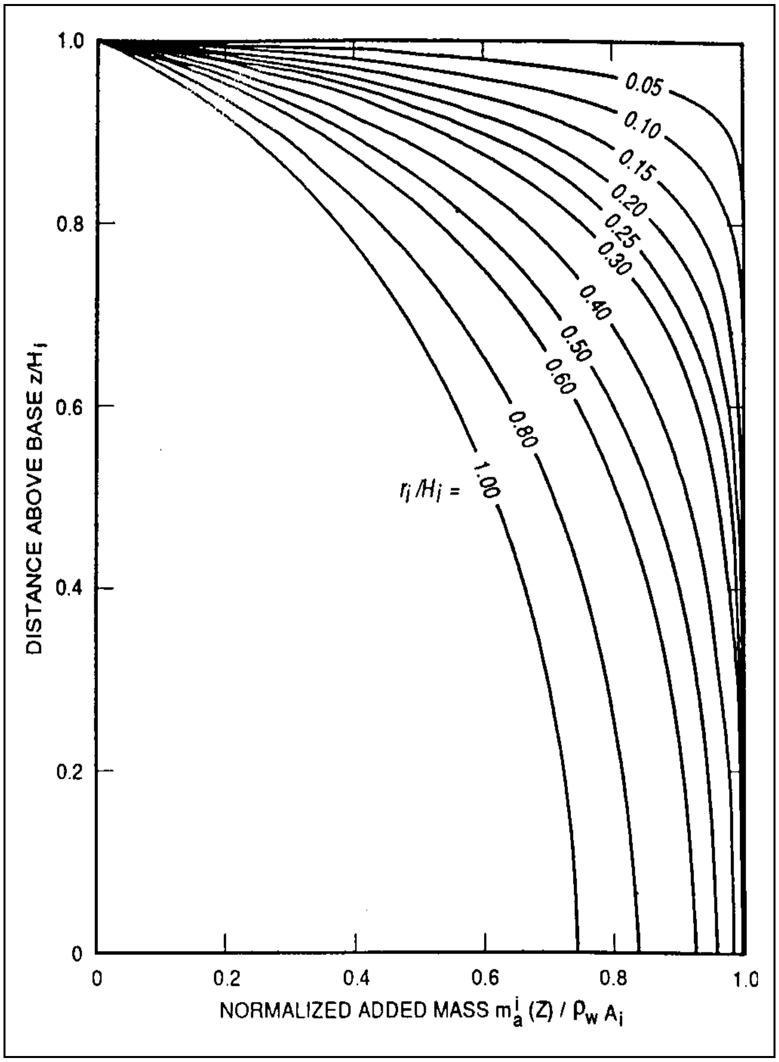


Figure B-5. Hydrodynamic added mass m_a^i from the two-mode analysis procedure

$$L_n \sum_{0}^{L} M_n \phi_n \tag{B-7}$$

and the generalized mass m^* is given by

$$m^* = \sum_{0}^{L} M_n \phi_n^2 \tag{B-8}$$

where M_n is found in step 1, and ϕ_n is the distance from the center line of the tower to the center of the deflected segment n.

(4) Determine the natural period of vibration T in seconds, the natural frequency ω in radians per second, and the mode shapes for the first two shape functions in both directions:

$$T = 2\pi\sqrt{\frac{m*}{k^*}} \tag{B-9}$$

and

$$\omega = \sqrt{\frac{k^*}{m^*}} \tag{B-10}$$

(5) Determine the spectral pseudo-acceleration S_A from the response spectrum. For the spectral pseudo-acceleration S_A, compute the pseudo-displacement S_D, where

$$S_D = \frac{S_A}{\omega^2} \tag{B-11}$$

Physically, the spectral pseudo-displacement S_D times the normalization value 1 is the normalized displacement at the top of the tower.

(6) Determine the lateral displacement, and the inertial forces, shears, and moments at the center of each segment.

(a) The actual lateral displacement of any mass M_n is given by

$$Y_n = (L_n/m^*)S_D\phi_n \tag{B-12}$$

(b) The inertial force acting on any mass M_n is computed as

$$F_n = M_n a_n = (L_n/m^*)M_n S_A \phi_n \tag{B-13}$$

where a_n is the actual acceleration of the mass M_n.

(c) The shear at the center of any segment is the sum of all inertial forces above that level:

$$V_n = \sum_{n+1}^{L} F_{n+1} \tag{B-14}$$

(d) The moment at the center of any segment n is the sum of moments above that level:

$$M_n = M_{n+1} + V_{n+1}(Z_{n+1} - Z_n)L \tag{B-15}$$

where Z_m is the nth coordinate measured along the height of the tower

b. Computer analysis using beam elements.

(1) Typically, the output from a beam modal analysis is in terms of moments, shears, and thrusts that can be directly used to design or evaluate reinforced concrete sections. Beam or finite element modals can be used with response spectra modal analyses as deemed appropriate by the structural engineer.

(2) Alternatively, the structure may be modeled using beam elements, as per the two-mode approximate method, and analyzed using a structural analysis computer program with dynamic analysis capability.

Computer programs are capable of analyzing as many modes of vibration as there are degrees of freedom, although for regular towers only the first two or three modes of vibration need usually be considered. Bending, shear, and torsional stiffness can be considered in a computer analysis. Some computer programs can also model structure-foundation interaction and structure-reservoir interaction. The use of beam elements is satisfactory for towers that are regular in plan and elevation, but it may not be appropriate for irregular towers. The added-mass concept described in paragraph B-2*a*(1) can be used to account for the hydrodynamic effects of the internal and surrounding water. The refined procedure of Appendix D for computing hydrodynamic added mass remains valid for beam element analyses.

c. Computer analysis using finite elements. The finite element response spectrum approach can be used for intake towers that cannot be adequately modeled with beam elements (e.g., irregular towers). RSMA of intake towers can also be performed by using finite elements such as plate, shell, and three-dimensional solid elements for either preliminary or final design using commercially available software with dynamic analysis capability (SAP, ANSYS, ADINA, GTSTRUDL, STAAD, etc.). Dynamic analysis performed by commercially available software can include the effects of both flexural and shear deformation and can consider the effects of any number of modes of vibration.

(1) The primary advantage of this approach lies in its ability to evaluate three-dimensional systems, to include torsional effects, and to pinpoint areas of localized stress concentrations. The finite element method warrants consideration when the tower is irregular in plan and/or elevation (e.g., torsion, vertically soft areas, major openings, etc.); or when soil-structure interaction is important (e.g., embedment greater than one third of the total tower height, siltation surrounding the tower, nonrigid foundation, etc.). Multiple analyses are often used to bracket their effects on the response of the tower.

(2) Two problems exist when finite elements are used to model the foundation of a structure subjected to seismic input.

(a) First, the sides and base of the elements do not allow the wave motion to permeate the boundaries (base and sides). The waves are, therefore, reflected back into the finite element grid, which distorts the results.

(b) Second, it is inappropriate to excite the base of the foundation model with an earthquake motion recorded at the ground surface. Several computer programs have a feature called a transmitting boundary that alleviates this problem. Otherwise, the foundation can be modeled as massless. This is equivalent to using massless springs, and the motion at the base of the foundation is equivalent to the motions at the surface. The size of the foundation model should be no less than three times the longest base dimension of the structure in width and two times the longest base dimension in depth. The model size and influence can be checked statically by increasing the size of the foundation modal and checking the difference in the static stresses at the base of the structure. The correct dimensions are found once the difference in the stresses becomes negligible. Most commercial finite element programs in structural analysis can perform a response spectrum analysis, but the resulting analysis provides only the *absolute maximum* stresses. Therefore, actual stress distributions are not provided by the analysis. The structural engineer must make some assumptions regarding the nature and shape of the distributions. Using these assumptions, the engineer then converts the results into the moments, shears, and axial loads required for the ultimate strength design (see Section 2-8 of EM 1110-2-6050).

(3) The procedure for a finite element response spectrum analysis by finite elements is as follows:

(a) Determine the added mass. The added-mass concept as explained in paragraph B-2*a*(1) remains valid in the finite element approach. The total added mass is distributed to the elements in a way that approximates the actual location and effects of the water.

(b) Generate the structural mesh. The finite element mesh should be designed to represent adequately the structural configuration and stiffness. The elements (beam, plates, and/or solids) should be chosen and sized to represent both the shear and bending effects of the seismic event adequately.

(c) Generate the soil/foundation mesh. If the foundation of the tower is very stiff relative to the tower, or if the effect of a more flexible foundation causes a smaller pseudo-acceleration to be chosen from the design spectra, the base may be modeled as rigid, which will produce conservative results. The more flexible the base, the larger the period; therefore, it is conservative to use a rigid base when the period is on the descending portion of the acceleration response spectrum curve. If the foundation cannot be modeled as rigid, it can be modeled with the use of springs or finite elements. Springs have often been used to model soil-structure interaction (Hall and Radhakrishnan 1983). The difficulty lies in choosing spring constants that reflect the vertical, horizontal, and rotational stiffnesses of the foundation with adequate accuracy. These constants are dependent on the size of the structure, the design loads, the loading history, etc., all of which are quite difficult to quantify. The spring constants are usually given upper and lower bounds and are used in multiple analysis to bracket their effects on the response of the tower.

Table B-1
Displacements for Step-Tapered Towers for the First Shape Function

First Shape Function										
I_{BASE} / I_{TOP}	1	2	3	4	5	6	7	8	9	10
With or without added mass, Coeff. of k^*	3.091	5.394	7.391	9.194	10.86	12.41	13.88	15.27	16.60	17.87
When there is no added mass:										
Coeff. of m^*	0.250	0.253	0.254	0.254	0.254	0.254	0.254	0.254	0.254	0.254
L_n/m^*	1.566	1.633	1.677	1.710	1.737	1.760	1.781	1.799	1.815	1.830
Coeff. of T	1.787	1.360	1.164	1.045	0.962	0.900	0.851	0.811	0.777	0.749
Value of The Shape Function at The Tenth Points (ϕ)										
1.0L	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.9L	0.862	0.856	0.853	0.849	0.847	0.845	0.843	0.841	0.840	0.838
0.8L	0.725	0.714	0.706	0.701	0.696	0.692	0.689	0.685	0.683	0.680
0.7L	0.591	0.575	0.565	0.557	0.551	0.545	0.540	0.536	0.533	0.529
0.6L	0.461	0.443	0.431	0.422	0.415	0.409	0.404	0.400	0.396	0.392
0.5L	0.340	0.321	0.309	0.301	0.294	0.288	0.283	0.279	0.275	0.272
0.4L	0.230	0.214	0.204	0.197	0.191	0.186	0.182	0.179	0.176	0.173
0.3L	0.136	0.125	0.118	0.113	0.109	0.106	0.103	0.101	0.098	0.097
0.2L	0.064	0.058	0.054	0.051	0.049	0.048	0.046	0.045	0.044	0.043
0.1L	0.017	0.015	0.014	0.014	0.013	0.013	0.012	0.012	0.012	0.011
0.0L	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Stiffness $k^* = \text{coeff.} \times \dfrac{EI_{TOP}}{L^3}$ for all values of $\dfrac{I_{BASE}}{L_{TOP}}$

With no added mass: m_{TOP} = mass of top step
Mass $m^* = \text{coeff.} \times m_{TOP}\, L$

$$\text{Period } T = 2\pi\sqrt{\frac{m^*}{k^*}}$$

L = overall height of tower

E = modulus of elasticity

I_{TOP} = moment of inertia of the top step

When an added mass exists, see paragraph B-2*a*(4) for the calculation of period *T*.

Table B-2
Displacements for Step-Tapered Towers for the Second Shape Function

Second Shape Function										
$\frac{I_{base}}{I_{top}}$	1	2	3	4	5	6	7	8	9	10
with or without added mass, coeff. of *k**	121.4	137.4	147.3	154.7	160.6	165.6	169.9	173.7	177.1	180.3
when there is no added mass:										
coeff. of *m**	0.250	0.216	0.198	0.187	0.180	0.174	0.169	0.165	0.162	0.159
l_n/m^*	-0.868	-0.939	-0.976	-0.999	-1.015	-1.027	-1.036	-1.043	-1.048	-1.053
coeff. of *t*	0.285	0.249	0.231	0.219	0.210	0.204	0.198	0.194	0.190	0.187
Value Of The Shape Function At The Tenth Points (ϕ)										
1.0l	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.9l	0.524	0.545	0.556	0.563	0.568	0.572	0.574	0.577	0.579	0.580
0.8l	0.070	0.117	0.141	0.156	0.167	0.176	0.183	0.188	0.193	0.197
0.7l	-0.317	-0.241	-0.201	-0.176	-0.157	-0.143	-0.131	-0.121	-0.113	-0.106
0.6l	-0.589	-0.483	-0.428	-0.392	-0.366	-0.346	-0.329	-0.316	-0.304	-0.294
0.5l	-0.714	-0.585	-0.520	-0.477	-0.446	-0.422	-0.402	-0.386	-0.372	-0.360
0.4l	-0.683	-0.552	-0.486	-0.443	-0.413	-0.389	-0.370	-0.354	-0.341	-0.329
0.3l	-0.526	-0.416	-0.361	-0.326	-0.301	-0.282	-0.267	-0.255	-0.244	-0.235
0.2l	-0.301	-0.233	-0.200	-0.179	-0.165	-0.153	-0.145	-0.137	-0.131	-0.126
0.1l	-0.093	-0.071	-0.060	-0.054	-0.049	-0.046	-0.043	-0.041	-0.039	-0.037
0.0l	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Stiffness $k^* = \text{coeff.} \times \dfrac{EI_{TOP}}{l^3}$ for all values of $\dfrac{I_{BASE}}{I_{TOP}}$

with no added mass:
mass $m^* = \text{coeff.} \times m_{top} l$ $\qquad$ *mtop* = mass of top step

$$\text{Period } T = 2\pi\sqrt{\frac{m^*}{k^*}}$$

l = overall height of tower

E = modulus of elasticity

I_{top} = moment of inertia of the top step

When an added mass exists, see paragraph B-2*a*(4) for the calculation of period *T*.