CHAPTER 4

FLEXURAL AND AXIAL LOADS

4-1. Design Assumptions and General Requirements

a. The assumed maximum usable strain ε_c at the extreme concrete compression fiber should be equal to 0.003 in accordance with ACI 318.

b. Balanced conditions for hydraulic structures exist at a cross section when the tension reinforcement ρ_b reaches the strain corresponding to its specified yield strength f_y just as the concrete in compression reaches its design strain ε_c.

c. Concrete stress of $0.85f'_c$ should be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain.

d. Factor β_1 should be taken as specified in ACI 318.

e. The eccentricity ratio e'/d should be defined as

$$\frac{e'}{d} = \frac{M_u/P_u + d - h/2}{d} \qquad \text{(4-11)*}$$

where e' = eccentricity of axial load measured from the centroid of the tension reinforcement

4-2. Flexural and Compressive Capacity - Tension Reinforcement Only

a. The design axial load strength ϕP_n at the centroid of compression members should not be taken greater than the following:

$$\phi P_{n(\max)} = 0.8\phi\left[0.85f'_c(A_g - \rho bd) + f_y\rho bd\right] \qquad \text{(4-12)}$$

b. The strength of a cross section is controlled by compression if the load has an eccentricity ratio e'/d no greater than that given by Equation 4-3 and by tension if e'/d exceeds this value.

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* P_u is considered positive for compression and negative for tension.

$$\frac{e_b'}{d} = \frac{2k_b - k_b^2}{2k_b - \dfrac{\rho f_y}{0.425 f_c'}} \tag{4-13}$$

where

$$k_b = \frac{\beta_1 E_s \varepsilon_c}{E_s \varepsilon_c + f_y} \tag{4-14}$$

c. Sections controlled by tension should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u - \rho f_y\right) bd \tag{4-15}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u - \rho f_y\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \tag{4-16}$$

where k_u should be determined from the following equation:

$$k_u = \sqrt{\left(\frac{e'}{d} - 1\right)^2 + \left(\frac{\rho f_y}{0.425 f_c'}\right)\frac{e'}{d}} - \left(\frac{e'}{d} - 1\right) \tag{4-17}$$

d. Sections controlled by compression should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u - \rho f_s\right) bd \tag{4-18}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u - \rho f_s\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \tag{4-19}$$

where

$$f_s = \frac{E_s \varepsilon_c (\beta_1 - k_u)}{k_u} \geq -f_y \tag{4-20}$$

and k_u should be determined from the following equation by direct or iterative method:

$$k_u^3 + 2\left(\frac{e'}{d} - 1\right) k_u^2 + \left(\frac{E_s \varepsilon_c \rho e'}{0.425 f_c' d}\right) k_u - \frac{\beta_1 E_s \varepsilon_c \rho e'}{0.425 f_c' d} = 0 \tag{4-21}$$

e. The balanced load and moment can be computed using either Equations 4-5 and 4-6 or Equations 4-8 and 4-9 with $k_u = k_b$ and $\frac{e'}{d} = \frac{e_b'}{d}$. The values of e_b'/d and k_b are given by Equations 4-3 and 4-4, respectively.

4-3. Flexural and Compressive Capacity - Tension and Compression Reinforcement

a. The design axial load strength ϕP_n of compression members should not be taken greater than the following:

$$\phi P_{n(\max)} = 0.8\phi \left\{ 0.85 f_c' \left[A_g - (\rho + \rho') bd \right] + f_y (\rho + \rho') bd \right\} \tag{4-22}$$

b. The strength of a cross section is controlled by compression if the load has an eccentricity ratio e'/d no greater than that given by Equation 4-13 and by tension if e'/d exceeds this value.

$$\frac{e_b'}{d} = \frac{2k_b - k_b^2 + \dfrac{\rho' f_s' \left(1 - \dfrac{d'}{d}\right)}{0.425 f_c'}}{2k_b - \dfrac{\rho f_y}{0.425 f_c'} + \dfrac{\rho' f_s'}{0.425 f_c'}} \tag{4-23}$$

The value k_b is given in Equation 4-4 and f_s' is given in Equation 4-16 with $k_u = k_b$.

c. Sections controlled by tension should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_y\right) bd \tag{4-24}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_y\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] bd^2 \tag{4-25}$$

where

$$f_s' = \frac{\left(k_u - \beta_1 \dfrac{d'}{d}\right)}{\left(\beta_1 - k_u\right)} E_s \varepsilon_y \le f_y \tag{4-26}$$

and k_u should be determined from the following equation by direct or iterative methods:

$$\begin{aligned} k_u^3 &+ \left[2\left(\frac{e'}{d} - 1\right) - \beta_1\right] k_u^2 - \left\{\frac{f_y}{0.425 f_c'}\left[\rho'\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \frac{\rho e'}{d}\right]\right. \\ &\left. + 2\beta_1\left(\frac{e'}{d} - 1\right)\right\} k_u + \frac{f_y \beta_1}{0.425 f_c'}\left[\frac{\rho' d'}{d}\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \frac{\rho e'}{d}\right] \\ &= 0 \end{aligned} \tag{4-27}$$

d. Sections controlled by compression should be designed so

$$\phi P_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_s\right) bd \tag{4-28}$$

and

$$\phi M_n = \phi\left(0.85 f_c' k_u + \rho' f_s' - \rho f_s\right)\left[\frac{e'}{d} - \left(1 - \frac{h}{2d}\right)\right] b d^2 \qquad (4\text{-}29)$$

where

$$f_s = \frac{E_s \varepsilon_c \left(\beta_1 - k_u\right)}{k_u} \geq -f_y \qquad (4\text{-}30)$$

and

$$f_s' = \frac{E_s \varepsilon_c \left[k_u - \beta_1\left(\frac{d'}{d}\right)\right]}{k_u} \leq f_y \qquad (4\text{-}31)$$

and k_u should be determined from the following equation by direct or iterative methods:

$$k_u^3 + 2\left(\frac{e'}{d} - 1\right) k_u^2 + \frac{E_s \varepsilon_c}{0.425 f_c'}\left[\left(\rho + \rho'\right)\left(\frac{e'}{d}\right) - \rho'\left(1 - \frac{d'}{d}\right)\right] k_u - \frac{\beta_1 E_s \varepsilon_c}{0.425 f_c'}\left[\rho'\left(\frac{d'}{d}\right)\left(\frac{e'}{d} + \frac{d'}{d} - 1\right) + \rho\left(\frac{e'}{d}\right)\right] = 0 \qquad (4\text{-}32)$$

Design for flexure utilizing compression reinforcement is discouraged. However, if compression reinforcement is used in members controlled by compression, lateral reinforcement shall be provided in accordance with the ACI Building Code.

e. The balanced load and moment should be computed using Equations 4-14, 4-15, 4-16, and 4-17 with $k_u = k_b$ and $\frac{e'}{d} = \frac{e_b'}{d}$. The values of e_b'/d and k_b are given by Equations 4-13 and 4-4, respectively.

4-4. Flexural and Tensile Capacity

a. The design axial strength ϕP_n of tensile members should not be taken greater than the following:

$$\phi P_{n(\max)} = 0.8\phi(\rho + \rho')f_y bd \tag{4-33}$$

b. Tensile reinforcement should be provided in both faces of the member if the load has an eccentricity ratio e'/d in the following range:

$$\left(1 - \frac{h}{2d}\right) \geq \frac{e'}{d} \geq 0$$

The section should be designed so

$$\phi P_n = \phi(\rho f_y + \rho' f_s')bd \tag{4-24}$$

and

$$\phi M_n = \phi(\rho f_y + \rho' f_s')\left[\left(1 - \frac{h}{2d}\right) - \frac{e'}{d}\right]bd^2 \tag{4-25}$$

with

$$f_s' = f_y \frac{\left(k_u + \frac{d'}{d}\right)}{(k_u + 1)} \geq -f_y \tag{4-26}$$

and k_u should be determined from the following equation:

$$k_u = \frac{\rho' \frac{d'}{d}\left(1 - \frac{d'}{d} - \frac{e'}{d}\right) - \rho\frac{e'}{d}}{\rho\frac{e'}{d} - \rho'\left(1 - \frac{d'}{d} - \frac{e'}{d}\right)} \tag{4-27}$$

c. Sections subjected to a tensile load with an eccentricity ratio $e'/d < 0$ should be designed using Equations 4-5 and 4-6. The value of k_u is

$$k_u = -\left(\frac{e'}{d} - 1\right) - \sqrt{\left(\frac{e'}{d} - 1\right)^2 + \left(\frac{\rho f_y}{0.425 f_c'}\right)\frac{e'}{d}} \tag{4-28}$$

d. Sections subject to a tensile load with an eccentricity ratio $e'/d < 0$ should be designed using Equations 4-14, 4-15, 4-16, and 4-17 if $A'_s > 0$ and $c > d'$.

4-5. Biaxial Bending and Axial Load

a. Provisions of paragraph 4-5 shall apply to reinforced concrete members subjected to biaxial bending.

b. For a given nominal axial load $P_n = P_u/\phi$, the following nondimensional equation shall be satisfied:

$$(M_{nx}/M_{ox})^K + (M_{ny}/M_{oy})^K \leq 1.0 \tag{4-29}$$

where

M_{nx}, M_{ny} = nominal biaxial bending moments with respect to the x and y axes, respectively

M_{ox}, M_{oy} = uniaxial nominal bending strength at P_n about the x and y axes, respectively

K = 1.5 for rectangular members
= 1.75 for square or circular members
= 1.0 for any member subjected to axial tension

c. M_{ox} and M_{oy} shall be determined in accordance with paragraphs 4-1 through 4-3.