APPENDIX D

DESIGN EXAMPLES

D-1. Design Procedure

For convenience, a summary of the steps used in the design of the examples in this appendix is provided below. This procedure may be used to design flexural members subjected to pure flexure or flexure combined with axial load. The axial load may be tension or compression.

Step 1 - Compute the required nominal strength M_n, P_n where M_u and P_u are determined in accordance with paragraph 4-1.

$$M_n = \frac{M_u}{\phi} \qquad P_n = \frac{P_u}{\phi}$$

Note: Step 2 below provides a convenient and quick check to ensure that members are sized properly to meet steel ratio limits. The expressions in Step 2a are adequate for flexure and small axial load. For members with significant axial loads the somewhat more lengthy procedures of Step 2b should be used.

Step 2a - Compute d_d from Table D-1. The term d_d is the minimum effective depth a member may have and meet the limiting requirements on steel ratio. If $d \geq d_d$ the member is of adequate depth to meet steel ratio requirements and A_s is determined using Step 3.

Step 2b - When significant axial load is present, the expressions for d_d become cumbersome and it becomes easier to check the member size by determining M_{DS}. M_{DS} is the maximum bending moment a member may carry and remain within the specified steel ratio limits.

$$M_{DS} = 0.85 f_c' \, a_d b (d - a_d/2) - (d - h/2) P_n \qquad \text{(D-1)}$$

where

$$a_d = K_d d \qquad \text{(D-2)}$$

and K_d is found from Table D-1.

Step 3 - Singly Reinforced - When $d \geq d_d$ (or $M_n \leq M_{DS}$) the following equations are used to compute A_s.

$$K_u = 1 - \sqrt{1 - \frac{M_n + P_n(d - h/2)}{0.425 f_c' b d^2}} \tag{D-3}$$

$$A_s = \frac{0.85 f_c' K_u b d - P_n}{f_y} \tag{D-4}$$

Table D-1

Minimum Effective Depth

f_c' (psi)	f_y (psi)	$\frac{\rho^*}{\rho_b}$	K_d	d_d (in.)
3000	60	0.25	0.125765	$\sqrt{\frac{3.3274 M_n^{**}}{b}}$
4000	60	0.25	0.125765	$\sqrt{\frac{2.4956 M_n^{*}}{b}}$
5000	60	0.25	0.118367	$\sqrt{\frac{2.1129 M_n^{*}}{b}}$

* See Section 3-5. Maximum Tension Reinforcement

** M_n units are inch-kips.

where

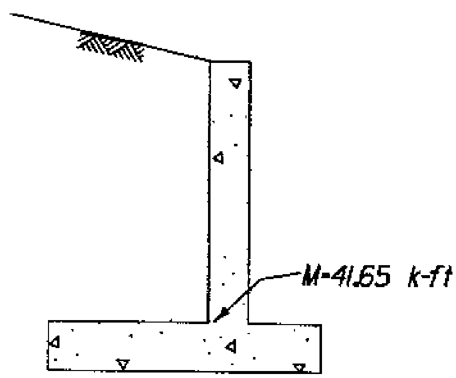
$$K_d = \frac{\left(\frac{\rho}{\rho_b}\right)\beta_1 \varepsilon_c}{\varepsilon_c + \frac{f_y}{E_s}}$$

$$d_d = \sqrt{\frac{M_n}{0.85 f_c' k_d b \left(1 - \frac{k_d}{2}\right)}}$$

D-2. Singly Reinforced Example

The following example demonstrates the use of the design procedure outlined in paragraph D-1 for a Singly Reinforced Beam with the recommended steel ratio of 0.25 ρ_b. The required area of steel is computed to carry the moment at the base of a retaining wall stem.

Given: M = 41.65 k-ft
(where M = moment from <u>unfactored</u> dead and live loads)
f_c' = 3.0 ksi
f_y = 60 ksi
d = 20 in.



First compute the required strength, M_u.

$$M_u = 1.7\ H_f(D + L)$$

$$M_u = (1.7)(1.3)(41.65) = 92.047 \text{ k-ft}$$

<u>Step</u> <u>1</u>. $M_n = M_u/\phi$ = 92.047/0.90 = 102.274 k-ft

Step 2. $d_d = \sqrt{\frac{3.3274 M_n}{b}} = 18.45$ in. (Table D-1)

$d > d_d$ therefore member size is adequate

Step 3. $K_u = 1 - \sqrt{1 - \frac{M_n + P_n(d - h/2)}{0.425 f_c' b d^2}}$ (D-3)

$$K_u = 1 - \sqrt{1 - \frac{(102.274)(12)}{(0.425)(3.0)(12)(20)^2}} = 0.10587$$

$$A_s = \frac{0.85 f_c' K_u b d}{f_y} = \frac{(0.85)(3.0)(0.10587)(12)(20)}{60} \quad \text{(D-4)}$$

$A_s = 1.08$ sq in.

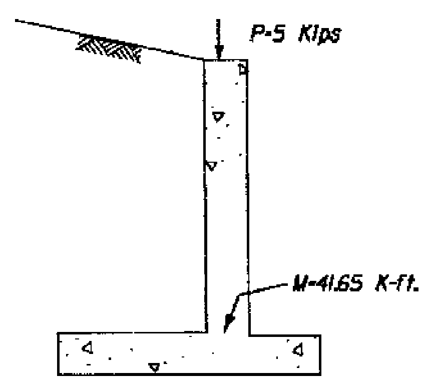
D-3. Combined Flexure Plus Axial Load Example

The following example demonstrates the use of the design procedure outlined in paragraph D-1 for a beam subjected to flexure plus small axial compressive load. The amount of tensile steel required to carry the moment and axial load at the base of a retaining wall stem is found.

Given: $M = 41.65$ k-ft

$P = 5$ kips (weight of stem)

where M and P are the moment and axial load from an unfactored analysis.



$f_c' = 3.0$ ksi

$f_y = 60$ ksi

d = 20 in.

h = 24 in.

First compute the required strength, M_u, P_u

$$M_u = 1.7\ H_f(D + L)$$

$$M_u = (1.7)(1.3)(41.65) = 92.047\ k\text{-}ft$$

$$P_u = 1.7\ H_f(D + L)$$

$$P_u = (1.7)(1.3)(5.0) = 11.05 \text{ kips}$$

Since axial load is present a value must be found for ϕ.

For small axial load $\phi \cong 0.9 - [(0.20\ P_u)/(0.10 f_c' A_g)]$

$$\phi \cong 0.88$$

<u>Step</u> <u>1</u>. $M_n = M_u/\phi = 92.047/0.88 = 104.60\ k\text{-}ft$

$$P_n = P_u/\phi = 11.05/0.88 = 12.56 \text{ kips}$$

<u>Step</u> <u>2</u>. $a_d = K_d d$ (D-2)

$$a_d = (0.12577)(20) = 2.515$$

$$M_{DS} = 0.85 f_c'\ a_d\ b(d - a_d/2.0) - (d - h/2.0)P_n \qquad \text{(D-1)}$$

$$M_{DS} = (0.85)(3.0)(2.515)(12)(20 - 1.258) - (20 - 12)(12.56)$$

$$M_{DS} = 1341.9\ k\text{-}in. \text{ or } 111.82\ k\text{-}ft$$

$M_{DS} > M_n$ therefore member size is adequate

<u>Step</u> <u>3</u>. $K_u = 1 - \sqrt{1 - \frac{M_n + P_n(d - h/2)}{0.425 f'_c b d^2}}$

$$K_u = 1 - \sqrt{1 - \frac{(12)104.6 + 12.56(20 - 12)}{(0.425)(3.0)(12)(20)^2}}$$

$$K_u = 0.11768$$

$$A_s = \frac{0.85 f'_c K_u b d - P_n}{f_y}$$

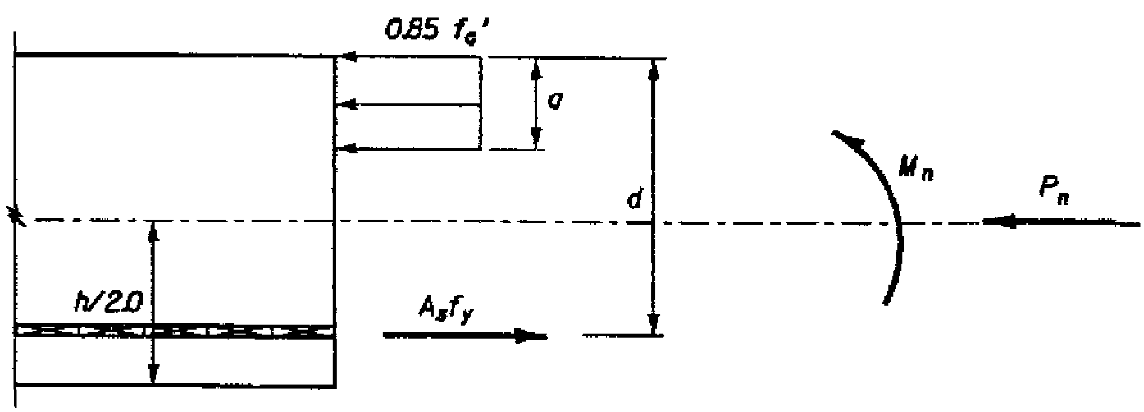
$$A_s = \frac{(0.85)(3.0)(0.11768)(12)(20) - 12.56}{60} \qquad \text{(D-4)}$$

$$A_s = 0.99 \text{ sq in.}$$

D-4. Derivation of Design Equations

The following paragraphs provide derivations of the design equations presented in paragraph D-1.

(1) Derivation of Design Equations for Singly Reinforced Members. The figure below shows the conditions of stress on a singly reinforced member subjected to a moment M_n and load P_n. Equations for design may be developed by satisfying conditions of equilibrium on the section.



By requiring the ΣM about the tensile steel to equal zero

$$M_n = 0.85 f'_c\, ab(d - a/2) - P_n(d - h/2) \qquad \text{(D-5)}$$

By requiring the ΣH to equal zero

$$A_s f_y = 0.85 f_c' \, ab - P_n \qquad \text{(D-6)}$$

Expanding Equation D-5 yields

$$M_n = 0.85 f_c' \, abd - 0.425 f_c' a^2 b - P_n (d - h/2)$$

Let $a = K_u d$ then

$$M_n = 0.85 f_c' K_u b d^2 - 0.425 f_c' K_u^2 d^2 b - P_n (d - h/2)$$

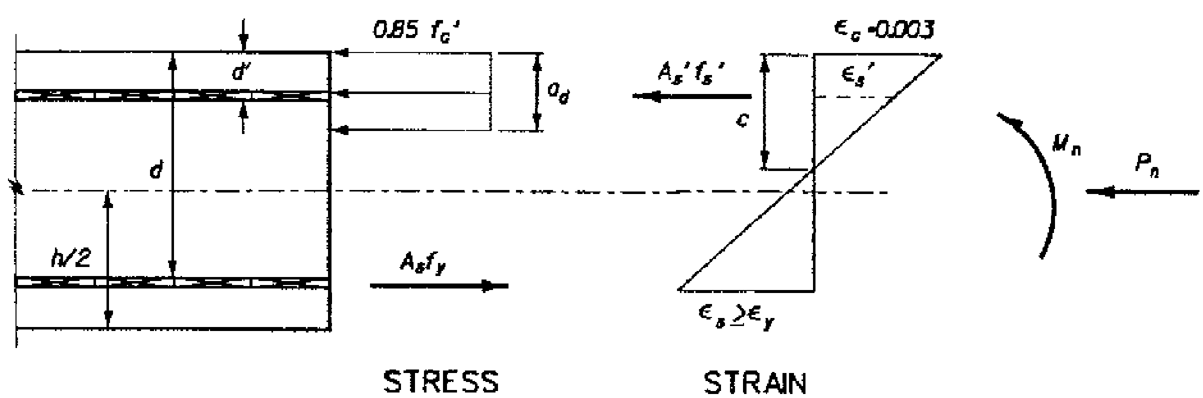
The above equation may be solved for K_u using the solution for a quadratic equation

$$K_u = 1 - \sqrt{1 - \frac{M_n + P_n (d - h/2)}{0.425 f_c' b d^2}} \qquad \text{(D-3)}$$

Substituting $K_u d$ for a in Equation D-6 then yields

$$A_s = \frac{0.85 f_c' \, K_u b d - P_n}{f_y}$$

(2) Derivation of Design Equations for Doubly Reinforced Members. The figure below shows the conditions of stress and strain on a doubly reinforced member subjected to a moment M_n and load P_n. Equations for design are developed in a manner identical to that shown previously for singly reinforced beams.



Requiring ΣH to equal zero yields

$$A_s = \frac{0.85 f_c' K_d b d - P_n + A_s' f_s'}{f_y} \qquad \text{(D-7)}$$

By setting $a_d = \beta_1 c$ and using the similar triangles from the strain diagram above, ε_s' and f_s' may be found:

$$f_s' = \frac{(a_d - \beta_1 d')\varepsilon_c E_s}{a_d}$$

An expression for the moment carried by the concrete (M_{DS}) may be found by summing moments about the tensile steel of the concrete contribution.

$$M_{DS} = 0.85 f_c' a_d b (d - a_d/2) - (d - h/2) P_n \qquad \text{(D-1)}$$

Finally, an expression for A_s' may be found by requiring the compression steel to carry any moment above that which the concrete can carry (M_n - M_{DS}).

$$A_s' = \frac{M_n - M_{DS}}{f_s'(d - d')} \qquad \text{(D-8)}$$

(3) Derivation of Expression of d_d. The expression for d_d is found by substituting $a_d = k_d d_d$ in the equation shown above for M_{DS} and solving the resulting quadratic expression for d_d.

$$d_d = \sqrt{\frac{M_{DS}}{\left[0.85 f_c' K_d b (1 - K_d/2)\right]}} \qquad \text{(D-9)}$$

D-5. Shear Strength Example for Special Straight Members

Paragraph 5.2 describes the conditions for which a special shear strength criterion shall apply for straight members. The following example demonstrates the application of Equation 5-1. Figure D-1 shows a rectangular conduit with factored loads, 1.7 H_f (dead load + live Load). The following parameters are given or computed for the roof slab of the conduit.

$f'_c = 4{,}000$ psi

$\ell_n = 10.0 \text{ ft} = 120 \text{ in.}$

$d = 2.0 \text{ ft} = 24 \text{ in.}$

$b = 1.0 \text{ ft (unit width)} = 12 \text{ in.}$

$N_u = 6.33(5) = 31.7 \text{ kips}$

$A_g = 2.33 \text{ sq ft} = 336 \text{ sq in.}$

$$V_c = \left[\left(11.5 - \frac{120 \text{ in.}}{24 \text{ in.}}\right)\sqrt{4{,}000}\sqrt{1 + \left(\frac{\frac{31{,}700 \text{ lb}}{336 \text{ sq in.}}}{5\sqrt{4{,}000}}\right)}\right](12 \text{ in.})(24 \text{ in.}) \qquad \text{(D-10, Eq. 5-1)}$$

$V_c = 134{,}906 \text{ lb} = 134.9 \text{ kips}$

Check limit $V_c = 10\sqrt{f'_c}\, bd = 10\sqrt{4{,}000}\,(12 \text{ in.})(24 \text{ in.}) = 182{,}147 \text{ lb}$

Compare shear strength with applied shear.

$\phi V_c = 0.85(134.9 \text{ kips}) = 114.7 \text{ kips}$

V_u at $0.15(\ell_n)$ from face of the support is

$$V_u = w\left(\frac{\ell_n}{2} - 0.15\,\ell_n\right)$$

$$= 15.0 \text{ kips/ft}\left[\left(\frac{10 \text{ ft}}{2}\right) - (0.15)(10 \text{ ft})\right]$$

$$= 52.5 \text{ kips} < \phi V_c \text{ ; shear strength adequate}$$

D-6. Shear Strength Example for Curved Members

Paragraph 5-3 describes the conditions for which Equation 5-3 shall apply. The following example applies Equation 5-3 to the circular conduit presented in Figure D-2. Factored loads are shown, and the following values are given or computed:

f'_c = 4,000 psi

b = 12 in.

d = 43.5 in.

A_g = 576 sq in.

N_u = 162.5 kips

V_u = 81.3 kips at a section 45 degrees from the crown

$$V_c = 4\sqrt{4{,}000}\left[\sqrt{1 + \left(\frac{\dfrac{162{,}500 \text{ lb}}{576 \text{ sq in.}}}{4\sqrt{4{,}000}}\right)}\right](12 \text{ in.})(43.5 \text{ in.})$$

$$V_c = 192{,}058 \text{ lb} = 192.1 \text{ kips}$$

Check limit $V_c = 10\sqrt{f'_c}\,bd = 10\sqrt{4{,}000}\,(12 \text{ in.})(43.5 \text{ in.}) = 330{,}142 \text{ lb}$

Compare shear strength with applied shear

ϕV_c = 0.85(192.1 kips) = 163.3 kips

$V_u < \phi V_c$; shear strength adequate

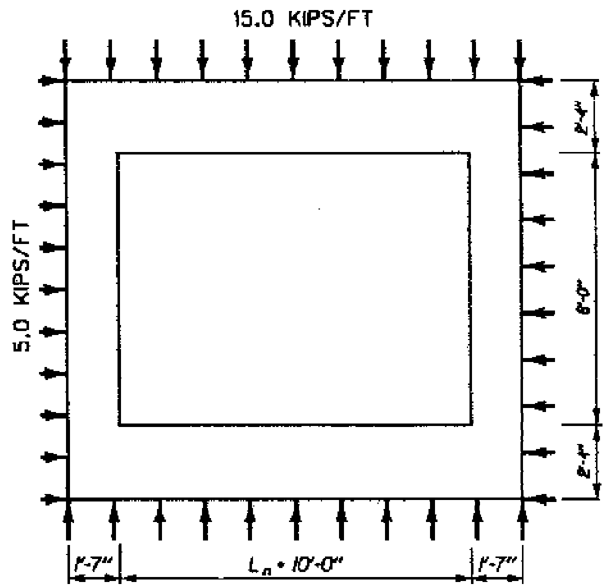


Figure D-1. Rectangular conduit

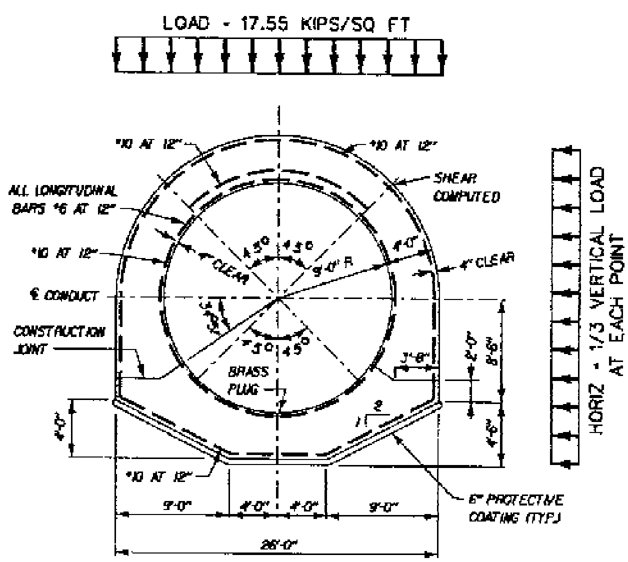


Figure D-2. Circular conduit