APPENDIX C

INVESTIGATION EXAMPLES

C-1. General

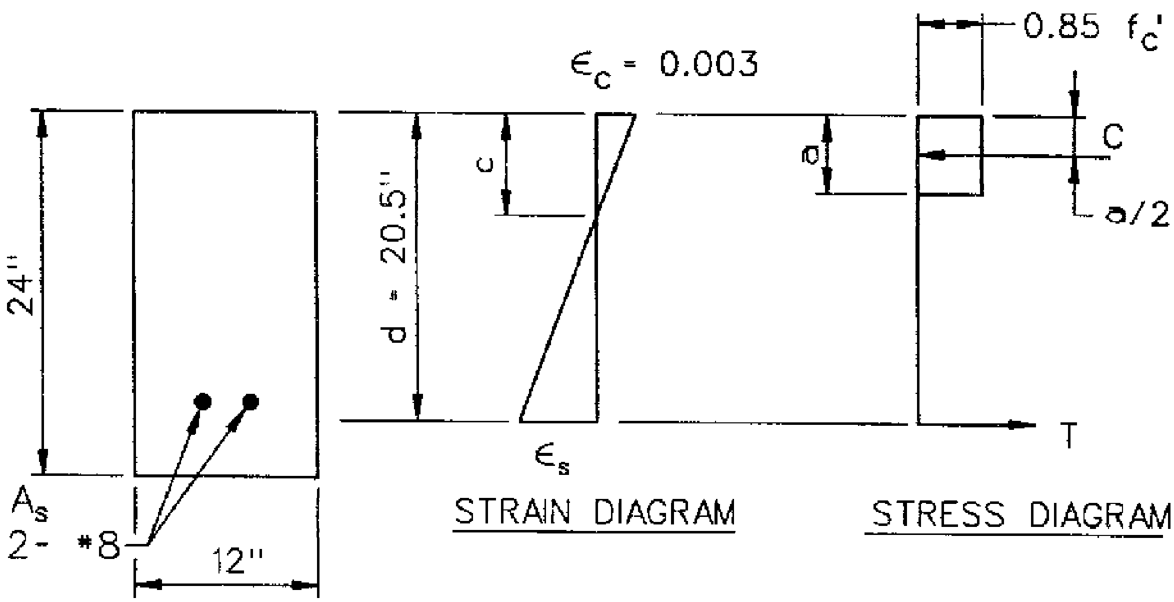
For the designer's convenience and reference, the following examples are provided to illustrate how to determine the flexural capacity of existing concrete sections in accordance with this Engineer Manual and ACI 318.

C-2. Analysis of a Singly Reinforced Beam

Given: f'_c = 3 ksi β_1 = 0.85

f_y = 60 ksi E_s = 29,000 ksi

A_s = 1.58 in.^{2}



Solution:

1. Check steel ratio

$$\rho_{act} = \frac{A_s}{bd}$$

$$= \frac{1.58}{12(20.5)}$$

$$= 0.006423$$

$$\rho_b = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{87,000}{87,000 + f_y}\right)$$

$$= 0.85\,(0.85)\left(\frac{3}{60}\right)\left(\frac{87,000}{87,000 + 60,000}\right)$$

$$= 0.02138$$

in accordance with Paragraph 3-5 check:

$$0.25\rho_b = 0.00534$$

$$0.375\rho_b = 0.00802$$

$$\rho_{act} = 0.00642$$

$$0.25\rho_b < \rho_{act} < 0.375\rho_b$$

ρ_{act} is greater than the recommended limit, but less than the maximum permitted upper limit not requiring special study or investigation. Therefore, no special consideration for serviceability, constructibility, and economy is required. This reinforced section is satisfactory.

2. Assume the steel yields and compute the internal forces:

$$T = A_s f_y = 1.58\,(60) = 94.8 \text{ kips}$$

$$C = 0.85\, f_c' ba$$

$$C = 0.85\,(3)(12)a = 30.6a$$

3. From equilibrium set $T = C$ and solve for a:

$$94.8 = 30.6a \longrightarrow a = 3.10 \text{ in.}$$

$$\text{Then, } a = \beta_1 c \longrightarrow c = \frac{3.10}{0.85} = 3.65 \text{ in.}$$

4. Check ε_s to demonstrate steel yields prior to crushing of the concrete:

$$\frac{\varepsilon_s}{20.5 - c} = \frac{0.003}{c}$$

$$\varepsilon_s = 16.85 \left(\frac{0.003}{3.65}\right) = 0.0138$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29{,}000} = 0.00207$$

$\varepsilon_s > \varepsilon_y$ Ok, steel yields

5. Compute the flexural capacity:

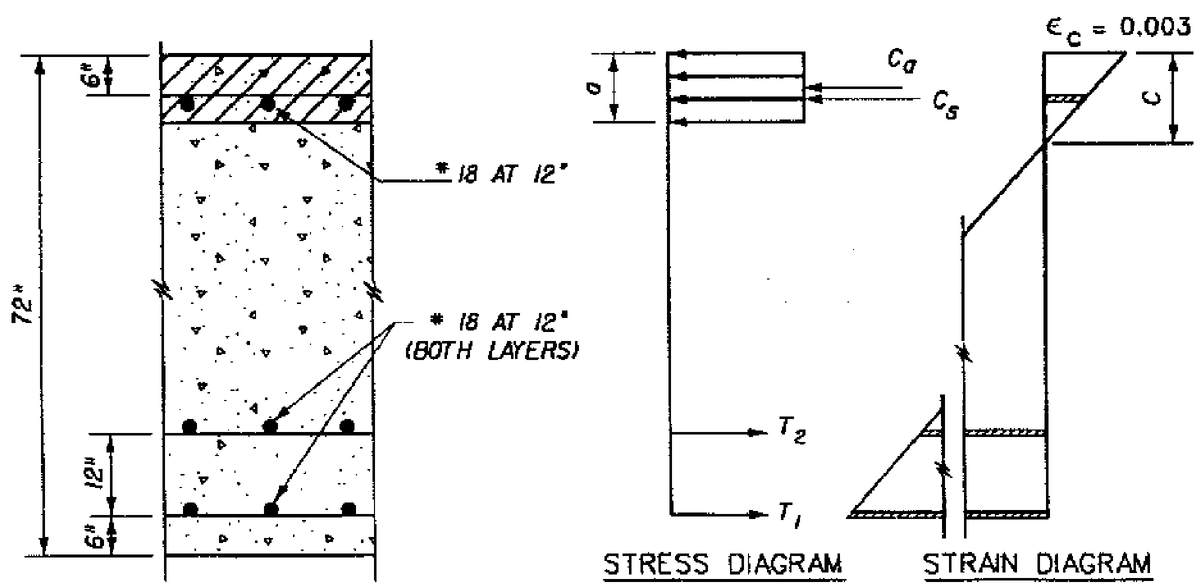
$$\phi M_n = \phi \left(A_s f_y\right) (d - a/2)$$

$$= 0.90\ (94.8)\left(20.5 - \frac{3.10}{2}\right)$$

$$= 1616.8 \text{ in.-k}$$

$$= 134.7 \text{ ft-k}$$

C-3. Analysis of an Existing Beam - Reinforcement in Both Faces



Given: f'_c = 3,000 psi ε_c = 0.003

f_y = 60,000 psi β_1 = 0.85

A_s = 8.00 in.^{2} E_s = 29,000,000 psi

A'_s = 4.00 in.^{2}

Solution:

1. First analyze considering steel in tension face only

$$\rho = \frac{A_s}{bd} = \frac{8}{(60)(12)} = 0.011$$

$$\rho_{bal} = 0.85\frac{\beta_1 f'_c}{f_y} = \frac{87{,}000}{87{,}000 + f_y} = 0.0214$$

$$\rho = \frac{0.011}{0.0214}\rho_{bal} = 0.51\rho_b$$

Note: ρ exceeds maximum permitted upper limit not requiring special study or investigation = 0.375 ρ_b. See Chapter 3.

$$T = A_s f_y$$

$$T = 8(60) = 480 \text{ kips}$$

then $C_c = 0.85 f'_c ba = 30.6a$

$$T = C_c$$

$\therefore$ a = 15.7 in. and c = 18.45 in.

By similar triangles, demonstrate that steel yields

$$\frac{\varepsilon_c}{18.45} = \frac{\varepsilon_{s(2)}}{54 - c} \Rightarrow \varepsilon_{s(2)} = 0.0057 > \varepsilon_y = 0.0021$$

ok; both layers of steel yield.

Moment capacity = 480 kips (d - a/2)

= 480 kips (52.15 in.)

M = 25,032 in.-k

2. Next analyze considering steel in compression face

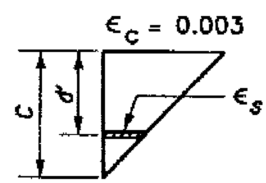
$$\rho' = \frac{4}{12(60)} = 0.0056$$

$$\rho - \rho' = 0.0054$$

$$= 0.85 \frac{\beta_1 f_c'}{f_y} \cdot \frac{d'}{d} \left(\frac{87,000}{87,000 - f_y} \right) = 0.016$$

$\rho - \rho' \le 0.0116$ $\quad \therefore$ compression steel does not yield, must do general analysis using $\sigma : \varepsilon$ compatability

Locate neutral axis



$$T = 480 \text{ kips}$$

$$C_c = 0.85 f_c ba = 30.6a$$

$$C_s = A_s' \, (f_s' - 0.85 f_c') = 4(f_s' - 2.55)$$

By similar triangles

$$\frac{\varepsilon_s'}{c - 6} = \frac{0.003}{c}$$

Substitute $c = \dfrac{a}{0.85} = 1.176a$

Then $\varepsilon_s' = 0.003 - \dfrac{0.0153}{a}$

Since $f_s' = E\varepsilon_s' \Rightarrow f_s' = \left(87 - \dfrac{443.7}{a}\right)$ksi

Then

$C_s = 4\left(87 - 2.55 - \frac{443.7}{a}\right)$kips

$T = C_c + C_s = 480$ kips

Substitute for C_c and C_s and solve for a

$30.6a + 337.8 - \frac{1774.8}{a} = 480$

$a^2 - 4.65a - 58 = 0$

Then $a = 10.3$ in.

and $c = 12.1$ in.

Check $\varepsilon_s' > \varepsilon_y$

By similar triangles $\frac{0.003}{12.1} = \frac{\varepsilon_s'}{d - 12.1}$

$$\varepsilon_s' = 0.0119 > 0.0021$$

$C_c = 30.6a \approx 315$ kips

$C_s = 4(41.37) \approx 165$ kips

$C_c + C_s = 480$ kips $= T$

Resultant of C_c *and* C_s $= \frac{315\left(\frac{10.3}{2}\right) + (165)(6)}{480} = 5.4$ in.

Internal Moment Arm = 60 – 5.4 = 54.6 in.

$M = 480(54.6) = 26{,}208$ in.-k

Comparison

	Tension Steel Only	Compression Steel
a	15.7 in.	10.3 in.
c	18.45 in.	12.1 in.
Arm	52.15 in.	54.6 in.
M	25,032 in.-k	26,208 in.-k ⇒ 4.7 percent increase