Chapter 4
Analysis of Single Wells

4-1. Assumptions

Analytical procedures for determining well flows and head distributions adjacent to single artesian relief wells are presented below. By definition, relief wells signify artesian conditions, and equations for artesian flow are applicable. In cases where wells are pumped, and gravity flow conditions exist, procedures for well analysis can be found in TM 5-818-5. It is assumed in the following analyses that all seepage flow is laminar or viscous, i.e., Darcy's Law is applicable. It is also assumed that steady state conditions prevail; the rate of seepage and rate of head reduction have reached equilibrium and are not time dependent. Unless otherwise indicated, the well is assumed to penetrate the full thickness of the aquifer.

4-2. Circular Source

Certain geologic or terrain conditions may require the assumption of a circular source of seepage. The formulas for a fully penetrating well located at the center of a circular source (see Figure 4-1) are

$$h_p = H - \frac{Q_w}{2\pi kD} \ln \frac{R}{r} \qquad (4\text{-}1)$$

$$h_w = H - \frac{Q_w}{2\pi kD} \ln \frac{R}{r_w} \qquad (4\text{-}2)$$

where

h_p = head at point p between the well and the source

H = head at the source

Q_w = well discharge

k, (k_f) = coefficient of permeability of pervious substratum

D = thickness of pervious foundation

R = radius of circular source (radius of influence)

h_w = head at well

r_w = radius of well

4-3. Noncircular Source

If geologic or terrain conditions indicate a noncircular source of seepage, the radius of influence, R, may be replaced by A_c, defined as an effective average of the distance from the well center to the external boundary. For a rectangular boundary of sides 2a and 2b, the value of A_c is

$$A_c = \sqrt{\frac{4ab}{\pi}} \qquad (4\text{-}3)$$

4-4. Infinite Line Source

Conditions may arise where the flow to the well originates from the bank of a river or canal reservoir or another body of water. In such cases, the bank or shoreline may act as an infinite line source of seepage. If leakage occurs through the top stratum, the effective distance to the infinite line source of seepage should be computed as discussed in Appendix B. The solutions for a single well adjacent to an infinite line source (see Figure 4-1) is determined using the method of images described by Muskat (1937), Todd (1980), and EM 1110-2-1901. The formulas are

$$h_p = H - \frac{Q_w}{2\pi kD} \ln \frac{r'}{r} \qquad (4\text{-}4)$$

$$h_w = H - \frac{Q_w}{2\pi kD} \ln \frac{2S}{r_w} \qquad (4\text{-}5)$$

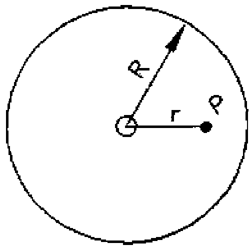
where

r' = distance from point p to image well

r = distance from point p to real well

S = distance from real well to line source

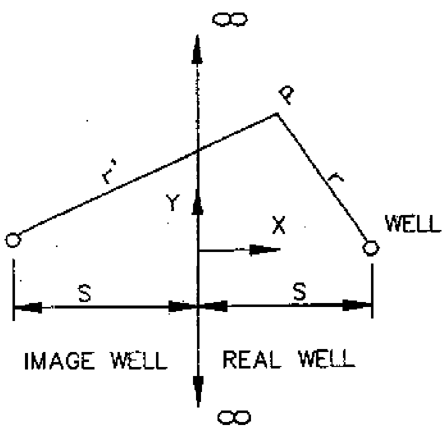
A solution for h_p is also presented in terms of x and y coordinates in Figure 4-1 (Equation 4-6).



$$h_p = H - \frac{Q_w}{2\pi kD} \ln \frac{R}{r} \qquad (4\text{-}1)$$

$$h_w = H - \frac{Q_w}{2\pi kD} \ln \frac{R}{r_w} \qquad (4\text{-}2)$$

CIRCULAR SOURCE



$$h_p = H - \frac{Q_w}{2\pi kD} \ln \frac{r'}{r} \qquad (4\text{-}4)$$

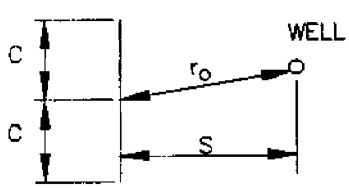
$$h_w = H - \frac{Q_w}{2\pi kD} \ln \frac{2S}{r_w} \qquad (4\text{-}5)$$

IN TERMS OF X AND Y COORDINATES

$$h_p = H - \frac{Q_w}{2\pi kD} \ln \left[\frac{y^2 + (x+S)^2}{y^2 + (x-S)^2}\right]^{\frac{1}{2}} \qquad (4\text{-}6)$$

INFINITE LINE SOURCE

$$h_w = H - \frac{Q_w}{2\pi kD} \ln \frac{4S}{r_w} \left[\frac{\left(c^2 - r_o^2\right)^2 + 4S^2c^2}{c^2 - r_o^2 + \sqrt{\left(c^2 - r_o^2\right)^2 + 4S^2c^2}}\right] \qquad (4\text{-}7)$$



FOR WELL ON PERPENDICULAR BISECTOR, $r_o = S$

$$h_w = H - \frac{Q_w}{2\pi kD} \ln \frac{2S}{r_w}\left(1 + \frac{S^2}{c^2}\right) \qquad (4\text{-}8)$$

FINITE LINE SOURCE

Figure 4-1. Summary of equations for artesian flow to single well

4-5. Finite Line Source

In cases where the length of the source of seepage is relatively small compared to its distance from the well, the source may be considered as a finite line source. The solution for a single well adjacent to a finite line source was developed by Muskat (1937). The formulas, which are available only in terms of head at the well, are shown in Figure 4-1 (Equations 4-7 and 4-8).

4-6. Infinite Line Source and Infinite Line Sink

As discussed in Appendix B, a semipervious landside blanket can be replaced by a totally impervious top stratum and a theoretical line sink at an appropriate equivalent distance from the well. The theoretical line sink, parallel to the infinite line source, is referred to as an infinite line sink. A solution, based also on the method of images, considering one of the infinite line sources as a sink, was developed by Barron (1948) and is shown in Figure 4-2.

4-7. Infinite Line source and Infinite Barrier

The method of images is an extremely powerful tool for developing solutions to wells for various boundary conditions. Solutions for various boundary conditions including barriers are presented by Ferris, Knowles, Brown, and Stellman (1962), Freeze and Cherry (1976), and Todd (1980). For example, a typical problem would be to calculate the discharge or heads for a single artesian well located between a river denoted by an infinite line source and a barrier such as a buried channel or rock bluff. In this case, the image well for the river would have a second image well with respect to the rock bluff which in turn would have an image with respect to the river and so on. A similar progression of image wells would be needed for the impermeable barrier (see EM 1110-2-1901). The image wells extend to infinity; however in practice, it is only necessary to include pairs of image wells closest to the real well because others have a negligible influence on the drawdown. A solution for this case was presented by Barron (1982) and is shown in Figure 4-3.

4-8. Complex Boundary Conditions

Oftentimes, geologic factors impose conditions which are difficult to simulate using circular or line sources and barriers. In such cases, flow net analyses or electrical analogy tests may be used to advantage especially when the aquifer thickness is irregular and three-dimensional analyses are required. The use of flow nets for the design of well systems is described by Mansur and Kaufman (1962). Methods for conducting three-dimensional electrical analogy tests are described by Duncan (1963), Banks (1965), and McAnear and Trahan (1972).

4-9. Partially Penetrating Wells

The previous equations are based on the assumption that the well fully penetrates the aquifer. For practical reasons, it is often necessary to use wells which only partially penetrate the aquifer. The ratio of flow from a partially penetrating artesian well to that for a fully penetrating well at the same drawdown is

$$\frac{Q_{wp}}{Q_w} = G_p \qquad (4\text{-}13)$$

or

$$Q_{wp} = G_p Q_w = \frac{2\pi k D (H - h_w) G_p}{\ln \frac{R}{r_w}} \qquad (4\text{-}14)$$

where

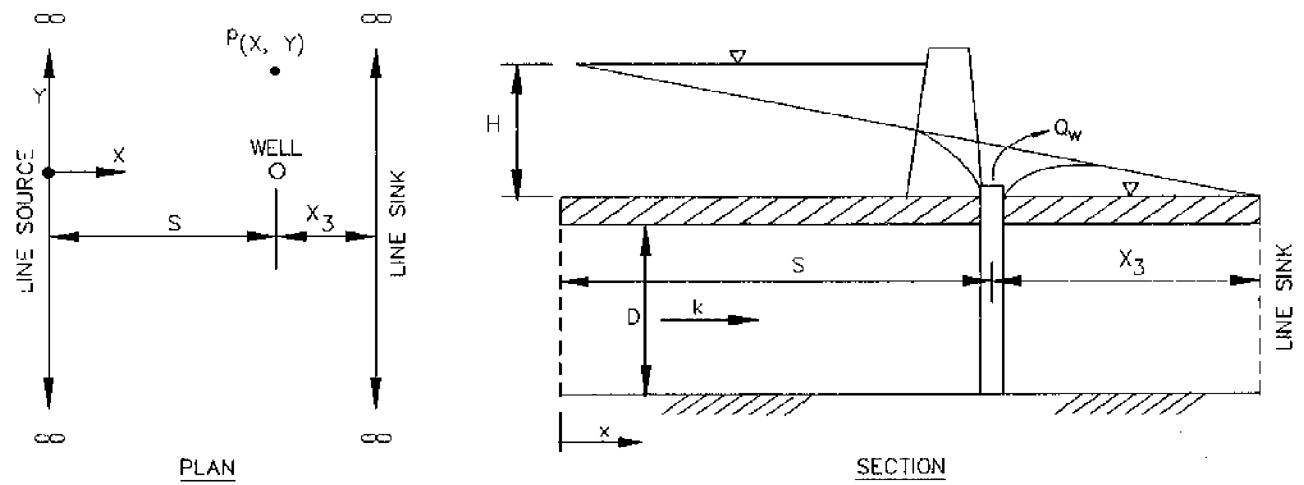
Q_{wp} = flow from partially penetrating well

G_p = flow correction factor for partially penetrating well

An approximate value of G_p can be obtained from the following equation developed by Kozeny (1933):

$$G_p = \frac{W}{D}\left(1 + 7\sqrt{\frac{r_w}{2w}}\cos\frac{\pi w}{2D}\right) \qquad (4\text{-}15)$$

where W/D is well penetration expressed as a decimal. An alternate equation developed by Muskat (1937) assuming a constant flow per unit length of well screen is

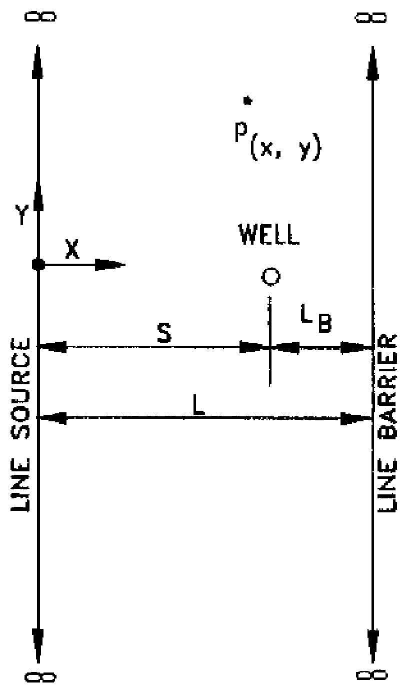


$$h_p = H\,\frac{S + x_3 - x}{S + x_3} - \frac{Q_w}{4\pi kD}\ln\left[\frac{\cosh\dfrac{\pi y}{S + x_3} - \cos\dfrac{\pi(x + S)}{S + x_3}}{\cosh\dfrac{\pi y}{S + x_3} - \cos\dfrac{\pi(x - S)}{S + x_3}}\right] \qquad (4\text{-}9)$$

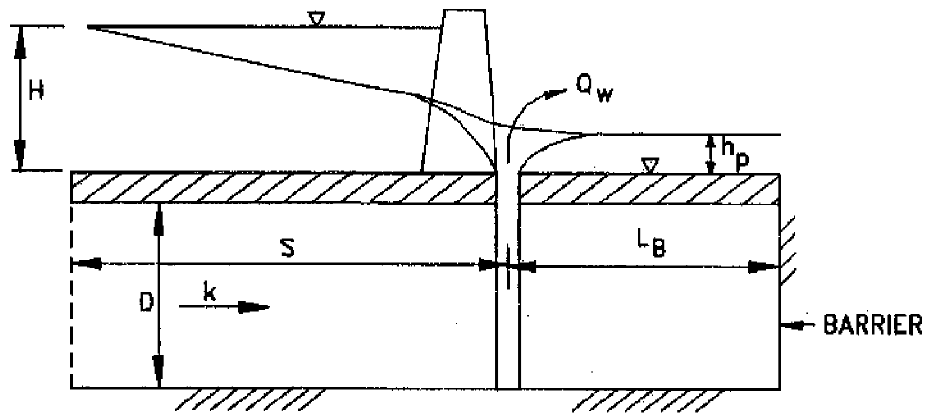
$$Q_w = \frac{4\pi kD\left(H - H\dfrac{S}{S + x_3}\right)}{\ln\left[\dfrac{2(S + x_3)^2\left(1 - \cos\dfrac{2\pi S}{S + x_3}\right)}{\pi^2 r_w^2}\right]} \qquad (4\text{-}10)$$

AFTER BARRON (1948)

Figure 4-2. Drawdown for well between infinite line source and downstream sink



PLAN



SECTION

$$h_p = H - \frac{Q_w}{4\pi kD} \ln \frac{\left(\sin\frac{\pi x}{2L}\cosh\frac{\pi y}{2L} + \sin\frac{\pi S}{2L}\right)^2 + \left(\cos\frac{\pi x}{2L}\sinh\frac{\pi y}{2L}\right)^2}{\left(\sin\frac{\pi x}{2L}\cosh\frac{\pi y}{2L} - \sin\frac{\pi S}{2L}\right)^2 + \left(\cos\frac{\pi x}{2L}\sinh\frac{\pi y}{2L}\right)^2} \qquad (4\text{-}11)$$

$$h_w = H - \frac{Q_w}{4\pi kD} \ln \frac{\left(\sin\frac{\pi S}{2L}\cosh\frac{\pi r_w}{2L} + \sin\frac{\pi S}{2L}\right)^2 + \left(\cos\frac{\pi S}{2L}\sinh\frac{\pi r_w}{2L}\right)^2}{\left(\sin\frac{\pi S}{2L}\cosh\frac{\pi r_w}{2L} - \sin\frac{\pi S}{2L}\right)^2 + \left(\cos\frac{\pi S}{2L}\sinh\frac{\pi r_w}{2L}\right)^2} \qquad (4\text{-}12)$$

Figure 4-3. Drawdown for well between infinite line source and infinite barrier

$$G_p = \frac{\ln \dfrac{R}{r_w}}{\dfrac{D}{2w}\left[2 \ln \dfrac{4D}{r_w} - G(\bar{T})\right] - \ln \dfrac{4D}{R}} \qquad (4\text{-}16)$$

where *G(T)* is a function of *W/D* and approximate values from Harr (1962) are given in Table 4-1.

Table 4-1
Partially Penetrating Well Function, $G(\bar{T})$

W/D	$G(\bar{T})$
0.1	6.4
0.2	5.0
0.3	4.3
0.4	3.5
0.5	2.9
0.6	2.4
0.7	1.9
0.8	1.3
0.9	0.7
1.0	0.0

Values of G_p based on the above values for a typical well (r_w = 1.0 ft) with a radius of 1,000 ft are plotted in Figure 4-4. An empirical method for calculating the head at any point for partially penetrating wells is described by Warriner and Banks (1977). Limitations of empirical formulas for determining flows from partially penetrating wells are discussed in TM 5-818-5.

4-10. Effective Well Penetration

In a stratified aquifer, the effective well penetration usually differs from that computed from the ratio of the length of well screen to total thickness of aquifer. To determine the required length of well screen W to achieve an effective penetration $\bar{W}$ in a stratified aquifer, the procedure shown in Figure 4-5 can be used. It is assumed that the individual strata are anisotropic and each stratum is transformed into an isotropic stratum in accordance with the following equation:

$$\bar{d} = d\sqrt{\frac{k_h}{k_v}} \qquad (4\text{-}17)$$

where

$\bar{d}$ = transformed vertical dimension

d = actual vertical dimension

k_h = permeability in the horizontal direction

k_v = permeability in the vertical direction

The horizontal dimension of the problem would remain unchanged in this transformation. The permeability of the transformed stratum to be used in all equations for flow or drawdown is as follows:

$$\bar{k} = \sqrt{k_h k_v} \qquad (4\text{-}18)$$

where $\bar{k}$ is the transformed coefficient of permeability.

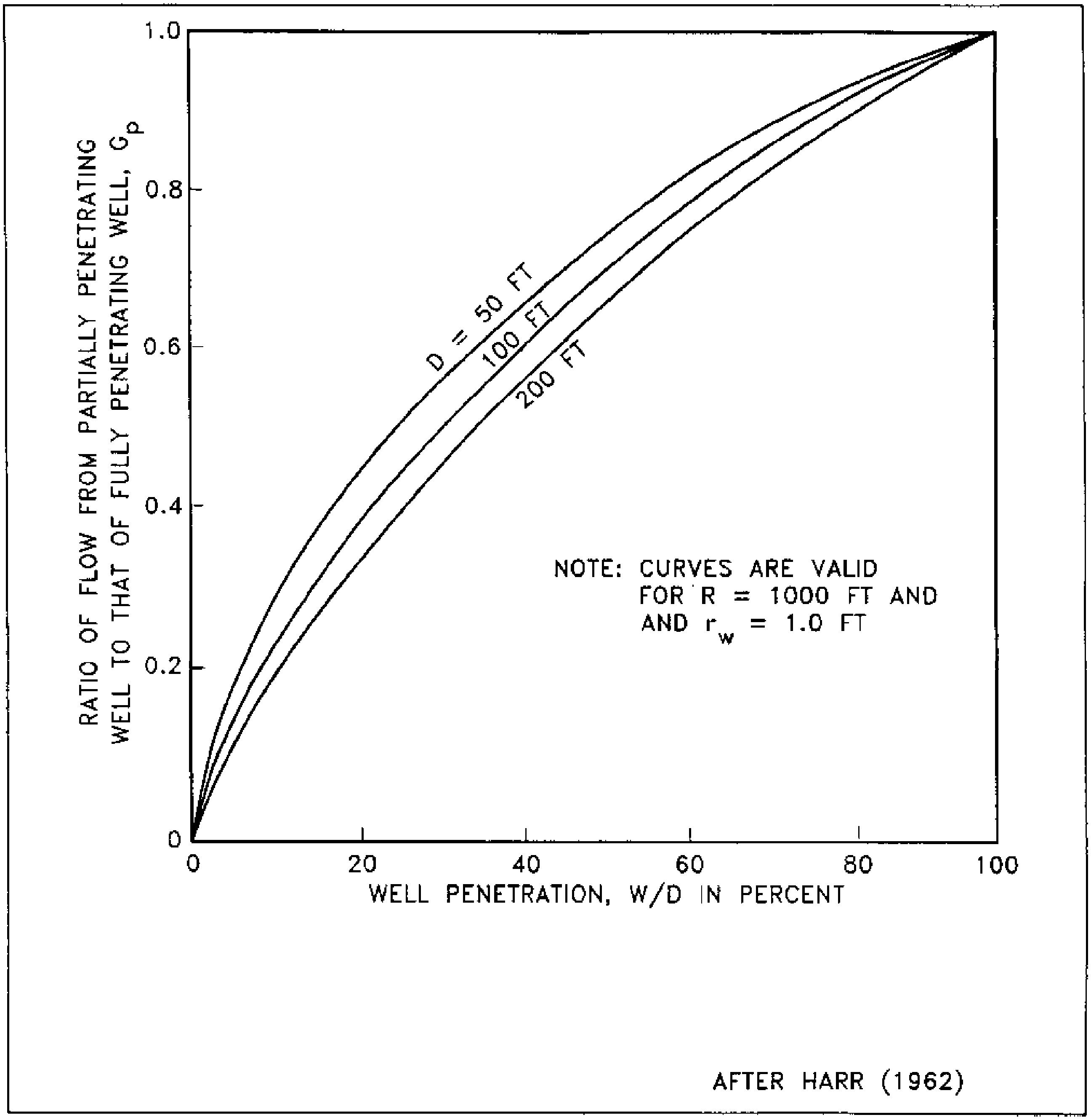
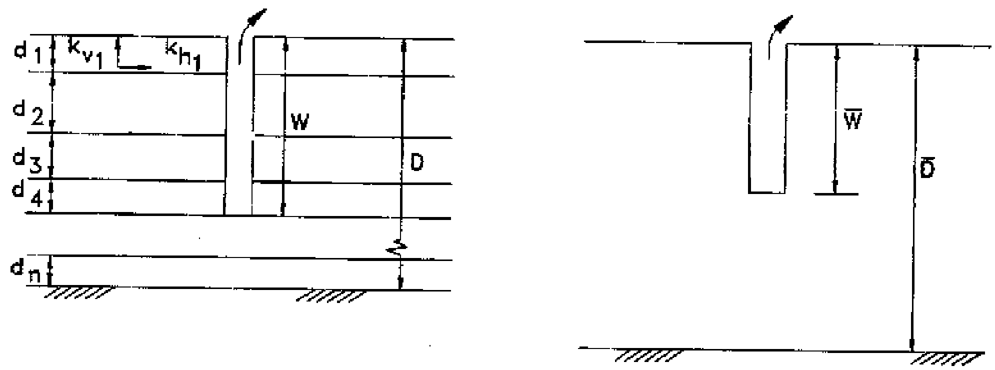


Figure 4-4. Flow to partially penetrating well with circular source



ACTUAL SECTION

TRANSFORMED SECTION

Actual well penetration = W
Effective well penetration = $\bar{W}$
Actual well penetration in percent = W/D × 100
Effective well penetration in percent = $\bar{W}/\bar{D}$ × 100

1. Transform each layer into an isotropic layer of thickness $\bar{d}$ and permeability $\bar{k}$

$$\bar{d} = d\sqrt{\frac{k_h}{k_v}} \qquad (4\text{-}17) \qquad\qquad \bar{k} = \sqrt{k_h\, k_v} \qquad (4\text{-}18)$$

2. Calculate thickness of the equivalent homogeneous, isotropic acquifer, $\bar{D}$

$$\bar{D} = \sqrt{\sum_{m=1}^{m=n} d_m k_{Hm} \sum_{m=1}^{m=n} d_m/k_{vm}} \qquad (4\text{-}19)$$

n = number of strata, numbered from top to bottom

3. Calculate the effective permeability of the transformed aquifer, $\bar{k}_e$

$$\bar{k}_e = \sqrt{\frac{\sum_{m=1}^{m=n} d_m k_{Hm}}{\sum_{m=1}^{m=n} d_m/k_{vm}}} \qquad (4\text{-}20)$$

4. Calculate the effective well screen penetration into the transformed aquifer, $\bar{W}/\bar{D}$

$$\frac{\bar{W}}{\bar{D}} = \frac{\sum_{o}^{W} \bar{d}\,\bar{k}}{\sum_{m=1}^{m=n} \bar{d}_m\,\bar{k}_m} = \frac{\sum_{o}^{W} \bar{d}\,\bar{k}}{\bar{D}\,\bar{k}_e} = \frac{\sum_{o}^{W} dk_H}{\sum_{m=1}^{m=n} dk_H} \qquad (4\text{-}21)$$

5. Determine actual well penetration required to achieve a given effective well penetration by successful trials

Figure 4-5. Determination of actual and effective well penetrations