

Appendix E Chart Solutions for Embankment Slopes

E-1. General Use and Applicability of Slope Stability Charts

a. Slope stability charts provide a means for rapid analysis of slope stability. They can be used for preliminary analyses, for checking detailed analyses, or for complete analyses. They are especially useful for making comparisons between design alternatives, because they provide answers so quickly. The accuracy of slope stability charts is usually as good as the accuracy with which shear strengths can be evaluated.

b. In this appendix, chart solutions are presented for four types of slopes:

- (1) Slopes in soils with $\phi = 0$ and uniform strength throughout the depth of the soil layer.
- (2) Slopes in soils with $\phi > 0$ and $c > 0$ and uniform strength throughout the depth of the soil layer.
- (3) Infinite slopes in soils with $\phi > 0$ and $c = 0$ and soils with $\phi > 0$ and $c > 0$.
- (4) Slopes in soils with $\phi = 0$ and strength increasing linearly with depth.

Using approximations in slope geometry and carefully selected soil properties, these chart solutions can be applied to a wide range of nonhomogenous slopes.

c. This appendix contains the following slope stability charts:

- Figure E-1: Slope stability charts for $\phi = 0$ soils (after Janbu 1968).¹
- Figure E-2: Surcharge adjustment factors for $\phi = 0$ and $\phi > 0$ soils (after Janbu 1968).
- Figure E-3: Submergence and seepage adjustment factors for $\phi = 0$ and $\phi > 0$ soils (after Janbu 1968).
- Figure E-4: Tension crack adjustment factors for $\phi = 0$ and $\phi > 0$ soils (after Janbu 1968).
- Figure E-5: Slope stability charts for $\phi > 0$ soils (after Janbu 1968).
- Figure E-6: Steady seepage adjustment factor for $\phi > 0$ soils (after Duncan, Buchianani, and DeWet 1987).
- Figure E-7: Slope stability charts for infinite slopes (after Duncan, Buchianani, and DeWet 1987).
- Figure E-8: Slope stability charts for $\phi = 0$ soils, with strength increasing with depth (after Hunter and Schuster 1968).

E-2. Averaging Slope Inclinations, Unit Weights and Shear Strengths

a. For simplicity, charts are developed for simple homogenous soil conditions. To apply them to nonhomogeneous conditions, it is necessary to approximate the real conditions with an equivalent homogenous slope. The most effective method of developing a simple slope profile for chart analysis is to begin with a cross section of the slope drawn to scale. On this cross section, using judgment, draw a geometrically simple slope that approximates the real slope as closely as possible.

¹ Reference information is presented in Appendix A.

b. To average the shear strengths for chart analysis, it is useful to know the location of the critical slip surface. The charts contained in the following sections of this appendix provide a means of estimating the position of the critical circle. Average strength values are calculated by drawing the critical circle, determined from the charts, on the slope. Then the central angle of arc subtended within each layer or zone of soil is measured with a protractor. The central angles are used as weighting factors to calculate weighted average strength parameters, c_{avg} and ϕ_{avg} are as follows:

$$c_{avg} = \frac{\sum \delta_i c_i}{\sum \delta_i} \quad (E-1)$$

$$\phi_{avg} = \frac{\sum \delta_i \phi_i}{\sum \delta_i} \quad (E-2)$$

where

c_{avg} = average cohesion (stress units)

ϕ_{avg} = average angle of internal friction (degrees)

δ_i = central angle of arc, measured around the center of the estimated critical circle, within zone i (degrees)

c_i = cohesion in zone i (stress units)

ϕ_i = angle of internal friction in zone i

c. One condition in which it is preferable not to use these averaging procedures is the case in which an embankment overlies a weak foundation of saturated clay, with $\phi = 0$. Using Equations E-1 and E-2 to develop average values of c and ϕ in such a case would lead to a small value of ϕ_{avg} (perhaps 2 to 5 degrees). With $\phi_{avg} > 0$, it would be necessary to use the chart shown in Figure E-5, which is based entirely on circles that pass through the toe of the slope. With $\phi = 0$ foundation soils, the critical circle usually goes below the toe into the foundation. In these cases, it is better to approximate the embankment as a $\phi = 0$ soil and to use the $\phi = 0$ slope stability charts shown in Figure E-1. The equivalent $\phi = 0$ strength of the embankment soil can be estimated by calculating the average normal stress on the part of the slip surface within the embankment (one-half the average vertical stress is usually a reasonable approximation of the normal stress on this part of the slip surface) and determining the corresponding shear strength at that point on the shear strength envelope for the embankment soil. This value of strength is treated as a value of S_u for the embankment, with $\phi = 0$. The average value of S_u is then calculated for both the embankment and the foundation using the same averaging procedure as described above.

$$(S_u)_{avg} = \frac{\sum \delta_i (S_u)_i}{\sum \delta_i} \quad (E-3)$$

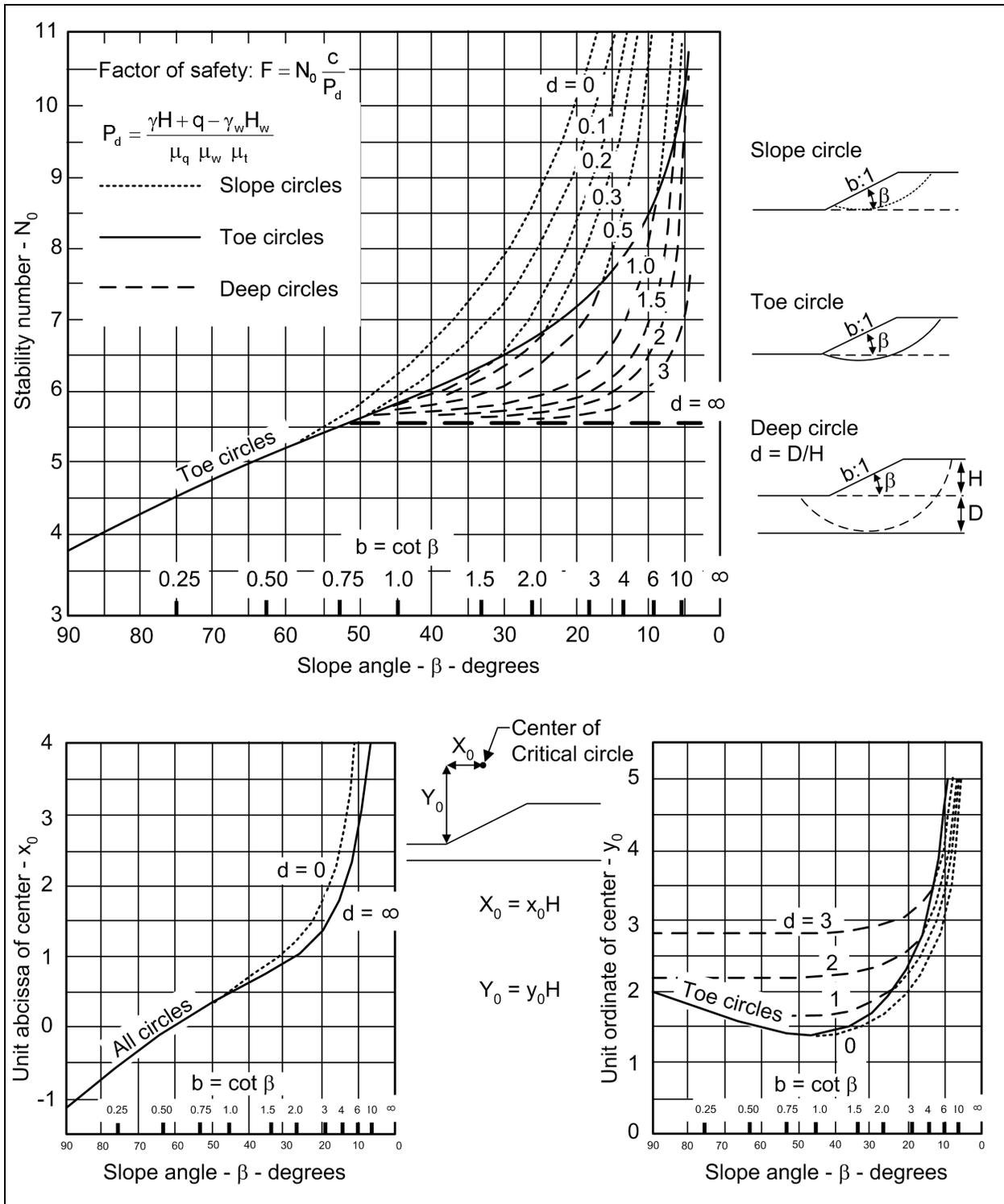


Figure E-1. Slope stability charts for $\phi = 0$ soils (after Janbu 1968)

where

$(S_u)_{avg}$ = average undrained shear strength (in stress units)

$(S_u)_i$ = S_u in layer i (in stress units)

δ_i = central angle of arc, measured around the center of the estimated critical circle, within zone i (degrees)

This average value of S_u is then used, with $\phi = 0$, for analysis of the slope.

d. To average unit weights for use in chart analysis, it is usually sufficient to use layer thickness as a weighting factor, as indicated by the following expression:

$$\gamma_{avg} = \frac{\sum \gamma_i h_i}{\sum h_i} \quad (E-4)$$

where

γ_{avg} = average unit weight (force per length cubed)

γ_i = unit weight of layer i (force per length cubed)

h_i = thickness of layer i (in length units)

e. Unit weights should be averaged only to the depth of the bottom of the critical circle. If the material below the toe of the slope is a $\phi = 0$ material, the unit weight should be averaged only down to the toe of the slope, since the unit weight of the material below the toe has no effect on stability in this case.

E-3. Soils with $\phi = 0$

The slope stability chart for $\phi = 0$ soils, developed by Janbu (1968), is shown in Figure E-1. Charts providing adjustment factors for surcharge loading at the top of the slope are shown in Figure E-2. Charts providing adjustment factors for submergence and seepage are shown in Figure E-3. Charts providing adjustment factors to account for tension cracks are shown in Figure E-4.

a. Steps for use of $\phi = 0$ charts:

(1) Using judgment, decide which cases should be investigated. For uniform soil conditions, the critical circle passes through the toe of the slope if the slope is steeper than about 1 (H) on 1 (V). For flatter slopes, the critical circle usually extends below the toe, and is tangent to some deep firm layer. The chart in Figure E-1 can be used to compute factors of safety for circles extending to any depth. Multiple possibilities should be analyzed, to be sure that the overall critical circle and overall minimum factor of safety have been found.

(2) The following criteria can be used to determine which possibilities should be examined:

(a) If there is water outside the slope, a circle passing above the water may be critical.

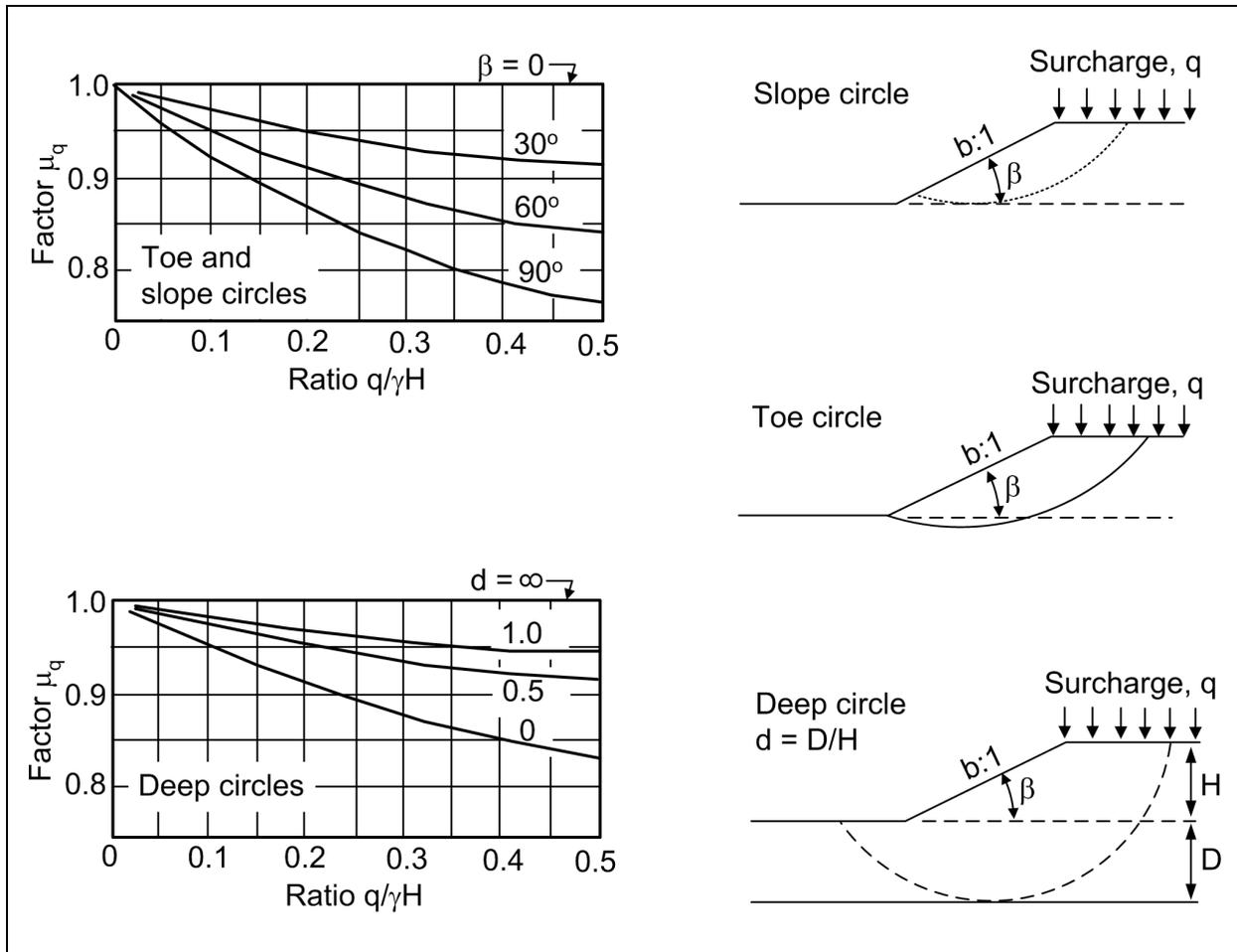


Figure E-2. Surcharge adjustment factors $\phi = 0$ and $\phi > 0$ soils (after Janbu 1968)

(b) If a soil layer is weaker than the one above it, the critical circle may be tangent to the base of the lower (weaker) layer. This applies to layers both above and below the toe.

(c) If a soil layer is stronger than the one above it, the critical circle may be tangent to the base of either layer, and both possibilities should be examined. This applies to layers both above and below the toe.

(3) The following steps are performed for each circle:

(a) Calculate the depth factor, d , using the formula:

$$d = \frac{D}{H} \tag{E-5}$$

where

D = depth from the toe of the slope to the lowest point on the slip circle (L ; length)

H = slope height above the toe of the slope (L)

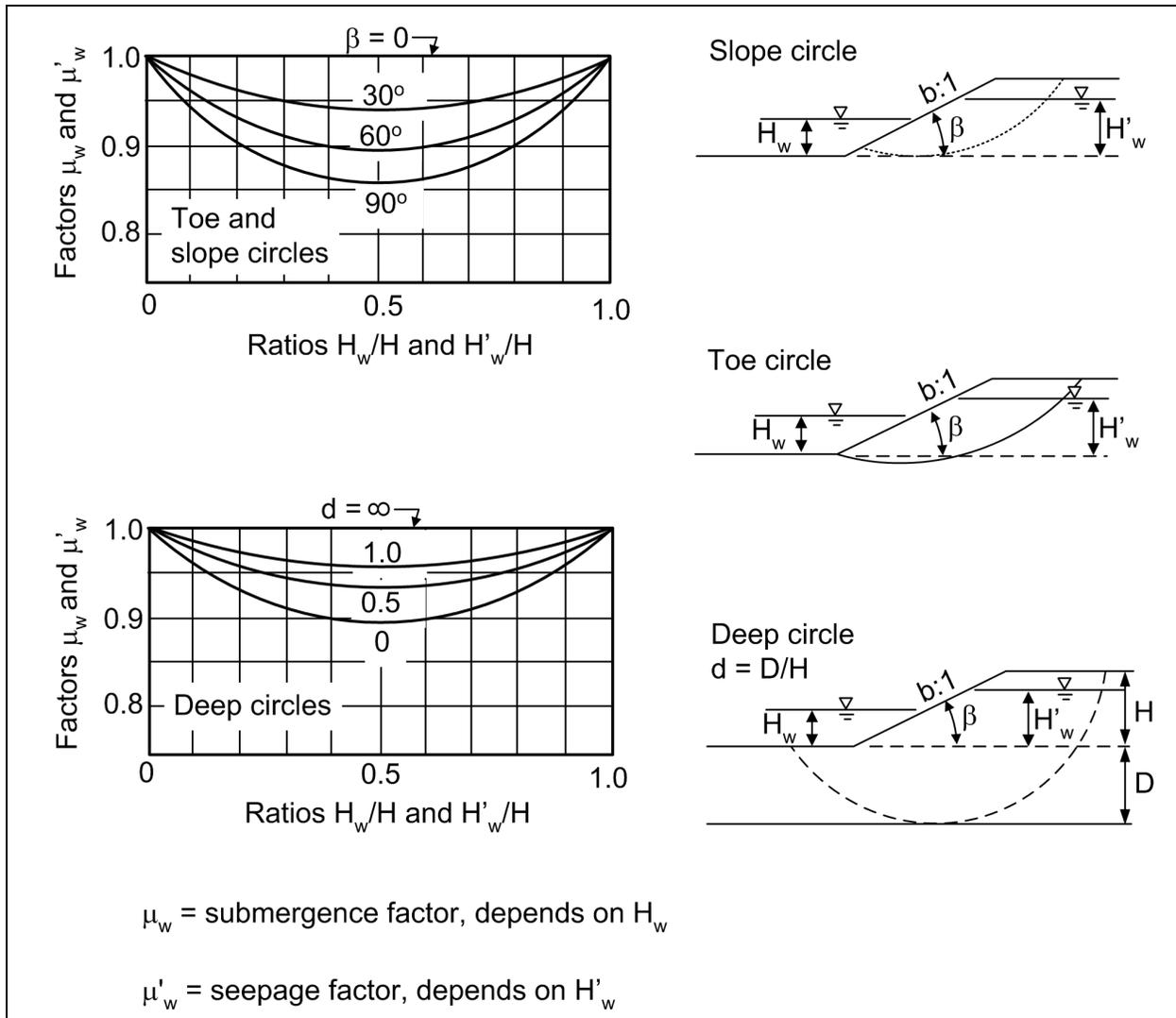


Figure E-3. Submergence and seepage adjustment factors $\phi = 0$ and $\phi > 0$ soils (after Janbu 1968)

The value of d is 0 if the circle does not pass below the toe of the slope. If the circle being analyzed is entirely above the toe, its point of interaction with the slope should be taken as an “adjusted toe,” and all dimensions like D , H , and H_w must be adjusted accordingly in the calculations.

(b) Find the center of the critical circle using the charts at the bottom of Figure E-1, and draw this circle to scale on a cross section of the slope.

(c) Determine the average value of the strength, $c = S_u$, for the circle, using Equation E-3.

(d) Calculate the quantity P_d using Equation E-6:

$$P_d = \frac{\gamma H + q - \gamma_w H_w}{\mu_q \mu_w \mu_t} \quad (\text{E-6})$$

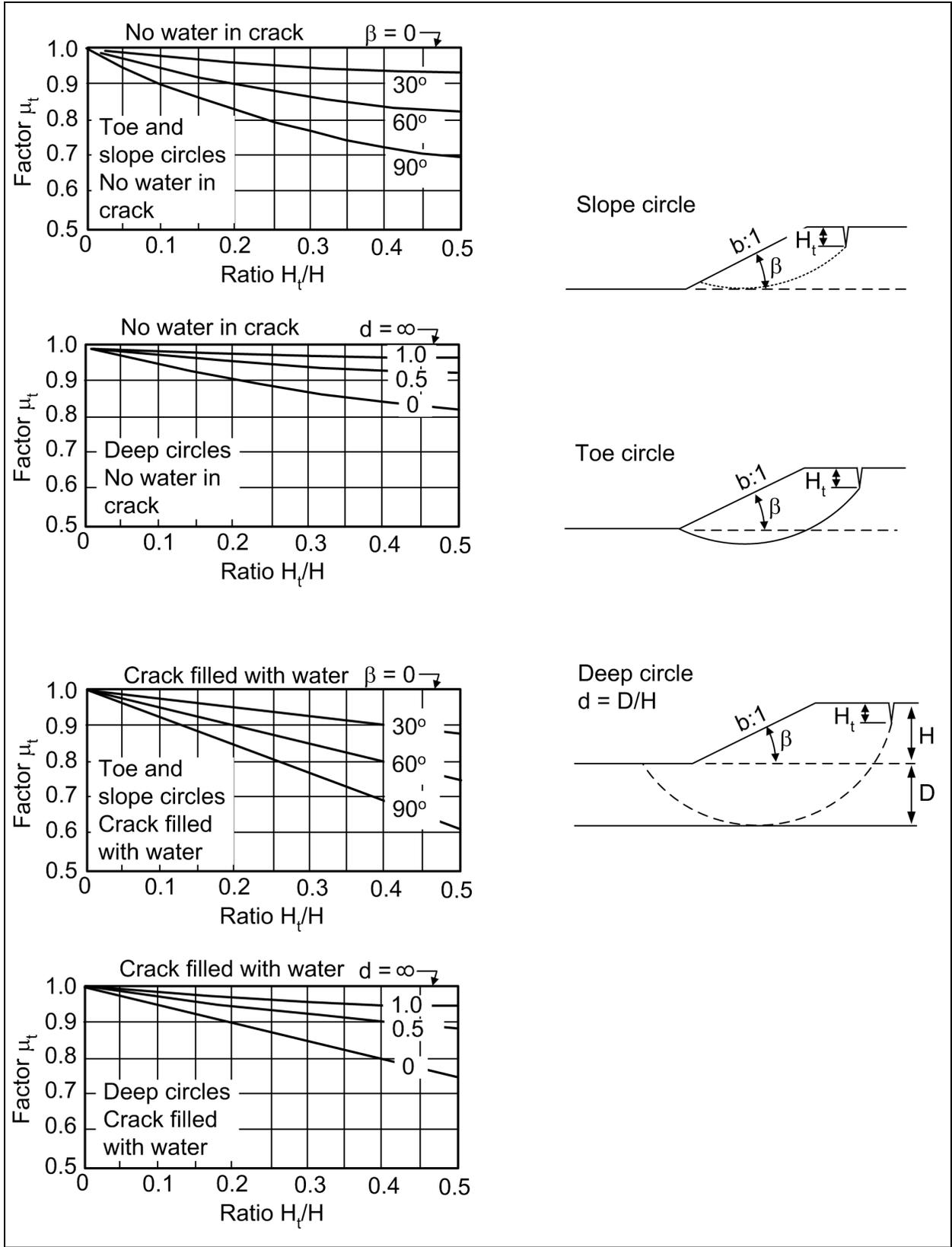


Figure E-4. Tension crack adjustment factors $\phi = 0$ and $\phi > 0$ soils (after Janbu 1968)

where

γ = average unit weight of soil (F/L^3)

H = slope height above toe (L)

q = surcharge (F/L^2)

γ_w = unit weight of water (F/L^3)

H_w = height of external water level above toe (L)

μ_q = surcharge adjustment factor (Figure E-2)

μ_w = submergence adjustment factor (Figure E-3)

μ_t = tension crack adjustment factor (Figure E-4)

If there is no surcharge, $\mu_q = 1$.

If there is no external water above toe, $\mu_w = 1$; and if there are no tension cracks, $\mu_t = 1$.

(e) Using the chart at the top of Figure E-1, determine the value of the stability number, N_o , which depends on the slope angle, β , and the value of d .

(f) Calculate the factor of safety, F , using Equation E-7:

$$F = \frac{N_o c}{P_d} \quad (E-7)$$

where

N_o = stability number

c = average shear strength = $(S_u)_{avg}$ (F/L^2)

b. The example problems in Figures E-9 and E-10 illustrate the use of these methods. Note that both problems involve the same slope, and that the only difference between the two problems is the depth of the circle analyzed.

E-4. Soils with $\phi > 0$

a. The slope stability chart for $\phi > 0$ soils, developed by Janbu (1968), is shown in Figure E-5.

b. Adjustment factors for surcharge are shown in Figure E-2. Adjustment factors for submergence and seepage are shown in Figure E-3. Adjustment factors for tension cracks are shown in Figure E-4.

c. The stability chart in Figure E-5 can be used for analyses in terms of effective stresses. The chart may also be used for total stress analysis of unsaturated slopes with $\phi > 0$.

d. Steps for use of charts are:

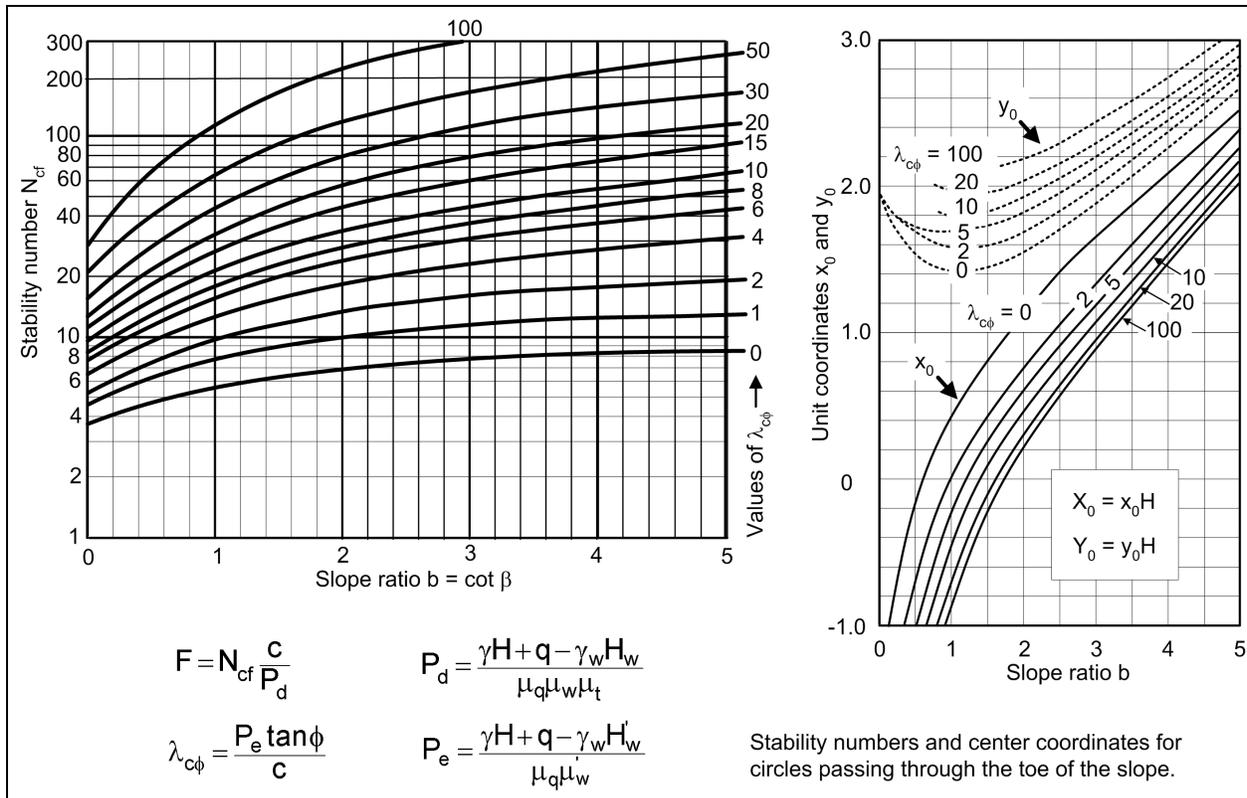


Figure E-5. Slope stability charts for $\phi > 0$ soils (after Janbu 1968)

(1) Estimate the location of the critical circle. For most conditions of slopes in uniform soils with $\phi > 0$, the critical circle passes through the toe of the slope. The stability numbers given in Figure E-5 were developed by analyzing toe circles. In cases where $c = 0$, the critical mechanism is shallow sliding, which can be analyzed as the infinite slope failure mechanism. The stability chart shown in Figure E-7 can be used in this case. If there is water outside the slope, the critical circle may pass above the water. If conditions are not homogeneous, a circle passing above or below the toe may be more critical than the toe circle. The following criteria can be used to determine which possibilities should be examined:

(a) If there is water outside the slope, a circle passing above the water may be critical.

(b) If a soil layer is weaker than the one above it, the critical circle may be tangent to the base of the lower (weaker) layer. This applies to layers both above and below the toe.

(c) If a soil layer is stronger than the one above it, the critical circle may be tangent to the base of either layer, and both possibilities should be examined. This applies to layers both above and below the toe.

The charts in Figure E-5 can be used for nonuniform conditions provided the values of c and ϕ used in the calculation represent average values for the circle considered. The following steps are performed for each circle.

(2) Calculate P_d using the formula:

$$P_d = \frac{\gamma H + q - \gamma_w H_w}{\mu_q \mu_w \mu_t} \tag{E-8}$$

where

γ = average unit weight of soil (F/L³)

H = $\sigma\lambda\omicron\pi\epsilon$ height above toe (L)

q = $\sigma\upsilon\rho\chi\eta\alpha\rho\gamma\epsilon$ (F/L²)

γ_w = unit weight of water (F/L³)

H_w = height of external water level above toe (L)

μ_q = surcharge reduction factor (Figure E-2)

μ_w = submergence reduction factor (Figure E-3)

μ_t = tension crack reduction factor (Figure E-4)

$\mu_q = 1$, if there is no surcharge

$\mu_w = 1$, if there is no external water above toe

$\mu_t = 1$, if there are no tension cracks

If the circle being studied passes above the toe of the slope, the point where the circle intersects the slope face should be taken as the toe of the slope for the calculation of H and H_w .

(3) Calculate P_e using the formula:

$$P_e = \frac{\gamma H + q - \gamma_w H_w'}{\mu_q \mu_w'} \quad (E-9)$$

where

H_w' = height of water within slope (L)

μ_w' = seepage correction factor (Figure E-3)

The other factors are as defined previously.

H_w' is the average level of the piezometric surface within the slope. For steady seepage conditions this is related to the position of the phreatic surface beneath the crest of the slope as shown in Figure E-6 (after Duncan, Buchignani, and DeWet 1987). If the circle being studied passes above the toe of the slope, H_w' is measured relative to the adjusted toe.

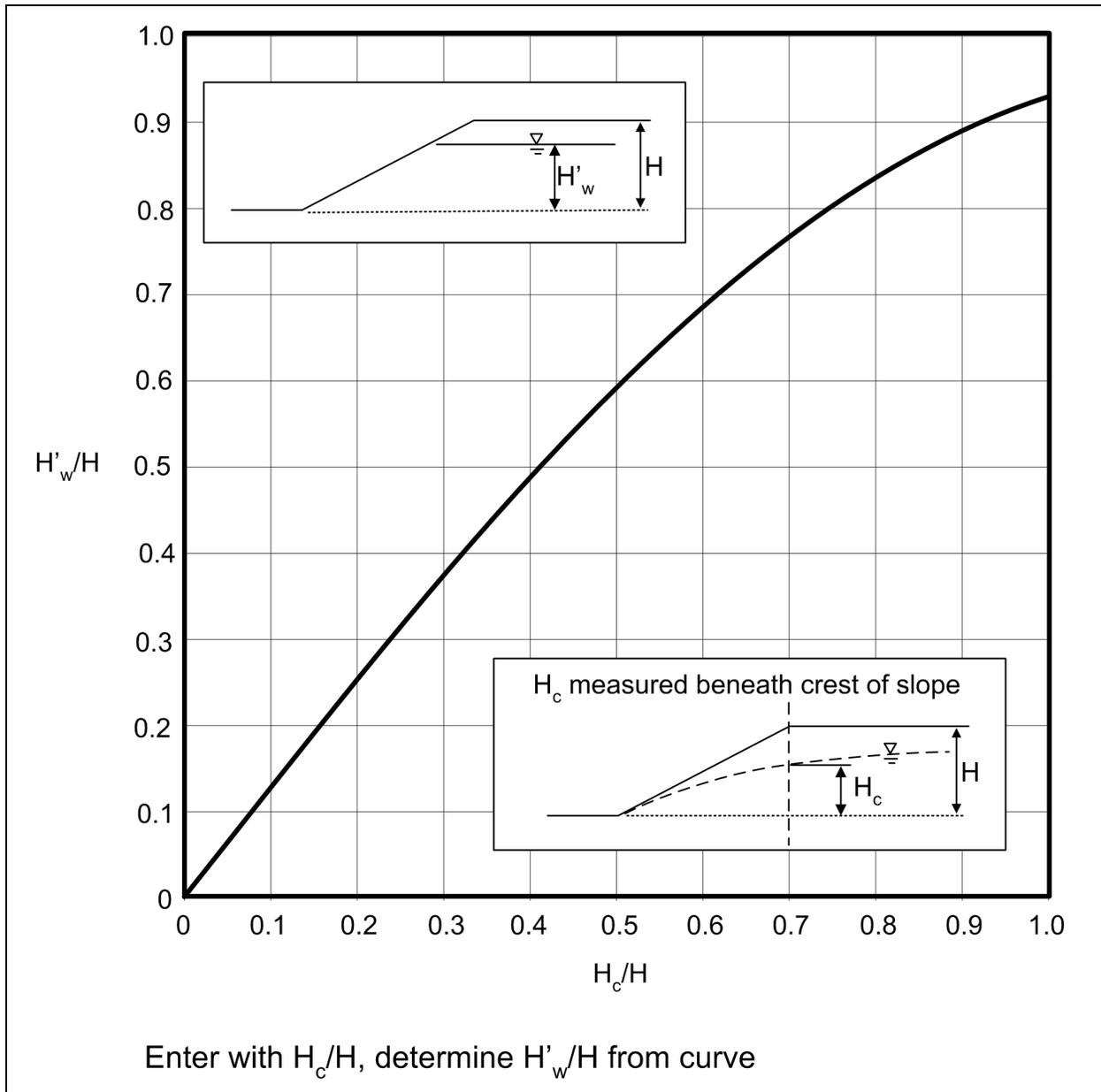


Figure E-6. Steady seepage adjustment factor $\phi > 0$ soils (after Duncan, Buchianani, and DeWet 1987)

$\mu_w' = 1$, if there is no seepage

$\mu_q = 1$, if there is no surcharge

In a total stress analysis, internal pore water pressure is not considered, so $H_w' = 0$ and $\mu_w' = 1$ in the formula for P_e .

(4) Calculate the dimensionless parameter $\lambda_{c\phi}$ using the formula:

$$\lambda_{c\phi} = \frac{P_e \tan \phi}{c} \quad (\text{E-10})$$

where

ϕ = average value of ϕ

c = average value of c (F/L^2)

For $c = 0$, $\lambda_{c\phi}$ is infinite. Use the charts for infinite slopes in this case. Steps 4 and 5 are iterative steps. On the first iteration, average values of $\tan \phi$ and c are estimated using judgment rather than averaging.

(5) Using the chart at the top of Figure E-5, determine the center coordinates of the circle being investigated.

(a) Plot the critical circle on a scaled cross section of the slope, and calculate the weighted average values of ϕ and c using Equations E-1 and E-2.

(b) Return to Step 4 with these average values of the shear strength parameters, and repeat this iterative process until the value of $\lambda_{c\phi}$ becomes constant. Usually one iteration is sufficient.

(6) Using the chart at the left side of Figure E-5, determine the value of the stability number N_{cf} , which depends on the slope angle, β , and the value of $\lambda_{c\phi}$.

(7) Calculate the factor of safety, F , using the formula:

$$F = N_{cf} \frac{c}{P_d} \quad (E-11)$$

The example problems in Figures E-11 and E-12 illustrate the use of these methods for total stress and effective stress analyses.

E-5. Infinite Slope Analyses

a. Two types of conditions can be analyzed using the charts shown in Figure E-7: These are:

(1) Slopes in cohesionless materials, where the critical failure mechanism is shallow sliding or surface raveling.

(2) Slopes in residual soils, where a relatively thin layer of soil overlies firmer soil or rock, and the critical failure mechanism is sliding along a plane parallel to the slope, at the top of the firm layer.

b. Steps for use of the charts for effective stress analyses:

(1) Determine the pore pressure ratio, r_u , which is defined by the formula:

$$r_u = \frac{u}{\gamma H} \quad (E-12)$$

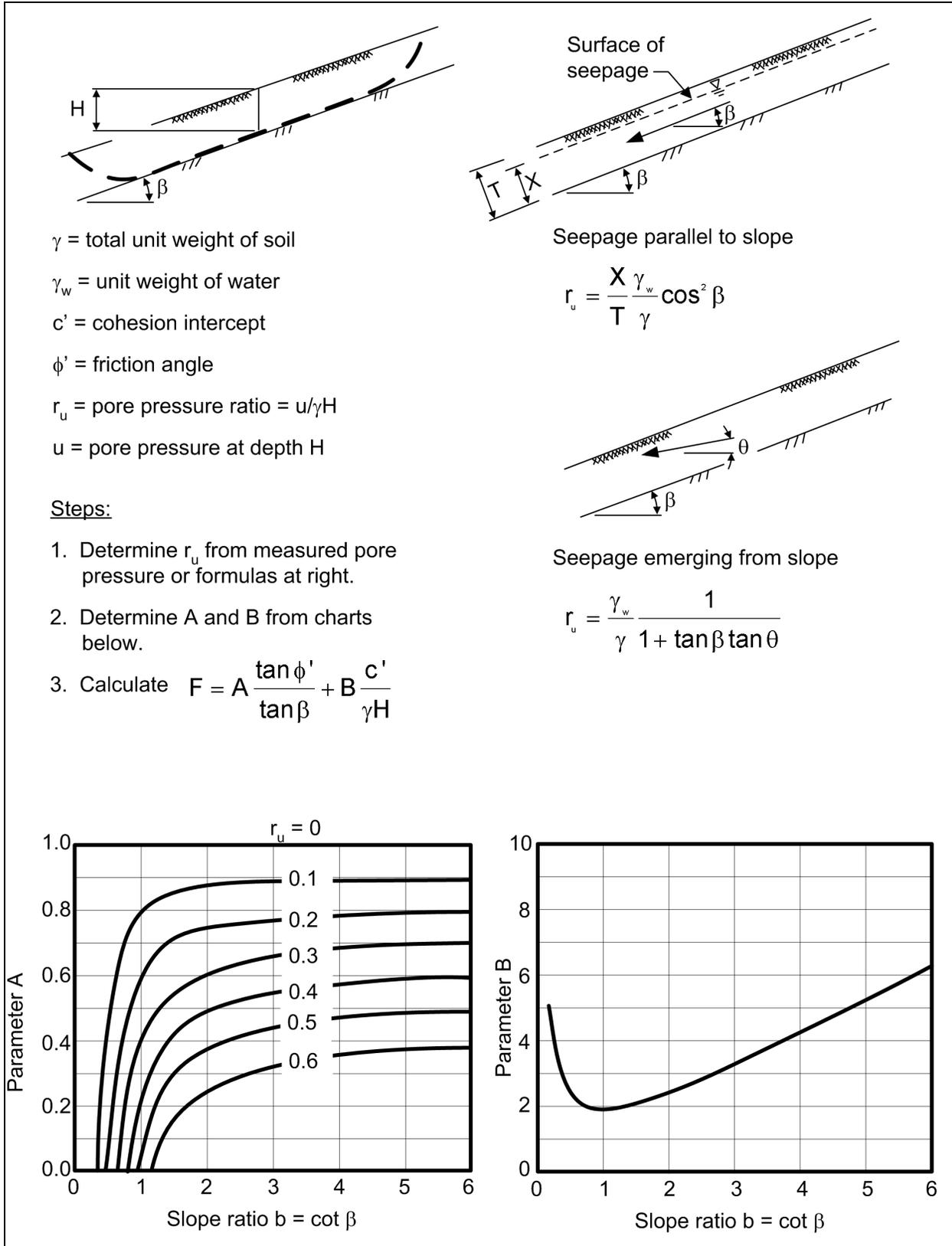


Figure E-7. Slope stability charts for infinite slopes (after Duncan, Buchianani, and DeWet 1987)

where

u = pore pressure (F/L^2)

γ = total unit weight of soil (F/L^3)

H = depth corresponding to pore pressure, u (L)

(a) For an existing slope, the pore pressure can be determined from field measurements, using piezometers installed at the depth of sliding, or estimated for the most adverse anticipated seepage condition.

(b) For seepage parallel to the slope, which is a condition frequently used for design, the value of r_u can be calculated using the following formula:

$$r_u = \frac{X}{T} \frac{\gamma_w}{\gamma} \cos^2 \beta \quad (E-13)$$

where

X = distance from the depth of sliding to the surface of seepage, measured normal to the surface of the slope (L)

T = distance from the depth of sliding to the surface of the slope, measured normal to the surface of the slope (L)

γ_w = unit weight of water (F/L^3)

γ = total unit weight of soil (F/L^3)

β = slope angle

(c) For seepage emerging from the slope, which is more critical than seepage parallel to the slope, the value of r_u can be calculated using the following formula:

$$r_u = \frac{\gamma_w}{\gamma} \frac{1}{1 + \tan \beta \tan \theta} \quad (E-14)$$

where

θ = angle of seepage measured from the horizontal direction

The other factors are as defined previously.

(1) Submerged slopes with no excess pore pressures can be analyzed using $\gamma = \gamma_b$ (buoyant unit weight) and $r_u = 0$.

(2) Determine the values of the dimensionless parameters A and B from the charts at the bottom of Figure E-7.

(3) Calculate the factor of safety, F , using Equation E-15:

$$F = A \frac{\tan \phi'}{\tan \beta} + B \frac{c'}{\gamma H} \quad (\text{E-15})$$

where

ϕ' = angle of internal friction in terms of effective stress

c' = cohesion intercept in terms of effective stress (F/L^2)

β = slope angle

H = depth of sliding mass measured vertically (L)

The other factors are as defined previously.

c. Steps for use of charts for total stress analyses:

(1) Determine the value of B from the chart in the lower right corner of Figure E-7.

(2) Calculate the factor of safety, F, using the formula:

$$F = \frac{\tan \phi}{\tan \beta} + B \frac{c}{\gamma H} \quad (\text{E-16})$$

where

ϕ = angle of internal friction in terms of total stress

c = cohesion intercept in terms of total stress (F/L^2)

The other factors are as defined previously.

The example in Figure E-13 illustrates use of the infinite slope stability charts.

E-6. Soils with $\phi = 0$ and Strength Increasing with Depth

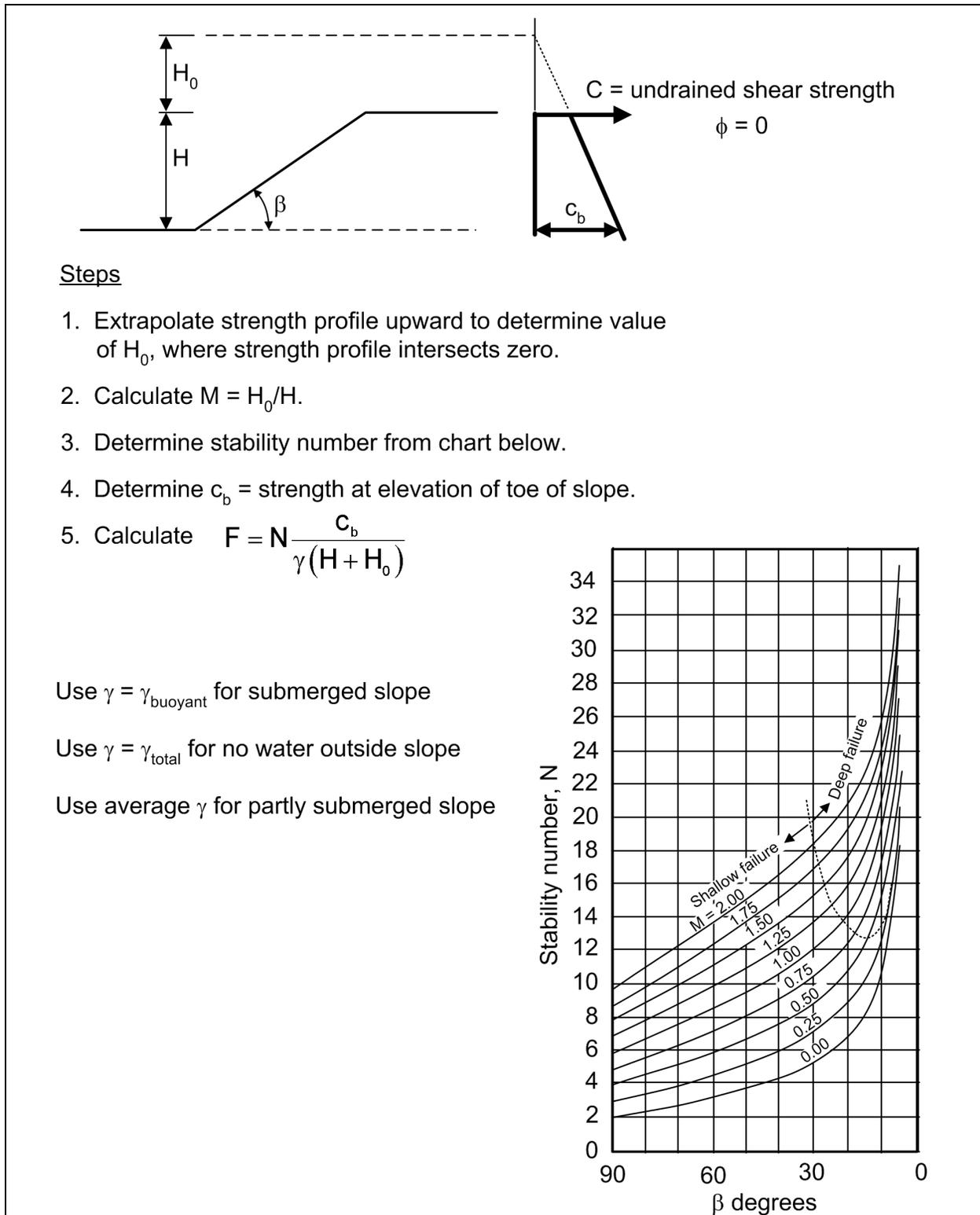
The chart for slopes in soils with $\phi = 0$ and strength increasing with depth is shown in Figure E-8. Steps for use of the chart are:

a. Select the linear variation of strength with depth that best fits the measured strength data. As shown in Figure E-8, extrapolate this straight line upward to determine H_0 , the height at which the straight line intersects zero.

b. Calculate $M = H_0/H$, where H = slope height.

c. Determine the dimensionless stability number, N , from the chart in the lower right corner of Figure E-8.

d. Determine the value of c_b , the strength at the elevation of the bottom (the toe) of the slope.



Steps

1. Extrapolate strength profile upward to determine value of H_0 , where strength profile intersects zero.
2. Calculate $M = H_0/H$.
3. Determine stability number from chart below.
4. Determine c_b = strength at elevation of toe of slope.
5. Calculate
$$F = N \frac{c_b}{\gamma(H + H_0)}$$

Use $\gamma = \gamma_{\text{buoyant}}$ for submerged slope

Use $\gamma = \gamma_{\text{total}}$ for no water outside slope

Use average γ for partly submerged slope

Figure E-8. Slope stability chart for $\phi = 0$ soils, with strength increasing with depth (after Hunter and Schuster 1968)

e. Calculate the factor of safety, F, using the formula:

$$F = N \frac{c_b}{\gamma(H + H_o)} \quad (E-17)$$

where

γ = total unit weight of soil for slopes above water

γ = buoyant unit weight for submerged slopes

γ = weighted average unit weight for partly submerged slopes

The example shown in Figure E-14 illustrates use of the stability chart shown in Figure E-8.

Example Problem E-1

Figure E-9 shows a slope in $\phi = 0$ soil. There are three layers with different strengths. There is water outside the slope.

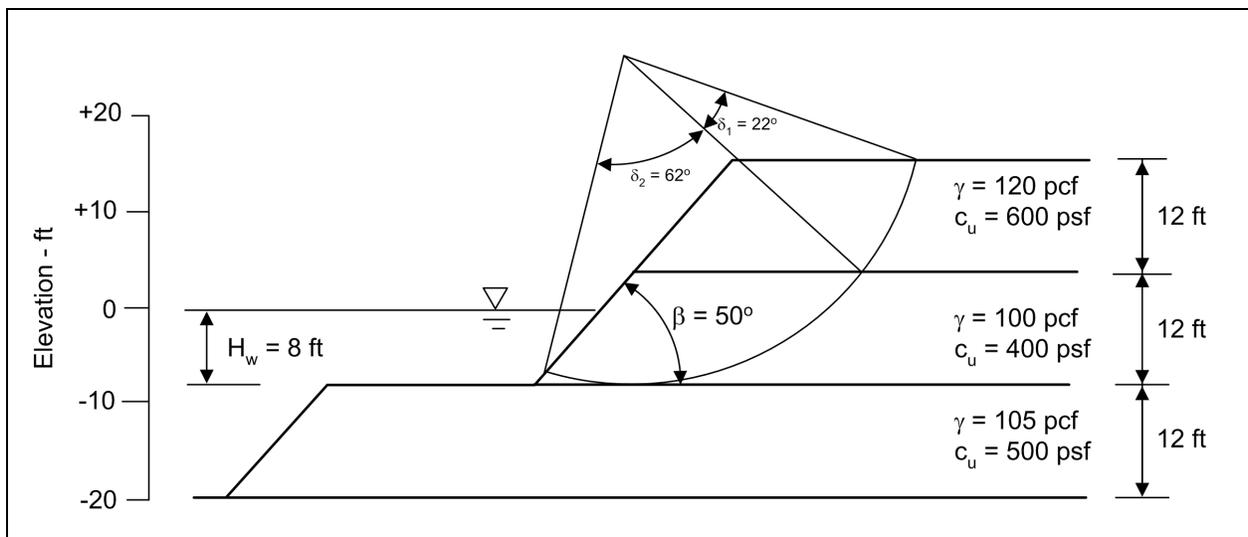


Figure E-9. Example for a circle tangent to elevation -8ft for cohesive soil with $\phi = 0$

Two circles were analyzed for this slope -- a shallow circle tangent to elevation -8 ft, and a deep circle tangent to elevation -20 ft.

The shallower circle, tangent to elevation -8 ft, is analyzed first.

For this circle:

$$d = \frac{D}{H} = \frac{0}{24} = 0$$

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$$\frac{H_w}{H} = \frac{8}{24} = 0.33$$

Using the charts at the top of Figure E-1, with $\beta = 50^\circ$ and $d = 0$:

$$x_o = 0.35 \text{ and } y_o = 1.4$$

$$X_o = (H)(x_o) = (24)(0.35) = 8.4 \text{ ft}$$

$$Y_o = (H)(y_o) = (24)(1.4) = 33.6 \text{ ft}$$

Plot the critical circle on the slope. The circle is shown in Figure E-9.

Measure the central angles of arc in each layer using a protractor. Calculate the weighted average strength parameter c_{avg} using Equation E-1.

$$c_{\text{avg}} = \frac{\sum \delta_i c_i}{\sum \delta_i} = \frac{(22)(600) + (62)(400)}{22 + 62} = 452 \text{ psf}$$

From Figure E-3, with $\beta = 50^\circ$ and $\frac{H_w}{H} = 0.33$, find $\mu_w = 0.93$.

Use layer thickness to average the unit weights. Unit weights are averaged only to the bottom of the critical circle.

$$\gamma_{\text{avg}} = \frac{\sum \gamma_i h_i}{\sum h_i} = \frac{(120)(10) + (100)(10)}{10 + 10} = 110$$

Calculate the driving force term P_d as follows:

$$P_d = \frac{\gamma H + q - \gamma_w H_w}{\mu_q \mu_w \mu_t} = \frac{(110)(24) + 0 - (62.4)(8)}{(1)(0.93)(1)} = 2302$$

From Figure E-1, with $d = 0$ and $\beta = 50^\circ$, find $N_o = 5.8$:

Calculate the factor of safety using Equation E-7:

$$F = \frac{N_o c}{P_d} = \frac{(5.8)(452)}{2302} = 1.14$$

Example Problem E-2

Figure E-10 shows the same slope as in Figure E-9.

The deeper circle, tangent to elevation -20 ft, is analyzed as follows:

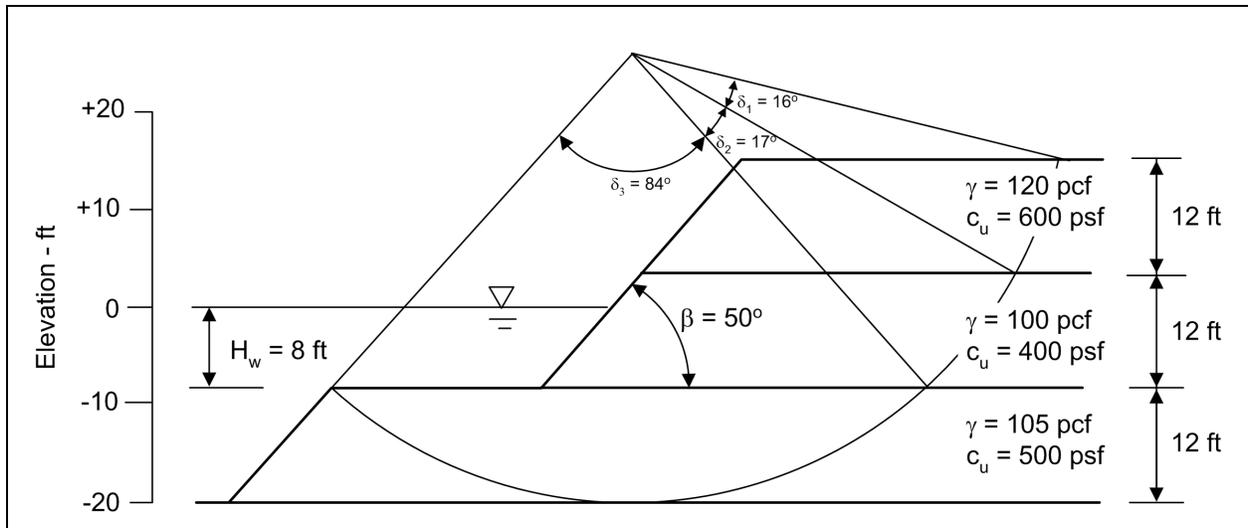


Figure E-10. Example for a circle tangent to elevation -20 ft for cohesive soil with $\phi = 0$

For this circle:

$$d = \frac{D}{H} = \frac{12}{24} = 0.5$$

$$\frac{H_w}{H} = \frac{8}{24} = 0.33$$

Using the charts at the bottom of Figure E-1, with $\beta = 50^\circ$ and $d = 0.5$:

$$x_o = 0.35 \text{ and } y_o = 1.5$$

$$X_o = (H)(x_o) = (24)(0.35) = 8.4 \text{ ft}$$

$$Y_o = (H)(y_o) = (24)(1.5) = 36 \text{ ft}$$

Plot the critical circle on the slope as shown in Figure E-10.

Measure the central angles of arc in each layer using a protractor. Calculate the weighted average strength parameter c_{avg} : using Equation E-1.

$$c_{\text{avg}} = \frac{\sum \delta_i c_i}{\sum \delta_i} = \frac{(16)(600) + (17)(400) + (84)(500)}{16 + 17 + 84} = 499 \text{ psf}$$

From Figure E-3, with $d = 0.5$ and $\frac{H_w}{H} = 0.33$:

$$\mu_w = 0.95$$

Use layer thickness to average the unit weights. Since the material below the toe of the slope is a $\phi = 0$ material, the unit weight is averaged only down to the toe of the slope. The unit weight below the toe has no influence on stability if $\phi = 0$.

$$\gamma_{\text{avg}} = \frac{\sum \gamma_i h_i}{\sum h_i} = \frac{(120)(10) + (100)(10)}{10 + 10} = 110$$

Calculate the driving force term P_d as follows:

$$P_d = \frac{\gamma H + q - \gamma_w H_w}{\mu_q \mu_w \mu_t} = \frac{(110)(24) + 0 - (62.4)(8)}{(1)(0.95)(1)} = 2253$$

From Figure E-1, with $d = 0.5$ and $\beta = 50^\circ$, $N_o = 5.6$:

Calculate the factor of safety using Equation E-7:

$$F = \frac{N_o c}{P_d} = \frac{(5.6)(499)}{2253} = 1.24$$

This circle is less critical than the circle tangent to elevation -8 ft, analyzed previously.

Example Problem E-3

Figure E-11 shows a slope in soils with both c and ϕ . There are three layers with different strengths. There is no water outside the slope.

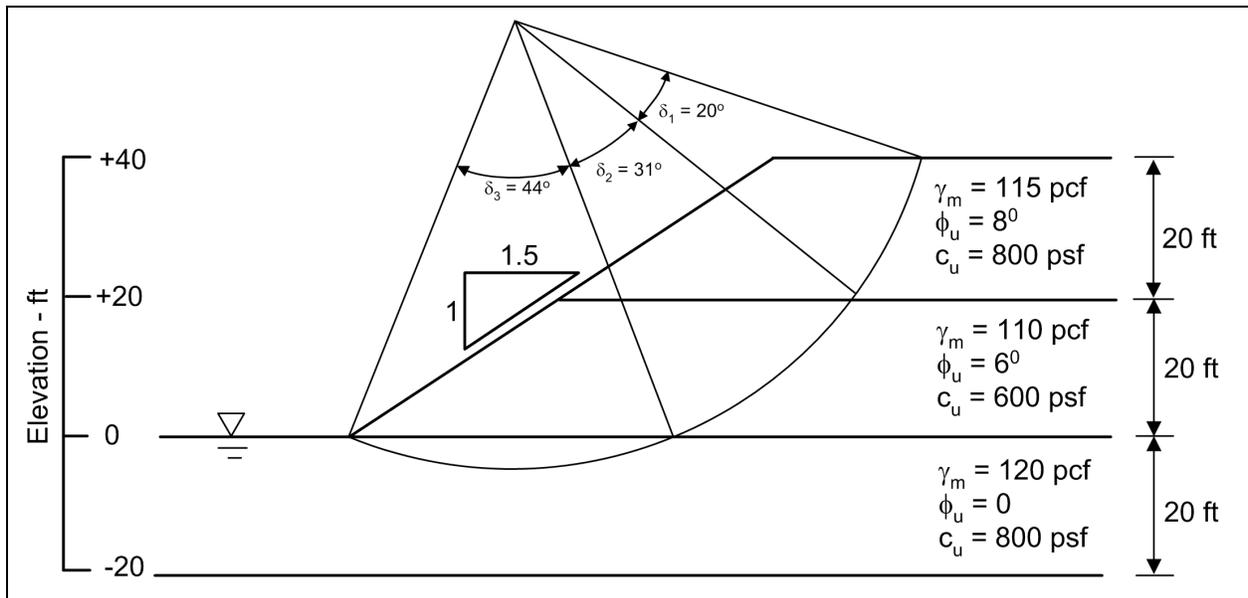


Figure E-11. Example for a total stress analysis of a toe circle in soils with both c and ϕ

The factor of safety for a toe circle is calculated as follows:

Use layer thickness to average the unit weights. Unit weights are averaged down to the toe of the slope, since the unit weight of the material below the toe has no effect on stability in this case.

$$\gamma_{\text{avg}} = \frac{\sum \gamma_i h_i}{\sum h_i} = \frac{(115)(10) + (110)(10)}{10 + 10} = 112.5$$

Since there is no surcharge, $\mu_q = 1$

Since there is no external water above toe, $\mu_w = 1$

Since there is no seepage, $\mu_w' = 1$

Since there are no tension cracks, $\mu_t = 1$

Calculate the driving force term P_d as follows:

$$P_d = \frac{\gamma H + q - \gamma_w H_w}{\mu_q \mu_w \mu_t} = \frac{(112.5)(40)}{(1)(1)(1)} = 4500 \text{ psf}$$

Calculate P_e as follows:

$$P_e = \frac{\gamma H + q - \gamma_w H_w'}{\mu_q \mu_w'} = \frac{(112.5)(40)}{(1)(1)} = 4500 \text{ psf}$$

Estimate $c_{\text{avg}} = 700$ psf and $\phi_{\text{avg}} = 7^\circ$, and calculate $\lambda_{c\phi}$ as follows:

$$\lambda_{c\phi} = \frac{P_e \tan \phi}{c} = \frac{(4500)(0.122)}{700} = 0.8$$

From Figure E-5, with $b = 1.5$ and $\lambda_{c\phi} = 0.8$:

$$x_o = 0.6 \text{ and } y_o = 1.5$$

$$X_o = (H)(x_o) = (40)(0.6) = 24 \text{ ft}$$

$$Y_o = (H)(y_o) = (40)(1.5) = 60 \text{ ft}$$

Plot the critical circle on the given slope, as shown in Figure E-11.

Calculate c_{avg} , $\tan \phi_{\text{avg}}$, and $\lambda_{c\phi}$ as follows:

$$c_{\text{avg}} = \frac{\sum \delta_i c_i}{\sum \delta_i} = \frac{(20)(800) + (31)(600) + (44)(800)}{20 + 31 + 44} = 735 \text{ psf}$$

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$$\tan \phi_{\text{avg}} = \frac{\sum \delta_i \tan \phi_i}{\sum \delta_i} = \frac{(20)(\tan 8^\circ) + (31)(\tan 6^\circ) + (44)(\tan 0^\circ)}{20 + 31 + 44} = 0.064$$

$$\lambda_{c\phi} = \frac{P_c \tan \phi}{c} = \frac{(4500)(0.064)}{735} = 0.4$$

From Figure E-5, with $b = 1.5$ and $\lambda_{c\phi} = 0.4$:

$$x_o = 0.65 \text{ and } y_o = 1.45$$

$$X_o = (H)(x_o) = (40)(0.65) = 26 \text{ ft}$$

$$Y_o = (H)(y_o) = (40)(1.45) = 58 \text{ ft}$$

This circle is close to the previous iteration, so keep $\lambda_{c\phi} = 0.4$ and $c_{\text{avg}} = 735$ psf

From Figure E-5, with $b = 1.5$ and $\lambda_{c\phi} = 0.4$

$$N_{cf} = 6$$

Calculate the factor of safety as follows:

$$F = N_{cf} \frac{c}{P_d} = 6.0 \frac{735}{4500} = 1.0$$

Example Problem E-4

Figure E-12 shows the same slope as shown in Figure E-11. Effective stress strength parameters are shown in the figure, and the analysis is performed using effective stresses. There is water outside the slope, and seepage within the slope.

Use layer thickness to average the unit weights. Unit weights are averaged only down to the toe of the slope.

$$\gamma_{\text{avg}} = \frac{\sum \gamma_i h_i}{\sum h_i} = \frac{(115)(10) + (115)(10)}{10 + 10} = 115$$

For this slope:

$$\frac{H_w}{H} = \frac{10}{40} = 0.25$$

$$\frac{H_w'}{H} = \frac{30}{40} = 0.75$$

Since there is no surcharge, $\mu_q = 1$

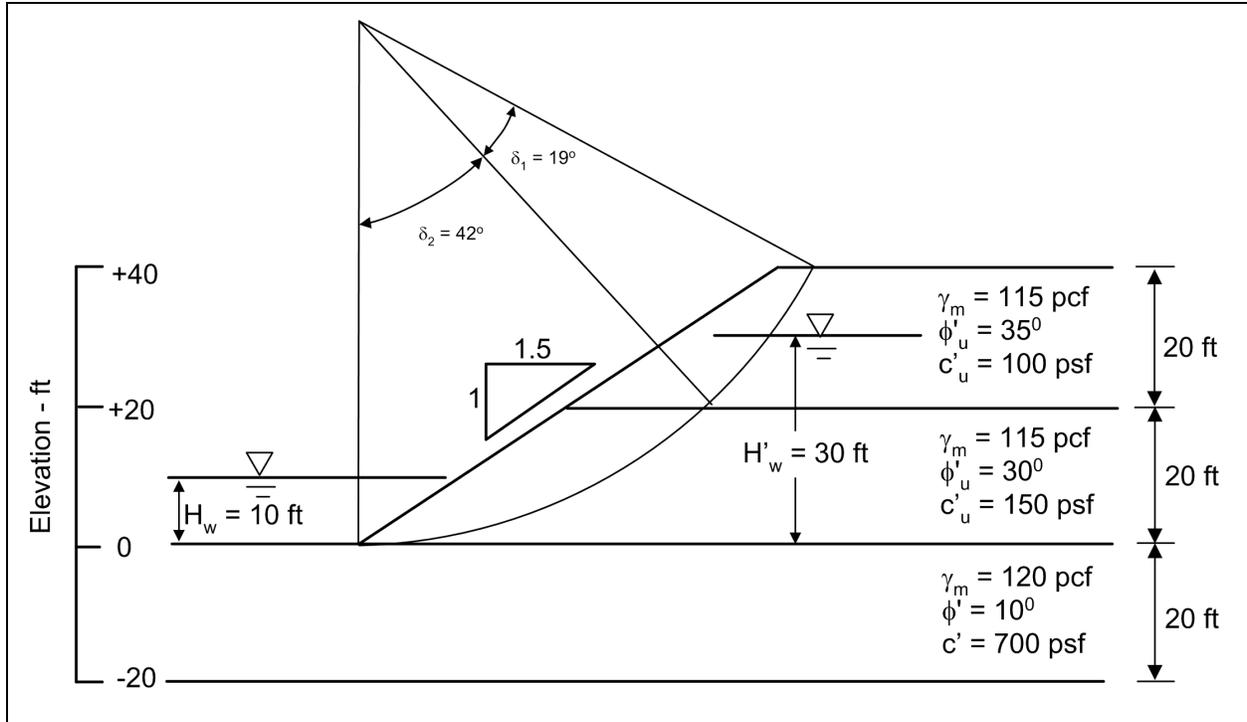


Figure E-12. Example for an effective stress analysis of a toe circle in soils with both c' and ϕ'

Using Figure E-3 for toe circles, with $H_w/H = 0.25$ and $\beta = 33.7^\circ$, find $\mu_w = 0.96$

Using Figure E-3 for toe circles, with $H'_w/H = 0.75$ and $\beta = 33.7^\circ$, find $\mu'_w = 0.95$

Since there are no tension cracks, $\mu_t = 1$

Calculate the driving force term P_d as follows:

$$P_d = \frac{\gamma H + q - \gamma_w H_w}{\mu_q \mu_w \mu_t} = \frac{(115)(40) + 0 - (62.4)(10)}{(1)(0.96)(1)} = 4141 \text{ psf}$$

Calculate P_e as follows:

$$P_e = \frac{\gamma H + q - \gamma_w H'_w}{\mu_q \mu'_w} = \frac{(115)(40) + 0 - (62.4)(30)}{(1)(0.95)} = 2870 \text{ psf}$$

Estimate $c_{\text{avg}} = 120 \text{ psf}$ and $\phi_{\text{avg}} = 33^\circ$

$$\lambda_{c\phi} = \frac{P_e \tan \phi}{c} = \frac{(2870)(0.64)}{120} = 15.3$$

From Figure E-5, with $b = 1.5$ and $\lambda_{c\phi} = 15.3$:

$x_o = 0$ and $y_o = 1.9$

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$$X_o = (H)(x_o) = (40)(0) = 0 \text{ ft}$$

$$Y_o = (H)(y_o) = (40)(1.9) = 76 \text{ ft}$$

Plot the critical circle on the given slope as shown in Figure E-12.

Calculate c_{avg} , $\tan \phi_{\text{avg}}$, and $\lambda_{c\phi}$ as follows:

$$c_{\text{avg}} = \frac{\sum \delta_i c_i}{\sum \delta_i} = \frac{(19)(100) + (42)(150)}{19 + 42} = 134 \text{ psf}$$

$$\tan \phi_{\text{avg}} = \frac{\sum \delta_i \tan \phi_i}{\sum \delta_i} = \frac{(19)(\tan 35^\circ) + (42)(\tan 30^\circ)}{19 + 42} = 0.62$$

$$\lambda_{c\phi} = \frac{(2870)(0.62)}{134} = 13.3$$

From Figure E-5, with $b = 1.5$ and $\lambda_{c\phi} = 13.3$:

$$x_o = 0.02 \text{ and } y_o = 1.85$$

$$X_o = (H)(x_o) = (40)(0.02) = 0.8 \text{ ft}$$

$$Y_o = (H)(y_o) = (40)(1.85) = 74 \text{ ft}$$

This circle is close to the previous iteration, so keep $\lambda_{c\phi} = 13.3$ and $c_{\text{avg}} = 134 \text{ psf}$

From Figure E-5, with $b = 1.5$ and $\lambda_{c\phi} = 13.3$:

$$N_{cf} = 35$$

Calculate the factor of safety as follows:

$$F = N_{cf} \frac{c}{P_d} = 35 \frac{134}{4141} = 1.13$$

Example Problem E-5

Figure E-13 shows a slope where a relatively thin layer of soil overlies firm soil. The critical failure mechanism for this example is sliding along a plane parallel to the slope, at the top of the firm layer. This slope can be analyzed using the infinite slope stability chart shown in Figure E-7.

Calculate the factor of safety for seepage parallel to the slope and for horizontal seepage emerging from the slope.

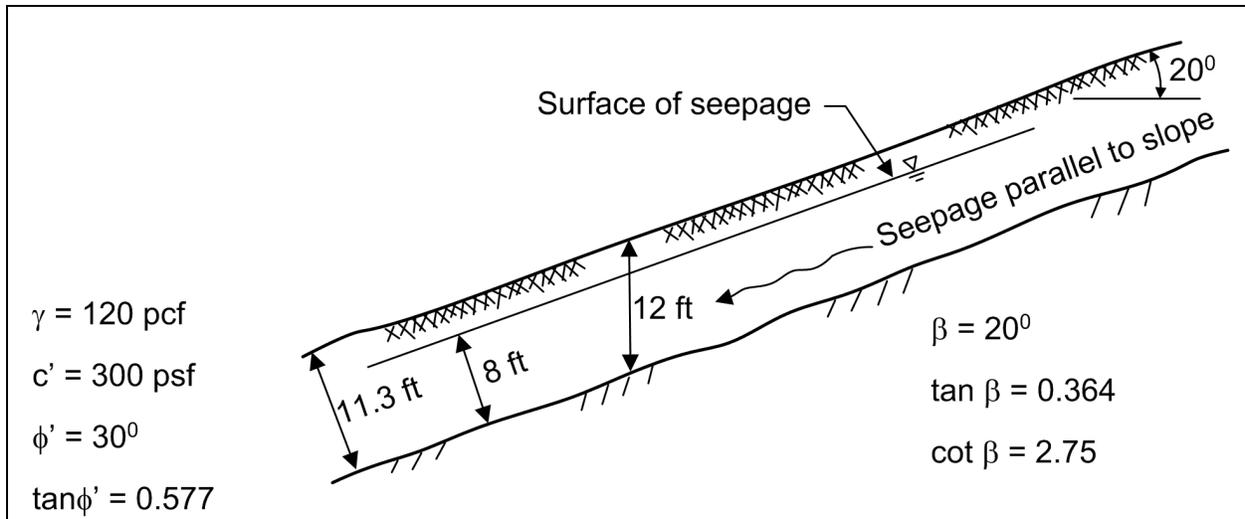


Figure E-13. Example of an infinite slope analysis

For seepage parallel to slope:

$X = 8 \text{ ft}$ and $T = 11.3 \text{ ft}$

$$r_u = \frac{X}{T} \frac{\gamma_w}{\gamma} \cos^2 \beta = \frac{8}{11.3} \frac{62.4}{120} (0.94)^2 = 0.325$$

From Figure E-7, with $r_u = 0.325$ and $\cot \beta = 2.75$:

$A = 0.62$ and $B = 3.1$

Calculate the factor of safety, as follows:

$$F = A \frac{\tan \phi'}{\tan \beta} + B \frac{c'}{\gamma H} = 0.62 \frac{0.577}{0.364} + 3.1 \frac{300}{(120)(12)} = 0.98 + 0.65 = 1.63$$

For horizontal seepage emerging from slope, $\theta = 0^\circ$

$$r_u = \frac{\gamma_w}{\gamma} \frac{1}{1 + \tan \beta \tan \theta} = \frac{62.4}{120} \frac{1}{1 + (0.364)(0)} = 0.52$$

From Figure E-7, with $r_u = 0.52$ and $\cot \beta = 2.75$:

$A = 0.41$ and $B = 3.1$

Calculate the factor of safety, as follows:

$$F = A \frac{\tan \phi'}{\tan \beta} + B \frac{c'}{\gamma H} = 0.41 \frac{0.577}{0.364} + 3.1 \frac{300}{(120)(12)} = 0.65 + 0.65 = 1.30$$

Note that the factor of safety for seepage emerging from the slope is smaller than the factor of safety for seepage parallel to the slope.

Example Problem E-6

Figure E-14 shows a submerged clay slope with $\phi = 0$ and strength is increasing linearly with depth.

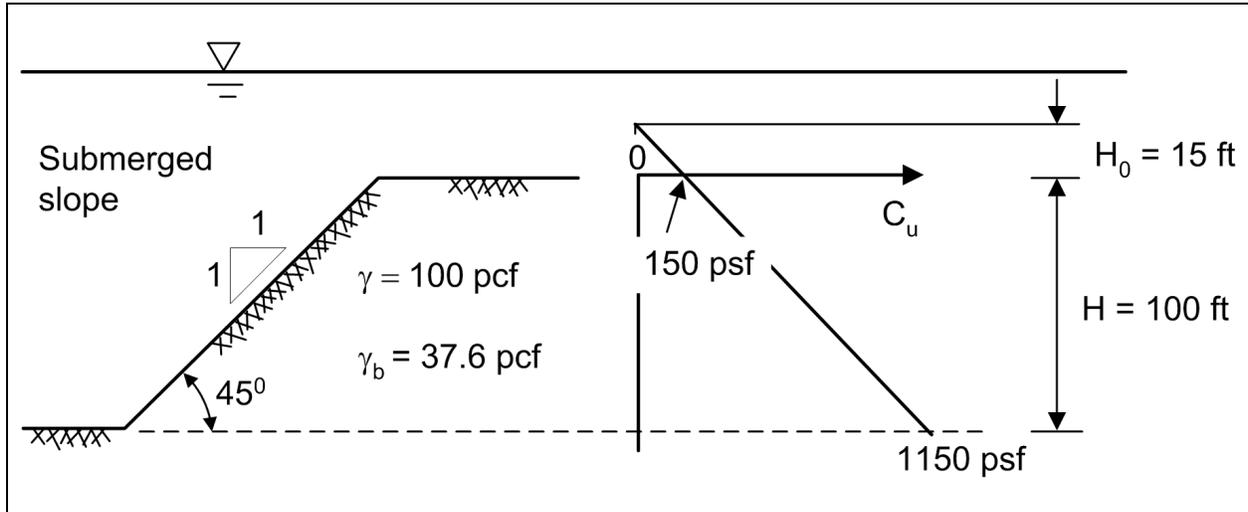


Figure E-14. Example of $\phi = 0$, and strength increasing with depth

The factor of safety is calculated using the slope stability chart shown in Figure E-8.

Extrapolating the strength profile up to zero gives $H_0 = 15$ ft

Calculate M as follows:

$$M = \frac{H_0}{H} = \frac{15}{100} = 0.15$$

From Figure E-8, with $M = 0.15$ and $\beta = 45^\circ$:

$$N = 5.1$$

From the soil strength profile, $c_b = 1150$ psf:

Calculate the factor of safety as follows:

$$F = N \frac{c_b}{\gamma(H + H_0)} = (5.1) \frac{1150}{(37.6)(115)} = 1.36$$