

## Appendix D Shear Strength Characterization

### D-1. Introduction

Selection of shear strength for use in slope stability computations is covered in this chapter. Shear strengths are usually determined from laboratory tests performed on specimens prepared by compaction in the laboratory or undisturbed samples obtained from exploratory soil borings. The laboratory test data may be supplemented with in situ field tests and correlations between shear strength parameters and other soil properties such as grain size, plasticity, and Standard Penetration Resistance (N) values. This chapter focuses primarily on shear strength selection from laboratory test data.

### D-2. Definition of Shear Strength

*a.* Shear strength for all of the slope stability analyses described in this manual is represented by a Mohr-Coulomb failure envelope that relates shear strength to either total or effective normal stress on the failure plane (Figure D-1). In the case of total stresses, the shear strength is expressed as:

$$s = c + \sigma \tan \phi \quad (\text{D-1})$$

where

$c$  and  $\phi$  = cohesion intercept and friction angle for the failure envelope

$\sigma$  = total normal stress on the failure plane

For effective stresses the shear strength is expressed as:

$$s = c' + (\sigma - u) \tan \phi' \quad (\text{D-2})$$

where

$c'$  and  $\phi'$  = intercept and slope angle for the failure envelope plotted in terms of effective stresses

$\sigma$  and  $u$  = total normal stress and pore water pressure, respectively, on the failure plane

The shear strength parameters,  $c$  and  $\phi$  or  $c'$  and  $\phi'$ , are determined from laboratory shear test data. The stresses from each test representing failure are plotted and a suitable failure envelope is drawn. The specific way in which the data are plotted, selection of the point representing failure (failure criteria), and whether effective or total stresses are used to plot the data depend on the type of test, loading conditions, and several other factors which are covered in the following sections of this appendix.

*b.* Theoretically the failure envelope is tangent to all of the Mohr's circles representing the stresses at failure (Figure D-2a). However, in actual practice there will be variations among samples tested, such that the failure envelope represents a "best-fit" to the data from several tests (Figure D-2b). Also, when the failure envelope is derived from direct shear tests, the complete state of stress is not known -- only the stresses on the horizontal plane are known. The horizontal plane is assumed to be the failure plane, and the failure envelope is drawn through the series of points representing the values of  $\tau$  and  $\sigma$  on the horizontal plane from each test.

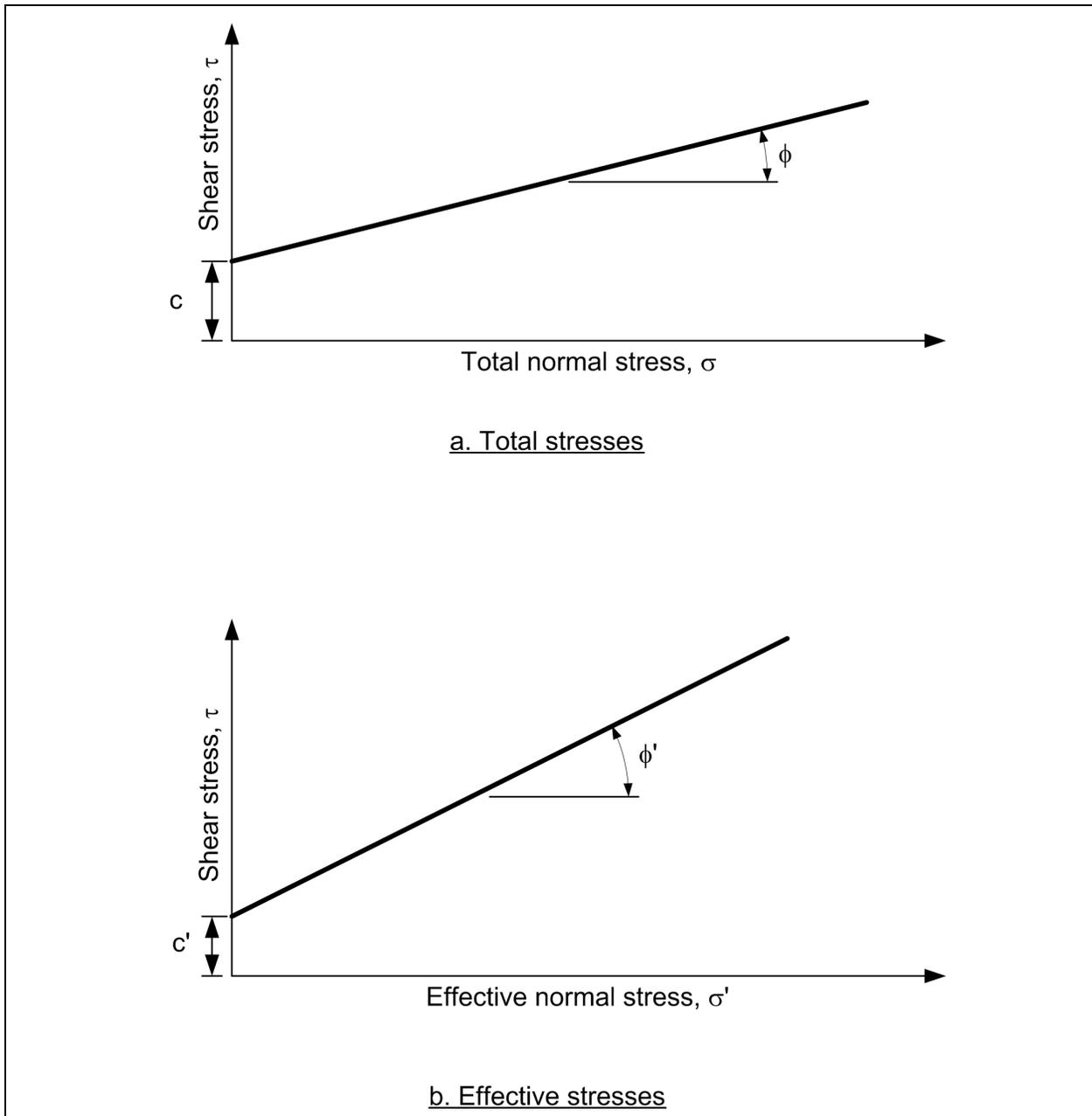


Figure D-1. Failure envelopes for total and effective stresses

c. Sometimes failure envelopes are curved, as shown in Figure D-3. Examples include the failure envelope obtained from Unconsolidated-Undrained tests on compacted soils and the residual shear strength envelope determined from consolidated-drained shear tests. In these cases the appropriate curved envelope, as illustrated in Figure D-3, is determined and used in the stability analyses, rather than values of cohesion and friction angle.

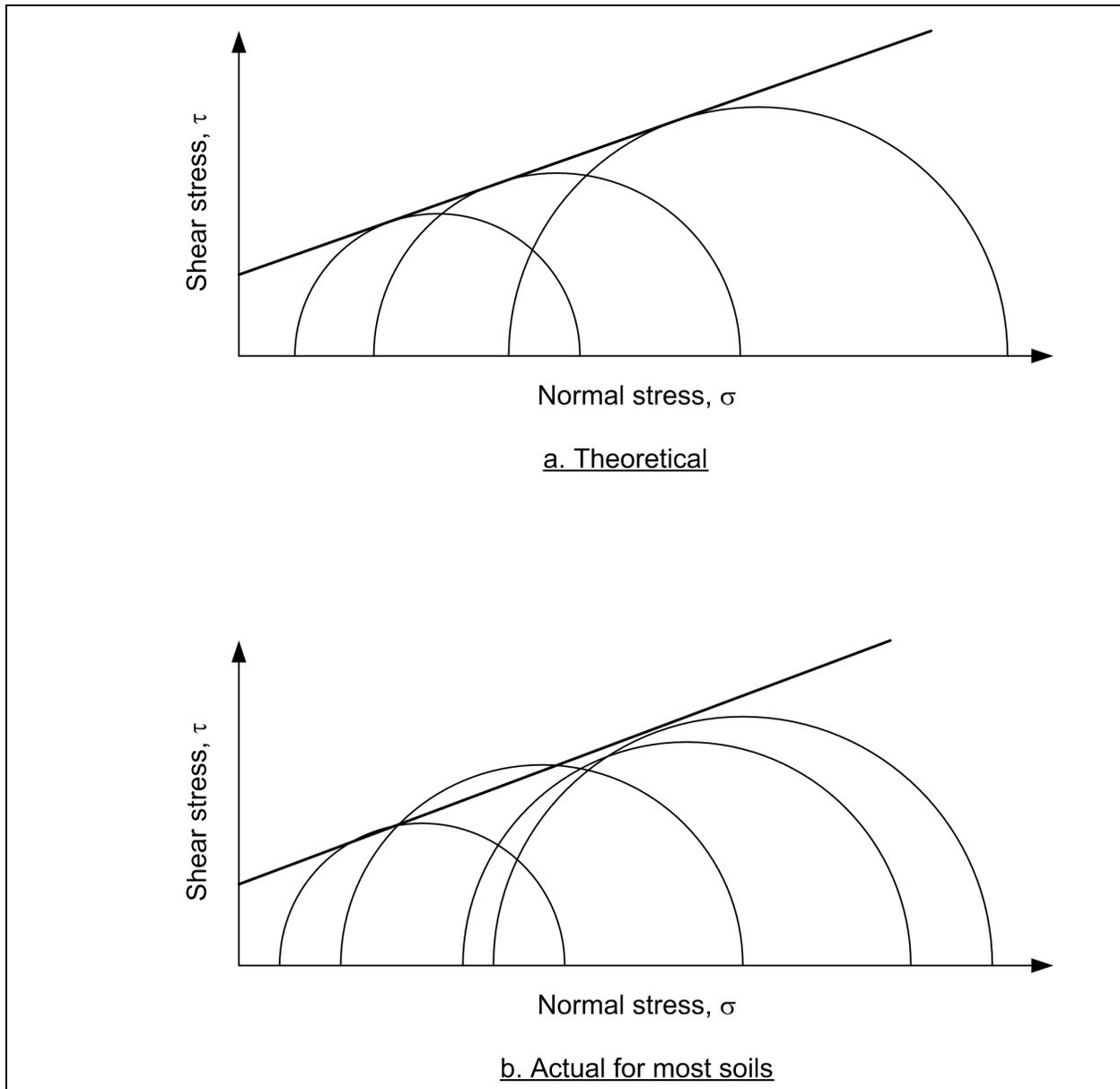


Figure D-2. Mohr's circles and failure envelopes

### D-3. Types of Laboratory Strength Test Procedures

*a.* Most laboratory tests are performed using either triaxial compression or direct shear test equipment. A two-stage loading procedure is used in each of these tests. In the first stage, a confining stress is applied. In the triaxial test, the confining stress is applied by increasing the cell, or all-round pressure on the sample. In the direct shear test, the confining pressure is applied by applying a vertical load to the horizontal plane, which becomes the eventual failure plane. The normal stress on the vertical plane in the direct shear device increases when the stress is applied to the horizontal plane, but the stress on the vertical plan is not known.

(1) The second stage of a strength test involves shearing the specimen. In the triaxial test, the axial load is gradually increased (load control), or the specimen is deformed slowly in the axial direction and the axial load is measured (deformation control), to shear the specimen. In the direct shear test, the horizontal shear

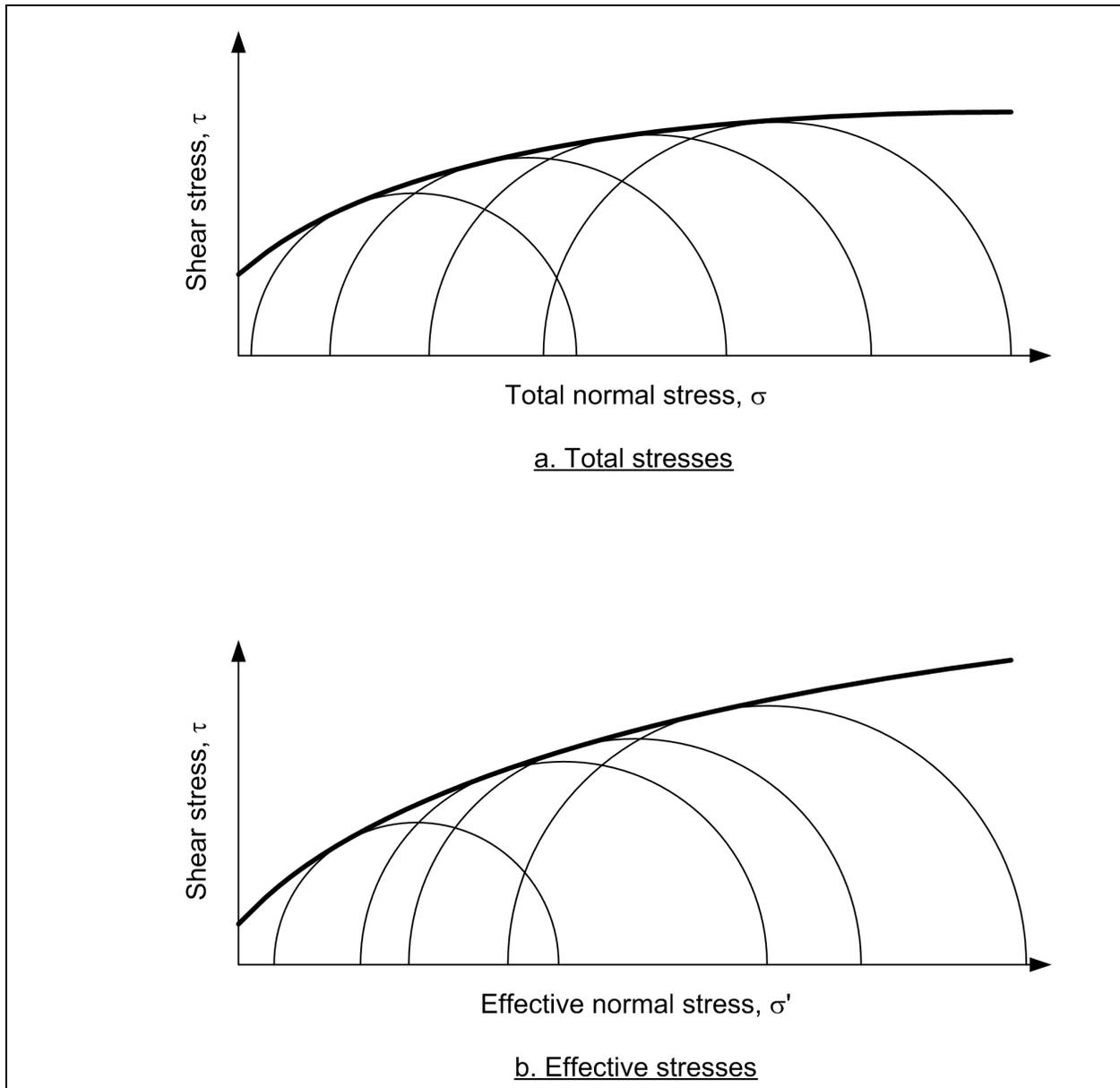


Figure D-3. Curved strength envelopes

load is gradually increased (load control) or the specimen is deformed horizontally by displacing the upper half of the shear box horizontally relative to the lower half and measuring the resulting load (deformation control).

(2) In the triaxial test, drainage of water into or out of the specimen can be controlled during application of both the confining stress and the shear stress. Depending on the drainage allowed in these phases of the test, three different types of test are possible -- Unconsolidated-Undrained (UU or Q), Consolidated-Undrained (CU or R), and Consolidated-Drained (CD or S). The three loading procedures are illustrated in Figure D-4. The loading procedures are intended in part to simulate conditions of loading and drainage in the field. The loading condition used in the laboratory test depends on the stability condition that strengths are being measured for, e.g., end-of-construction, steady-state seepage, or rapid drawdown.

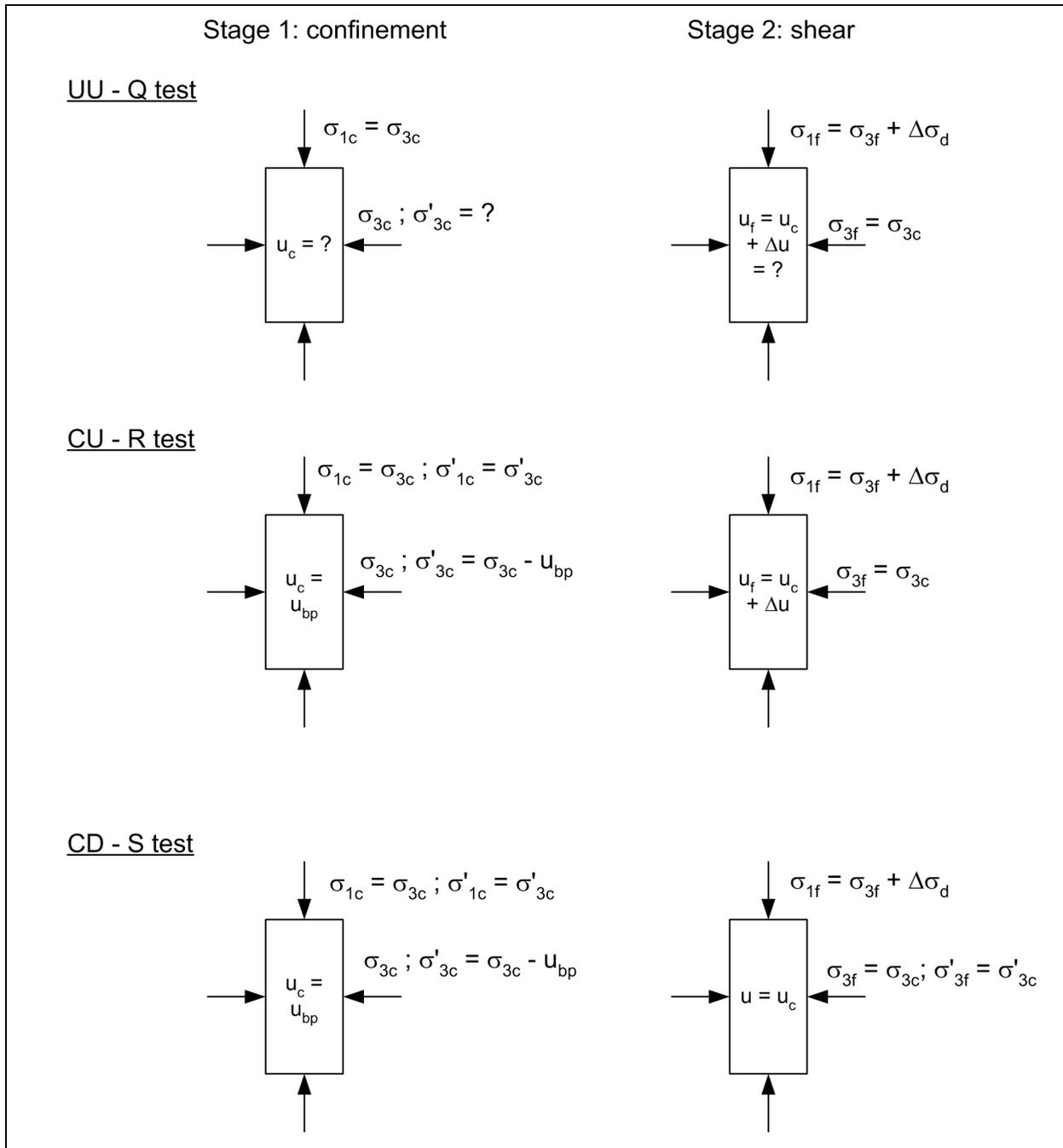


Figure D-4. Three types of triaxial shear tests

(3) In the direct shear test, drainage cannot be prevented. Thus, only the consolidated-drained (CD or S) loading procedure can be used. All three loading procedures (UU or Q, CU or R, and CD or S) are discussed in the following text.

*b. Unconsolidated-Undrained (UU or Q) test procedure.* No drainage is allowed in a UU test during the application of either the confining pressure or shear stress. The confining pressure is applied and the specimen is sheared shortly afterward. Rates of loading are relatively fast, and need only be slow enough to

avoid inertial effects and allow adequate time for recording data. Typical rates of loading for shear produce failure of the specimen in 10 to 20 minutes.

(1) The Unconsolidated-Undrained test procedure is used to measure the shear strength where there will be no drainage (no change in water content) when the soil is loaded. The objective of this test procedure is to measure the shear strength of the soil at the same water content that the soil will have in the field. It is important that the specimens being tested have the proper water content, which represents the field condition. Specimens of natural soils must be at the field water content and not allowed to dry out or absorb water between the time they are sampled and the time they are tested. Compacted specimens should be prepared at moisture content representing expected field conditions. The shear strength of compacted clays decrease with decreasing dry density and increasing water content. Test specimens should be compacted to the lowest dry density and highest water content that will be permitted under the specification, to ensure that the strength in the field will not be lower than that measured in the laboratory tests.

(2) Results from tests performed using Unconsolidated-Undrained loading procedures are always plotted using total stresses. Thus, the shear strength is expressed in terms of total stress, using  $c$  and  $\phi$ . Pore water pressures are not measured and are unknown.

*c. Consolidated-Undrained (CU or R) test procedure.* The Consolidated-Undrained test is used for several purposes and, depending on the purpose, pore water pressure may or may not be measured during shear. Each stage of the loading procedure is described further below, followed by discussion of how the test data are used.

(1) Consolidation stage. The first stage of Consolidated-Undrained loading is the consolidation stage, where the confining pressure is applied and the specimen is given time to consolidate fully. During this stage, the specimen is also saturated using back-pressure saturation techniques. Back-pressure saturation is done by increasing both the total confining stress and pore water pressure in equal increments until the specimen is saturated. Each increment of confining pressure is normally allowed to remain for some time to permit water to flow into the specimen and air to dissolve into the pore water. Increments should be small enough, and equilibration times long enough to avoid the specimen's being subjected to undesirably high effective stresses during back-pressure saturation. Until the specimen is saturated, increasing the confining pressure causes the effective stress to increase during the time required for the internal pore water pressures to equilibrate with the back pressure. It is desirable to back-pressure saturate specimens before consolidation to the final test confining pressures. In that way, reliable volume changes can be measured during consolidation by measuring the amount of water that flows into or out of the specimen as they consolidate. The volume changes should be recorded at suitable time intervals and plotted versus time to determine when consolidation is completed. The volume change-time data are also used to estimate the required times for shearing the specimen, as described in Section D-3.c(2) below.

(2) Shear stage. Once consolidation is complete, the drainage valve is closed to prevent further drainage as the axial load is increased to shear the specimen. In most Consolidated-Undrained shear tests, the pore water pressures developed during this stage of the test are measured. Pore water pressures are usually measured by measuring the water pressure in porous stones or disks at one or both ends of the specimen. Because of the end restraint at the two ends of the specimen, the strains in the specimen will not be uniform and, thus, the induced pore water pressures will not be entirely uniform over the height of the specimen. In order to measure representative values of the pore water pressures, the specimen should be sheared slowly enough for pore water pressures to equalize over the height of the specimen. To measure representative values of the pore water pressures, the specimen should be sheared slowly enough for pore water pressures to equalize over the height of the specimen. Suitable loading rates can be calculated from the time-volume change data recorded during the consolidation phase of the test. Axial load, axial deformation, and pore water pressure readings should be taken during shear. Specimens may be sheared using load control or deformation

control to increase the axial load on the specimens. Either method is adequate for measuring the peak load that the specimen can withstand. However, to measure the soil resistance beyond the point where the peak load is reached, it is necessary to control the deformation rate and measure the load. This is especially important for normally and slightly overconsolidated clays and some loose sands, where the peak effective stress shear strength parameters ( $c'$  and  $\phi'$ ) may be developed at strains larger than the strains at which the peak load in undrained shear is reached.

(3) Use of data. Shear strength data from Consolidated-Undrained tests are used in four different ways for slope stability computations:

- To determine the effective stress shear strength parameters for long-term, steady-state seepage analyses.
- To determine the relationship between undrained shear strength and effective consolidation pressure ( $\tau_{ff}$  vs.  $\sigma'_{fc}$ ) for analyses of rapid drawdown.
- To estimate undrained-shear strengths and reduce effects of sample disturbance for end-of-construction stability analyses.
- To estimate undrained shear strength for analyses of staged construction of embankments.

These uses are each discussed separately below.

(a) By plotting the effective stresses at failure from Consolidated-Undrained tests, the Mohr-Coulomb failure envelope for effective stresses ( $c'$  and  $\phi'$ ) can be determined. The failure envelope for effective stresses from Consolidated-Undrained tests is, for practical purposes, the same as the failure envelope from consolidated-drained (CD or S) tests. The failure envelope from either test can be used in slope stability computations for the long-term, steady-state seepage condition. Consolidated-Undrained (CU or R) tests are usually preferred over consolidated-drained (CD or S) tests for determining the effective stress failure envelope for clays, because Consolidated-Undrained tests can be performed more quickly. The time required for nonuniform pore water pressures to equalize in the specimen in a Consolidated-Undrained test is less than the time required for a specimen to fully drain during shear in a consolidated-drained test.

(b) Results of Consolidated-Undrained shear tests are also used to relate undrained shear strength to effective consolidation pressure for use in stability analyses for rapid drawdown. Further discussion of the plotting and use of the data for rapid drawdown analyses is presented in Appendix G.

(c) Data from Consolidated-Undrained shear tests can be used to estimate the undrained shear strength of saturated soils for use in analyses for end-of-construction stability. By reconsolidating specimens in the laboratory, it is possible to reduce some of the effects of sample disturbance. However, care must be used to avoid increasing the strength, and overestimating the undrained shear strength. When Consolidated-Undrained shear test procedures are used to estimate undrained shear strength the undrained shear strength is expressed as  $S_u = (\sigma_1 - \sigma_3)/2$  and is related to the effective consolidation pressure. Two approaches may be used to do this. One approach is the SHANSEP approach suggested by Ladd and Foott (1974);<sup>1</sup> the other is the “recompression” technique suggested by Bjerrum (1973). These are explained more fully below. The SHANSEP procedure suggested by Ladd and Foott (1974) involves the following steps:

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<sup>1</sup> Reference information is presented in Appendix A.

- Step 1: The variations of the present effective vertical stress and the maximum past pressure with depth are established. The present vertical effective stress is the current effective overburden pressure, calculated using unit weight and the groundwater level. The maximum past pressure is determined from consolidation tests on high-quality, undisturbed test specimens.
- Step 2: Consolidated-Undrained tests are performed using consolidation pressures that are higher than the maximum past pressure. If the clay is overconsolidated, the test specimens are allowed to swell after consolidation to achieve a suitable range of overconsolidation ratios, encompassing the range of values in the field. The test results are used to establish a relationship between the normalized shear strength  $[S_u/\sigma'_{vc} = \frac{1}{2} (\sigma_1 - \sigma_3)/\sigma'_{vc}]$  and overconsolidation ratio.
- Step 3: Undrained shear strengths applicable to the field are estimated by multiplying the normalized strength,  $S_u/\sigma'_{vc}$ , determined in Step 2, by the effective vertical stress,  $\sigma'_{vc}$ , determined in Step 1.

Once undrained shear strengths are determined in the manner described, they are represented in the slope stability computations as cohesion values,  $c$ , with  $\phi = 0$ .

- The SHANSEP procedure removes some of the effects of sample disturbance but also alters the structure of the soil. Alteration of the structure can lead to values of shear strength that are not representative of those in the field. This approach is not recommended for heavily overconsolidated soils, or for soils that have distinct structure or cementation bonds.
- The “recompression” technique proposed by Bjerrum (1973) involves reconsolidating specimens to the same effective stress that the specimens currently experience in the field. Although this approach results in specimens that have somewhat lower water contents than in the field (and therefore higher shear strengths), it produces less change in the soil structure than the SHANSEP approach. The recompression technique is recommended over the SHANSEP approach for heavily overconsolidated soils, but the approach may overestimate the shear strength even for these soils and should be used cautiously.
- Further details of the SHANSEP and “recompression” procedures can be found in the cited references. The shear strength obtained from Consolidated-Undrained tests using either the SHANSEP or recompression technique is assigned as a cohesion value with  $\phi = \text{zero}$  (these techniques apply only to saturated soils). The advantage of using Consolidated-Undrained tests, rather than Unconsolidated-Undrained tests to estimate the undrained shear strength is that some of the effects of sample disturbance can be reduced. However, care must be exercised to ensure that strengths are not overestimated. In addition, the offsetting effects of such factors as anisotropy and creep may need to be accounted for if the effects of sample disturbance are eliminated.

(d) The fourth use of Consolidated-Undrained shear tests is to measure shear strengths for use in analyses of staged construction of embankments. This is discussed in Section D-10.

*d. Consolidated-Drained (CD or S).* Complete drainage is allowed during the application of both the confining pressure and shear for consolidated-drained loading. Either the triaxial or direct shear apparatus may be used for testing. The two stages of loading (consolidation and shear) are described separately below, followed by a discussion of how the test data are used.

(1) Consolidation stage. The consolidation stage for consolidated-drained (CD or S) loading procedures is the same as the consolidation stage for Consolidation-Undrained (CU or R) test procedure. Back-pressure saturation is used to ensure complete saturation and to allow accurate measurements of volume change during

both consolidation and shear, by measuring the amount of water that flows into or out of the specimen. Volume change-time data from the consolidation phase of the test are used to estimate rates of loading and times to failure for the shear phase. Saturation is also important to eliminate effects of capillary stresses that would influence the strength measurements and their interpretation if the soil is partly saturated. The results of Consolidated-Drained shear tests are plotted using effective stresses that are equal to the total stresses minus the measured pore water pressures. One of the advantages of triaxial tests over direct shear test is that the specimen can be back-pressure saturated. Direct shear devices should only be used for testing soils which are either already saturated or will become saturated once placed in the direct shear apparatus and submerged.

(2) Shear stage. Specimens are sheared by slowly increasing either the axial load, in the case of triaxial tests, or the horizontal shear load, in the case of direct shear tests. It is very important to shear the specimen slowly enough that the soil can completely drain and no excess pore water pressures are developed. For some heavily overconsolidated clays and clay shales, the loading rates may need to be so slow that failure is reached only after several days or even weeks of shear. Suitable rates of loading to achieve complete drainage can be estimated from the volume change-time data recorded during the consolidation phase of the test. During triaxial shear the axial load, axial deformation, and volume changes of the specimen are recorded at various time intervals. The axial load-deformation data are used to determine the point where the specimen has failed, while the volume change information is used to make corrections for the change in cross-sectional area of the specimen that occurs during shear.

(3) Usage. Consolidated-Drained loading procedures are used to determine the effective stress shear strength parameters of freely draining soils. These soils will drain with relatively short testing times and the consolidated-drained loading procedure comes closest to representing the loading for long-term, drained conditions in the field. Consolidated-Drained tests procedures are also used to measure the residual shear strength of clays using direct shear or torsional shear equipment. The direct shear and torsional shear equipment allow for large strains, like those needed to measure residual shear strengths. When residual shear strengths are not needed and the soils are fine-grained, triaxial tests using Consolidated-Undrained (CU or R) procedures with pore water pressure measurements are preferred over Consolidated-Drained procedures because of the shorter testing times required for CU tests.

#### D-4. “Modified” Mohr-Coulomb Diagrams

Because the complete state of stress is known in the triaxial test, Mohr’s circles of stress can be plotted on a Mohr diagram. However, it can be difficult to judge what is a “best-fit” line tangent to a number of circles. A more convenient technique is to plot the data on a “modified” Mohr-Coulomb diagram where, rather than plotting circles, a point is plotted to represent the stresses at failure in each test. The diagrams used for this purpose are called “modified” Mohr-Coulomb diagrams. Several different forms of modified Mohr-Coulomb diagrams can be used. All modified Mohr-Coulomb diagrams are based on the fundamental relationship between the principal stresses and the Mohr-Coulomb shear strength parameters,  $c'$  and  $\phi'$  or  $c$  and  $\phi$ . Referring to Figure D-5 and the triangle formed by points,  $def$ , the following expression can be written:

$$\sin \phi = \frac{\frac{(\sigma_1 - \sigma_3)}{2}}{\frac{c}{\tan \phi} + \frac{(\sigma_1 + \sigma_3)}{2}} \quad (D-3)$$

Equation D-3 can be rearranged to obtain a number of different relationships between the principal stresses and the shear strength parameters,  $c$  and  $\phi$ . Two of the most useful forms of Equation D-3 and the resulting modified Mohr-Coulomb diagrams are described in the following text.

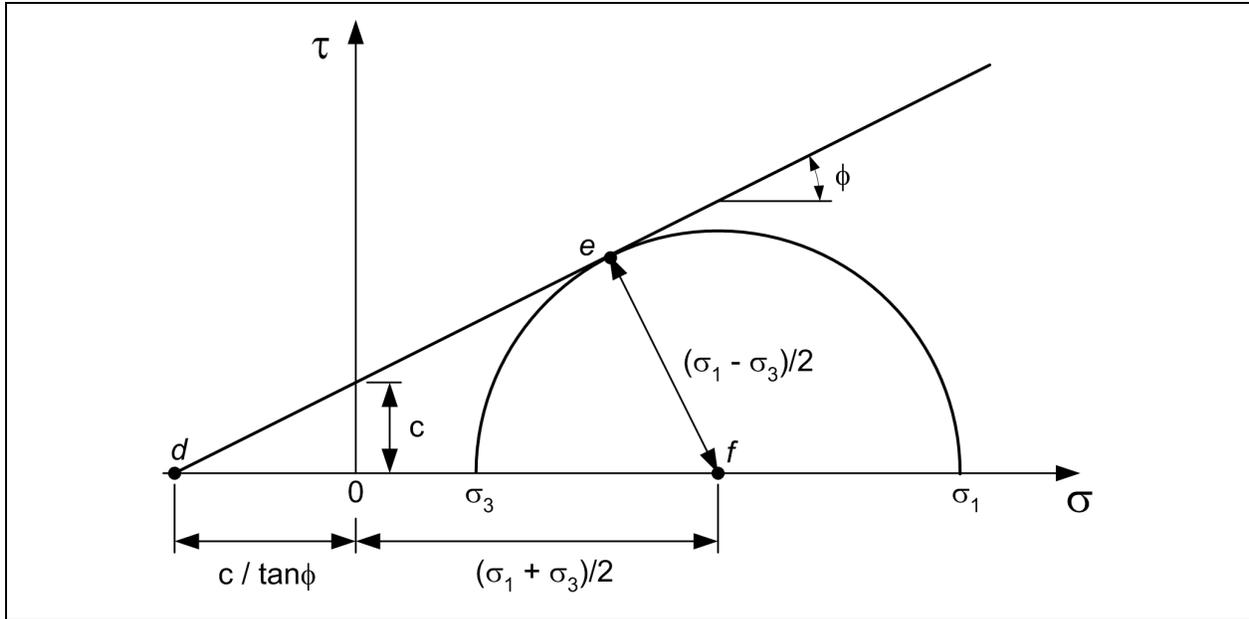


Figure D-5. Mohr's circle of stress used to derive equations for Modified Mohr-Coulomb diagram

a. "p-q" diagrams. One of the most commonly used modified Mohr-Coulomb diagrams is a "p-q" diagram. For this diagram,  $q = (\sigma_1 - \sigma_3)/2$  is plotted vs.  $p = (\sigma_1 + \sigma_3)/2$ , as shown in Figure D-6a. The basis for such a plot can be seen by rewriting Equation D-3 in the form:

$$q = c \cos \phi + p \sin \phi \quad (D-4)$$

which can also be written as:

$$q = d + p \tan \psi \quad (D-5)$$

Equation D-5 expresses a linear relationship between the quantities  $q$  and  $p$ . The parameter  $d$  is the intercept and  $\tan \psi$  is the slope of the line on the modified Mohr-Coulomb diagram shown in Figure D-6a. The slope,  $\tan \psi$ , is related to the friction angle,  $\phi$ , by the expression:

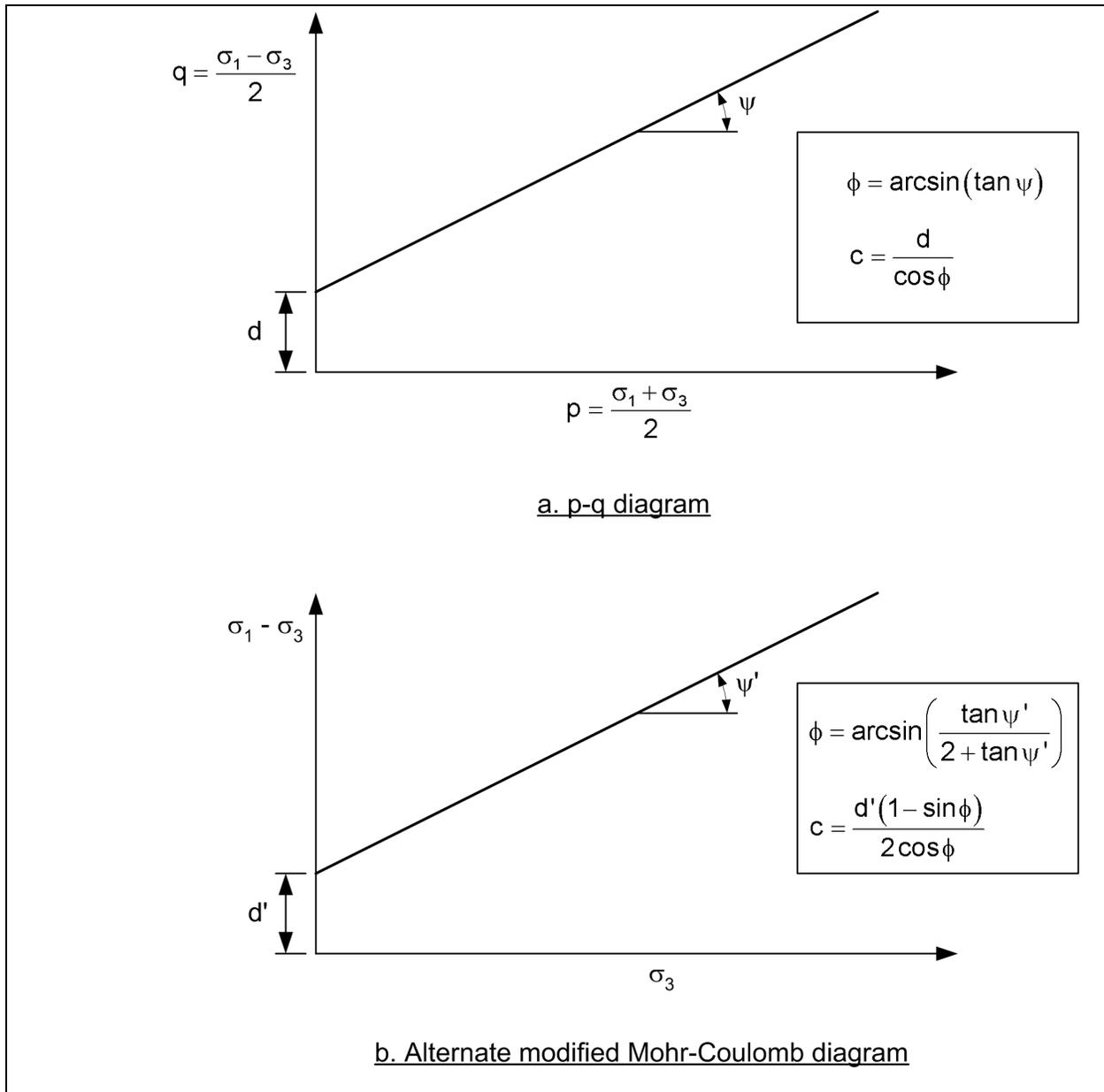
$$\tan \psi = \sin \phi \quad (D-6)$$

or

$$\phi = \arcsin (\tan \psi) \quad (D-7)$$

Similarly, the cohesion on a Mohr-Coulomb diagram is related to the friction angle ( $\phi$ ) and the intercept ( $d$ ) on a modified Mohr-Coulomb diagram by:

$$c = \frac{d}{\cos \phi} \quad (D-8)$$



**Figure D-6. Modified Mohr-Coulomb diagrams**

A p-q diagram can be used to plot the results of triaxial shear tests and determine the Mohr-Coulomb shear strength parameters. To do so, the values of  $q = (\sigma_1 - \sigma_3)$  and  $p = (\sigma_1 + \sigma_3)$  at failure are determined for each test and plotted on the diagram. A straight line is then drawn to fit the data and the slope ( $\tan \Psi$ ) and intercept ( $d$ ) are determined. Once the intercept and slope are found, Equations D-7 and D-8 are used to calculate the friction angle,  $\phi$ , and cohesion,  $c$ . Care must be exercised in presenting “p-q” diagrams and reviewing such diagrams prepared by others, because an entirely different set of axes and quantities from the ones described above are sometimes used and referred to as “p-q” diagrams. The alternative nomenclature defines  $p$  and  $q$  for triaxial compression tests as follows:

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3) \tag{D-9}$$

and

$$q = (\sigma_1 - \sigma_3) \quad (D-10)$$

This notation was suggested by Roscoe, Schofield, and Wroth (1958) and has appeared in a number of texts and reference books, e.g., Head (1986), Budhu (2000). To avoid confusion, any “p-q” diagram should always have the axes labeled so that they show the relationship to the principal stresses,  $q = (\sigma_1 - \sigma_3)/2$  and  $p = (\sigma_1 + \sigma_3)/2$ , instead of simply using the notation “p” and “q”, which is subject to ambiguity.

*b. Alternate modified Mohr-Coulomb diagram.* Another form of modified Mohr-Coulomb diagram that is useful is one in which the principal stress difference  $(\sigma_1 - \sigma_3)$  is plotted vs. the confining pressure,  $\sigma_3$ , as shown in Figure D-6b. The basis for such a plot can be seen by rewriting Equation D-3 as:

$$(\sigma_1 - \sigma_3) = \frac{2c \cos \phi}{1 - \sin \phi} + \frac{2 \sin \phi}{1 - \sin \phi} \sigma_3 \quad (D-11)$$

This requires somewhat more algebraic manipulation than is required to write Equation D-4, but Equation D-11 can be shown to be a valid form of Equation D-3. Equation D-11 can be written as:

$$(\sigma_1 - \sigma_3) = d' + \sigma_3 \tan \psi' \quad (D-12)$$

where

$$d' = \frac{2c \cos \phi}{1 - \sin \phi} \quad (D-13)$$

and

$$\tan \psi' = \frac{2 \sin \phi}{1 - \sin \phi} \quad (D-14)$$

From Equation D-14, the following equation can be written:

$$\phi = \arcsin \left( \frac{\tan \psi'}{2 + \tan \psi'} \right) \quad (D-15)$$

and from Equation D-13, the following equation can be written.

$$c = \frac{d' (1 - \sin \phi)}{2 \cos \phi} \quad (D-16)$$

By plotting the results of triaxial tests in the form of  $(\sigma_1 - \sigma_3)$  vs.  $\sigma_3$  and fitting a straight line through the data points, the cohesion and friction angle can be determined from the slope and intercept of the line using Equations D-15 and D-16. A modified Mohr-Coulomb diagram like the one shown in Figure D-6b is particularly useful and instructive for plotting stress paths from triaxial tests (Section D-5). The horizontal axis represents the confining pressure in the test,  $\sigma_3$ , while the vertical axis is directly related to the applied

axial load used to shear the specimen,  $(\sigma_1 - \sigma_3)$ . Thus, the two axes correspond to the two independently controlled and measured stresses in the triaxial test.

## D-5. Stress Paths

a. Stress paths are plots representing the successive states of stress in a laboratory test. Although stress paths may be drawn to represent the stresses during both consolidation and shear, stress paths are most useful for the shearing stage of the test. Although a number of different diagrams can be used to plot stress paths; the two types of Modified Mohr-Coulomb diagrams described in Section D-4 are probably the most widely used and useful. While stress paths can be plotted for all three types of loading (UU, CU, and CD), only stress paths from Consolidated-Undrained (CU or R) shear tests are useful, and only they are covered in this section.

b. Stress paths for Consolidated-Undrained shear tests can be plotted for either total or effective stresses. The stress paths for total stresses are shown in Figure D-7 on both p-q diagrams and the alternate  $(\sigma_1 - \sigma_3)$  vs.  $\sigma'_3$  diagram described earlier. On the p-q diagram, the total stress path is along a 45-degree line extending from the horizontal axis to the failure envelope. If the stresses decrease once failure is reached, the stress path will move back along the initial loading path. On the alternate modified Mohr-Coulomb diagram shown in Figure D-7, the total stress path rises vertically to the failure envelope because the total confining pressure does not change. If the strength drops off once failure is reached, the stress path will drop back vertically along the initial loading path.

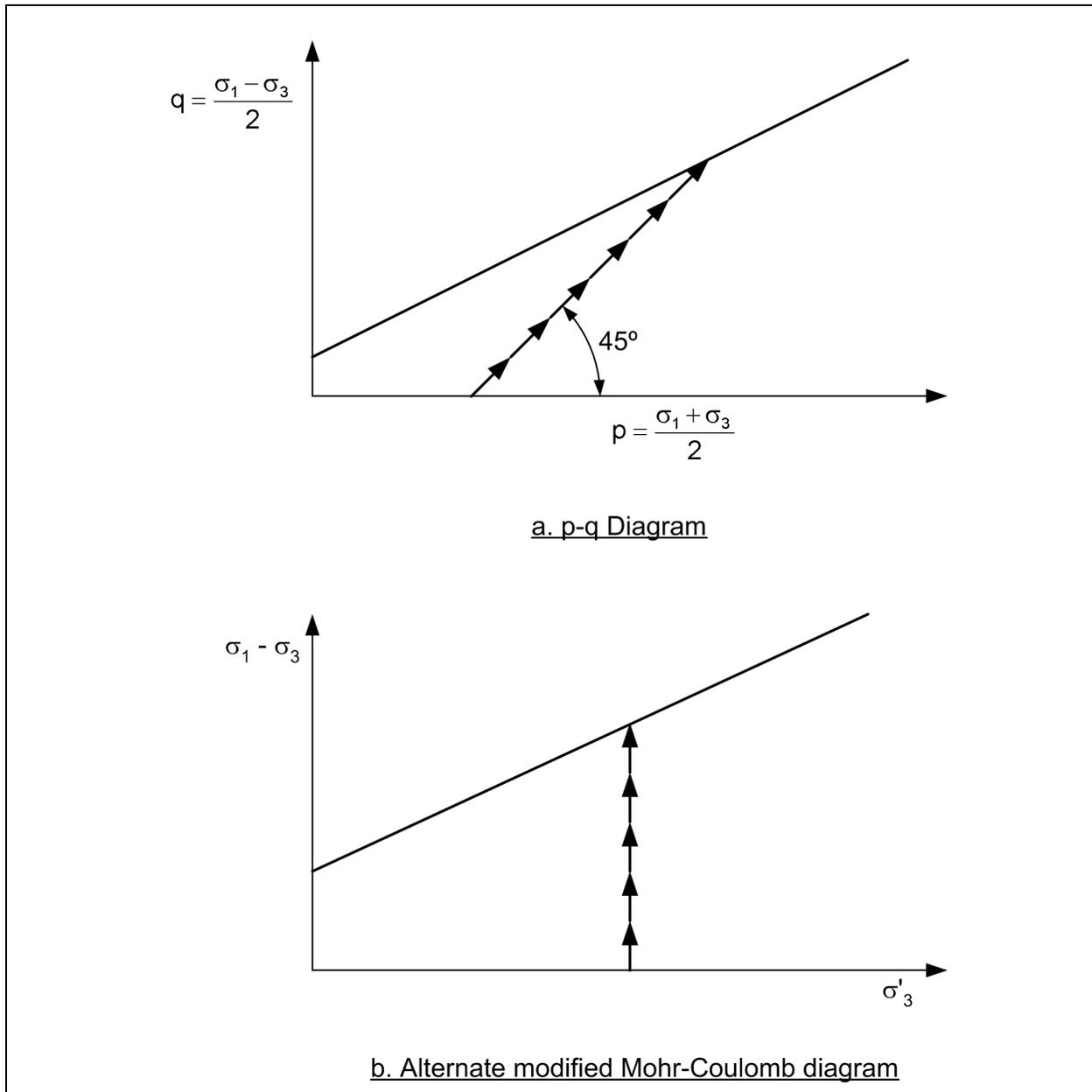
c. Effective stress paths during shear for Consolidated-Undrained loading are shown on a p-q diagram in Figure D-8 for a soil which tends to compress when sheared to failure (Figure D-8a) and for a soil which tends to dilate when sheared to failure (Figure D-8b). A broken 45-degree line extending from the initial stress point toward and across the failure envelope is also shown in each figure. The effective stress paths lie to the left of the 45-degree line when the pore water pressures increase during shear and to the right of the line when pore water pressures decrease during shear, i.e., when the soil tends to dilate. The horizontal distance between the 45-degree line and the stress path represents the change in pore water pressure  $\Delta u$  during shear.

d. Effective stress paths plotted on an alternate,  $(\sigma_1 - \sigma_3)$  vs.  $\sigma'_3$ , diagram are shown in Figure D-9. Stress paths are shown for a soil that compresses during shear and for a soil that dilates during shear. Broken lines are drawn on each diagram extending vertically from the initial stress point upward and across the failure envelope. Stress paths which lie to the left of these vertical lines represent stresses where the pore water pressure has increased during shear, while those to the right of the line represent decreases in pore water pressure. The horizontal distance between the vertical line and points on the stress paths represents the change in pore water pressure  $\Delta u$  during shear.

## D-6. Failure Criteria

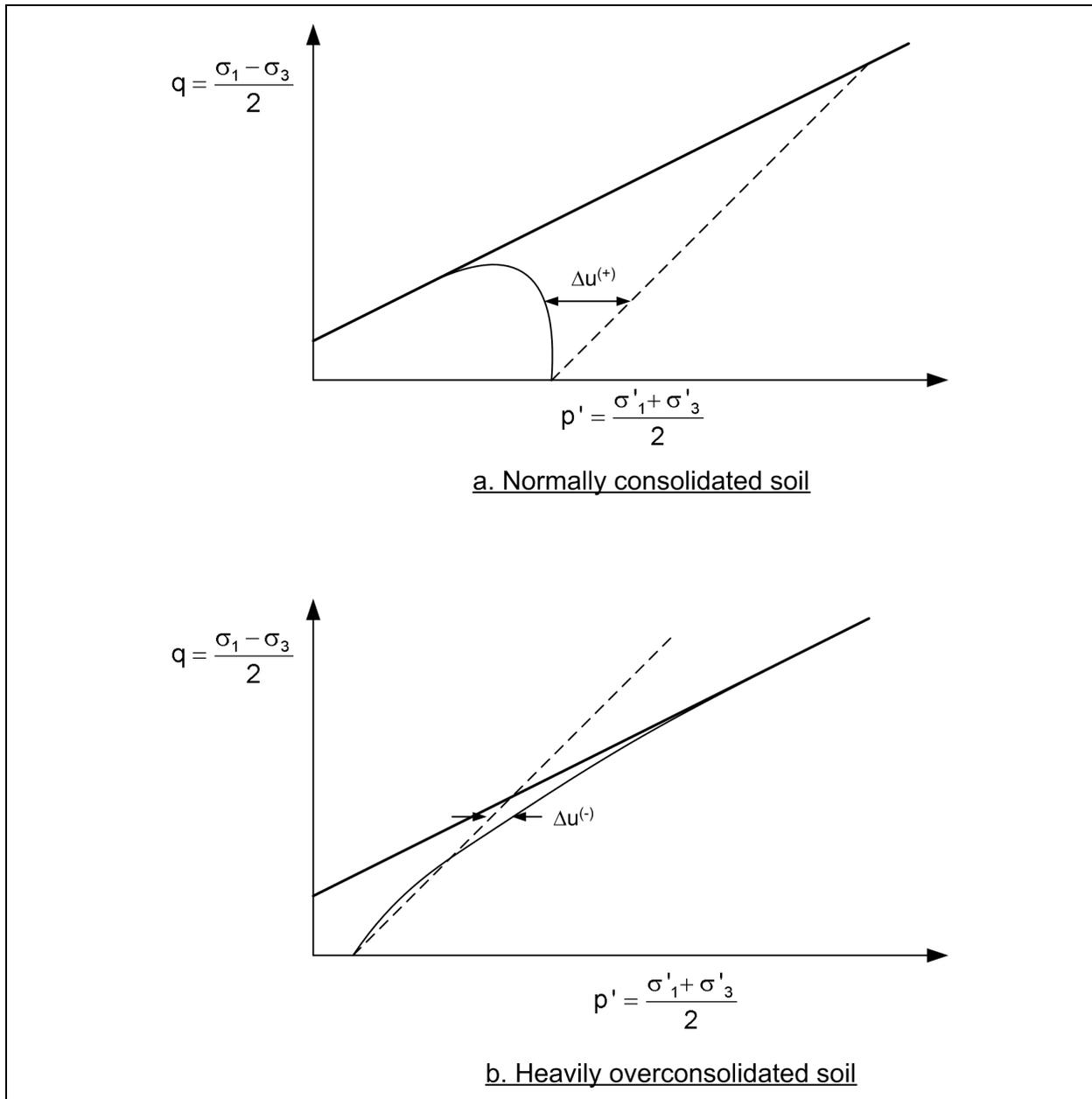
Mohr-Coulomb failure envelopes are determined by plotting stresses at failure and drawing a suitable line or curve either tangent to a series of circles or through a series of points. To define the stresses at failure, a suitable criterion that defines what is meant by “failure” must be established. The criterion chosen depends on the type of test, the type of soil, and the use that will be made of the failure envelope. Failure criteria are discussed for each type of test and loading condition in the sections below.

a. *Unconsolidated-Undrained (UU or Q) test.* For Unconsolidated-Undrained shear tests, failure is usually taken as the point of maximum axial stress,  $(\sigma_1 - \sigma_3)_{\max}$ . However, if large strains are required to reach a peak axial stress, or if the test data show no peak, it is appropriate to use some value of strain as the failure criterion. The ASTM Standard for Unconsolidated-Undrained shear tests suggests that the stress at



**Figure D-7. Total stress paths for shear plotted on modified Mohr-Coulomb diagrams**

15 percent axial strain should be taken as the stress at failure if no peak is reached prior to that point (ASTM 1999). This recommendation is reasonable and should be followed unless the use of stresses at larger strains can be justified. Stresses less than the peak stress may also be used as the failure stresses when strain compatibility is of concern (Section D-9). Ordinarily it will not be possible to draw the failure envelope so that it is precisely tangent to the Mohr's circles on a conventional Mohr diagram or precisely through the points that are plotted on a modified diagram. Consequently, the envelope should be drawn to fit the data in a manner that seems reasonable. Prior Corps of Engineers' practice has been to draw the strength envelope in a position such that data from two-thirds of the tests lie above the failure envelope. This recommendation is reasonable.



**Figure D-8. Effective stress paths for shear plotted on p-q diagrams**

b. *Consolidated-Undrained (CU or R) test.* How failure is defined for CU tests depends on the use that will be made of the results. Different criteria are appropriate depending on whether effective stress shear strength parameters or undrained shear strengths are being determined.

(1) Effective stress shear strength parameters. The appropriate failure stresses for determining effective stress shear strength parameters,  $c'$  and  $\phi'$ , are best determined by plotting the effective stress paths for the shear phase of the tests. A typical series of effective stress paths is shown on a modified Mohr-Coulomb diagram in Figure D-10. The failure envelope should be drawn such that it is approximately tangent to the stress paths, as shown in this figure. This criterion is referred to as “stress path tangency.” Although variations in soil and among the samples tested will probably make it impossible to draw a failure envelop

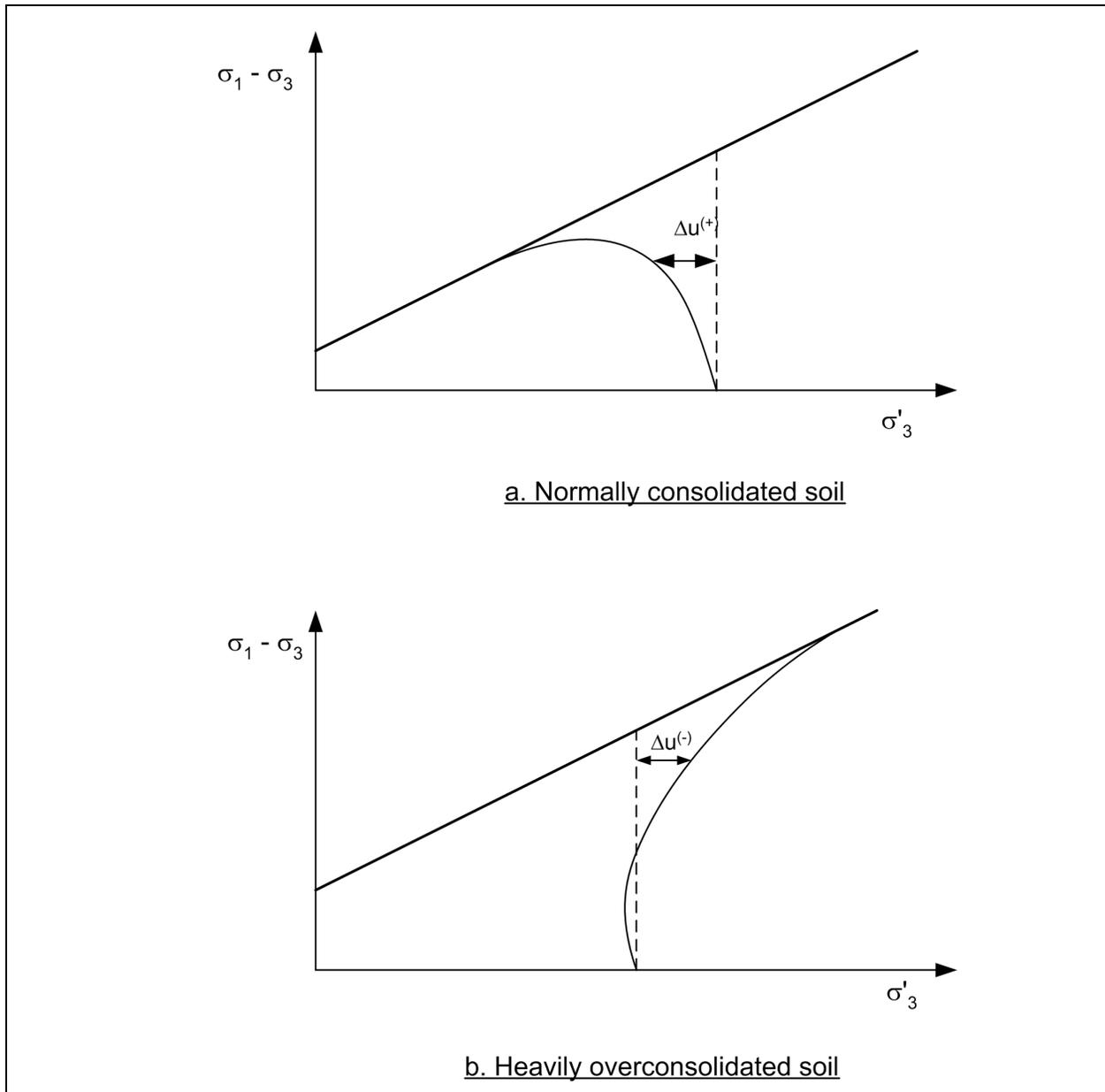
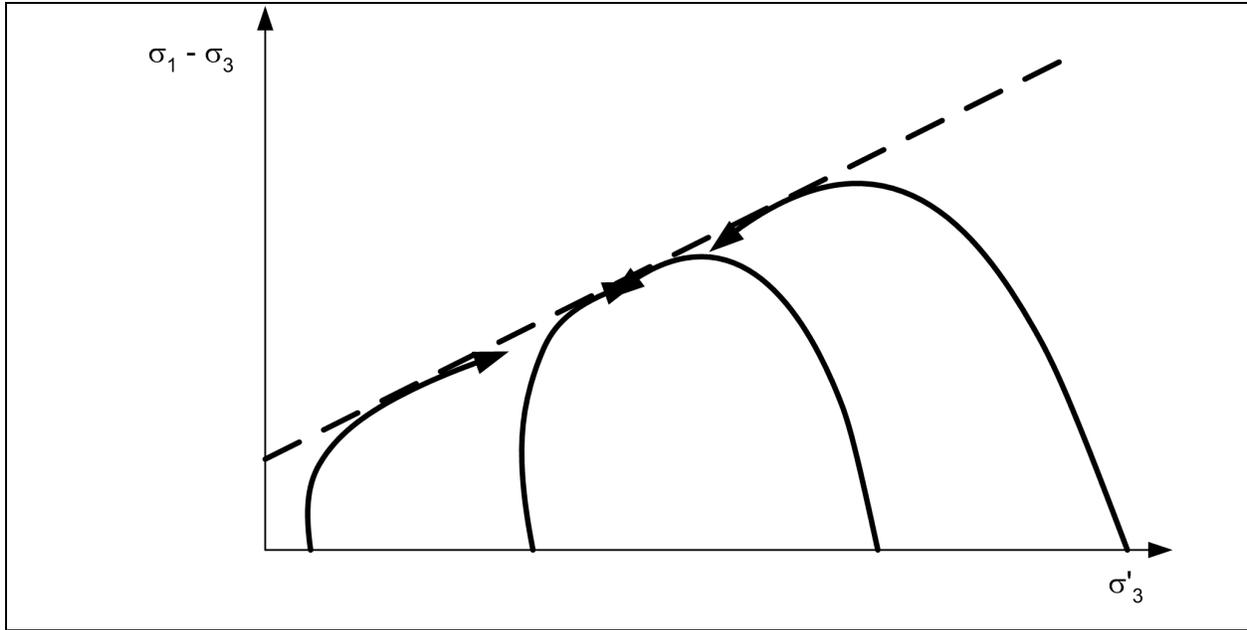


Figure D-9. Effective stress paths for shear plotted on alternate modified Mohr-Coulomb diagrams

that is precisely tangent to all stress paths, the envelope should be drawn as close to tangent as possible, with about two-thirds of the points of tangency above the line, and one-third below.

- The failure envelope may also be drawn by plotting the Mohr's circles of stress on a conventional Mohr-Coulomb diagram. In this case, it is much more difficult to determine when stress path tangency occurs; the particular set of stresses and Mohr's circle where stress path tangency occurs cannot be readily identified from the numerical test data. If several or all Mohr's circles representing the stresses at various stages of loading during each test are plotted, the number of circles becomes large and diagrams become complex and unclear.



**Figure D-10. Effective stress paths for Consolidated-Undrained shear tests plotted on a modified Mohr-Coulomb diagram**

- One way of determining where the peak effective stress shear strength parameters are developed without plotting stress paths or Mohr circles for all the stresses during loading is to compute and examine the effective principal stress ratios,  $\sigma'_1/\sigma'_3$ , during shear for each data point recorded during the test. If the failure envelope passes through the origin of the Mohr-Coulomb diagram ( $c' = 0$ ), the maximum value of the effective principal stress ratio,  $(\sigma'_1/\sigma'_3)_{\max}$ , coincides with the point of stress path tangency. By calculating  $\sigma'_1/\sigma'_3$  and determining where the maximum value occurs, the point of stress path tangency can be determined. The stresses at the point of tangency can then be used to plot the Mohr's circles on a Mohr-Coulomb diagram. However, this approach for determining the point of stress path tangency is only valid when the cohesion intercept,  $c'$ , is zero.
- When the Mohr-Coulomb failure envelope is curved, test data must be plotted on a conventional Mohr-Coulomb diagram of  $\tau$  vs.  $\sigma'$  in order to draw the failure envelope. This is necessary because no convenient means exists for transferring a curved envelope from a modified Mohr-Coulomb diagram back to the conventional diagram. However, even in instances where the failure envelope is curved, a modified Mohr-Coulomb diagram and stress paths may be drawn first to establish the point of stress-path tangency and failure. Once the point of stress path tangency is determined from stress-paths plotted on the modified Mohr-Coulomb diagram, the stresses can then be plotted on a conventional  $\tau$  vs.  $\sigma'$  diagram and the curved Mohr-Coulomb failure envelope can be drawn.
- For many normally consolidated clays and loose sands, the point of stress-path tangency occurs after the maximum axial load is reached. In order to capture the point of stress-path tangency, it is necessary to continue to shear specimens past the point where the maximum load is developed. In order to do so, deformation-controlled loading, rather than load-controlled loading, must be used.

(2) Undrained shear strengths. When Consolidated-Undrained loading procedures are used to determine undrained shear strengths, the failure criterion for plotting the data are the same as those used for UU tests. The peak stress or the stress at a limiting value of strain, e.g., 15 percent axial strain, is used as the failure criterion.

c. *Consolidated-Drained (CD or S) tests.* Failure in consolidated-drained tests is determined as the point of maximum principal stress difference,  $(\sigma_1 - \sigma_3)_{\max}$ , or at some limiting, maximum value of axial strain. Fifteen percent axial strain is a reasonable value to use as a failure criterion. Heavily overconsolidated, stiff-fissured clays and dense sands sometimes exhibit significant reduction in shearing resistance with strain beyond the peak. In these materials it is not possible to develop the peak strength simultaneously at all points along the shear surface. Also, in slopes where prior sliding has resulted in development of slickensided slip surfaces, the shear resistance has already declined to its residual value. In these instances adequate stability can only be ensured by using residual shear strengths in stability analyses.

## D-7 Generalized Stress-Strain-Strength Behavior

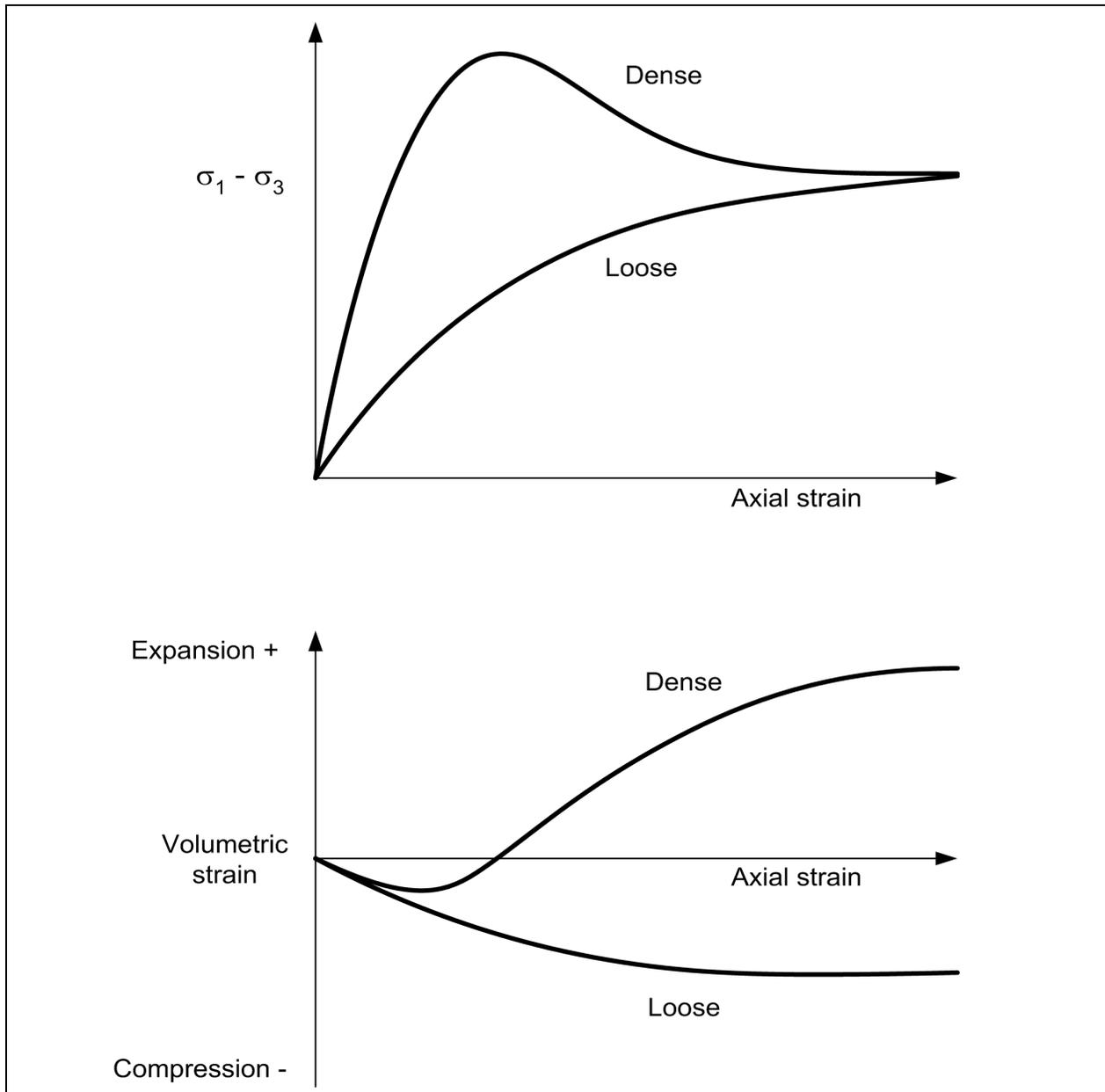
An understanding of the stress-strain response of soils is useful in interpreting the results of laboratory shear tests. The stress-strain response of soils in both drained and undrained shear tests is discussed below.

a. *Drained loading.* Typical stress-strain curves from triaxial shear tests on dense and loose sands are shown in Figure D-11. The upper portion of this figure shows the axial stress-strain curves, while the lower portion shows volumetric strain vs. axial strain curves. Loose sands tend to compress (volume decreases) during shear. The axial stress may increase with increasing strain up to 20 or 25 percent axial strain or even more. Dense sands also tend to compress initially when sheared, but they then expand as they are sheared to larger strains. In dense sands, peak load is reached at much smaller strains than for loose sands, and the stress may then decrease significantly as strains are further increased. If loose and dense specimens of the same sand are sheared to large strains at the same confining pressure, the strengths will become similar at large strains, regardless of the initial density. At large strains, the soil is said to reach a “critical state” or “critical void ratio,” and the shearing resistance at these large strains is largely independent of initial density. Normally and heavily overconsolidated clays tend to exhibit stress-strain response similar to those for loose and dense sands. Normally consolidated clays tend to compress throughout shear, developing a peak resistance at 10 to 20 percent axial strain. Heavily overconsolidated clays tend first to compress and then to dilate as they are sheared to large strains. Under drained loading, the peak resistance of heavily overconsolidated clays is usually developed at smaller strains than for normally consolidated clays.

(1) The response to shear of both clays and sands with different stress histories or densities can be illustrated and explained with the concept of a “critical void ratio” or “critical state” first suggested by Casagrande (1936) and later promoted for clays by Roscoe, Schofield, and Wroth (1958). This is illustrated by the diagram of void ratio vs. confining pressure,  $\sigma_3$ , shown in Figure D-12. The curve labeled “critical state” in Figure D-12 represents the void ratios which soils eventually reach when they are sheared to large strains at various confining pressures. If a soil is loose, such that it starts shear at a point above the “critical state” line, the soil will compress (void ratio will decrease), as suggested by the path a-c in Figure D-12. In contrast, if a soil is dense, such that it starts shear at a point below the critical state line, the soil will tend to dilate as large strains are reached and the soil will dilate (void ratio will increase), as suggested by path b-c in Figure D-12). Regardless of the initial density, two specimens of the same soil tested at the same confining stress will tend to reach a similar void ratio and have very similar shear strengths at large strains. The dense soil will, however, have a higher peak strength.

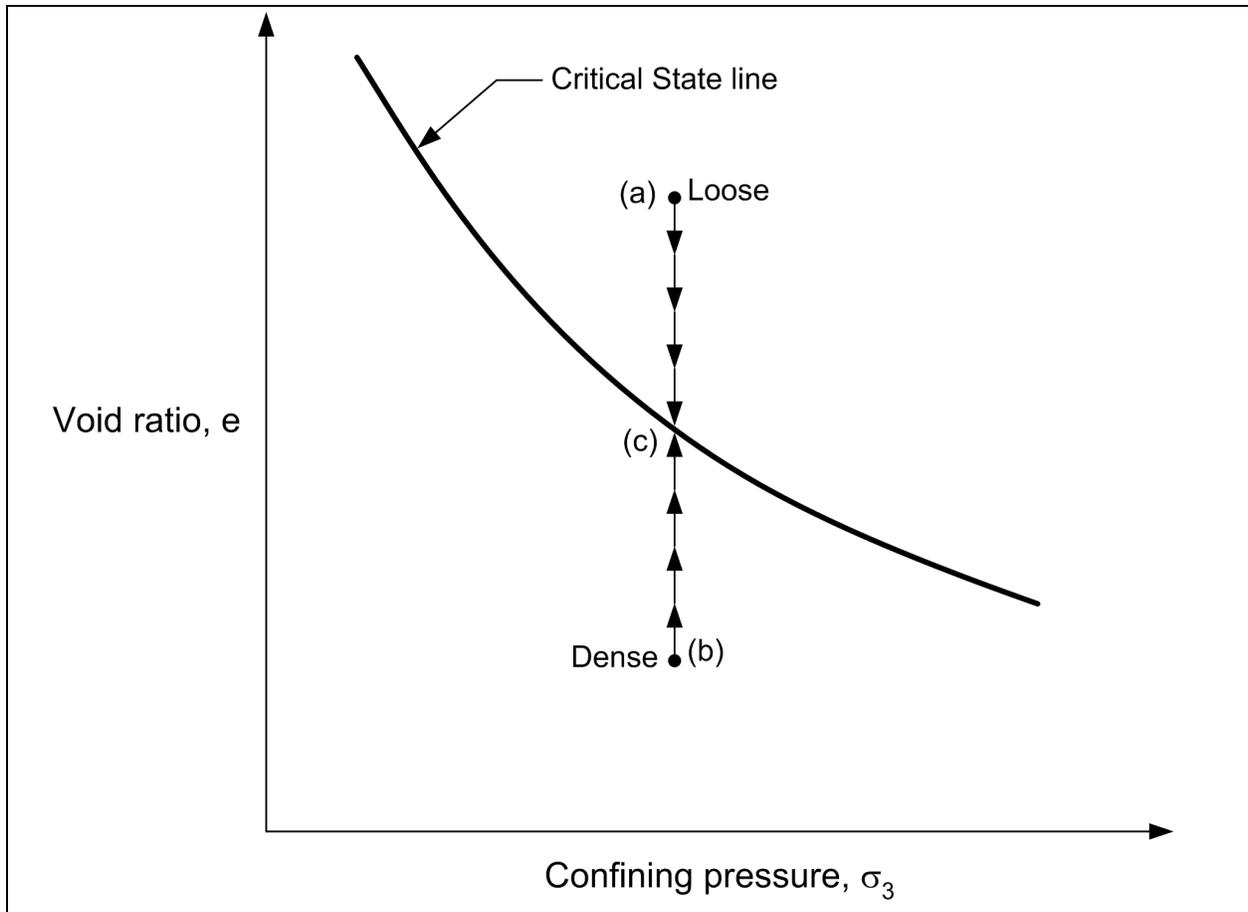
(2) For clays, a similar set of behavioral characteristics is observed. Normally consolidated clays tend to compress and reach a critical state when sheared, while heavily overconsolidated soils tend to expand as they reach the critical state at large strains.

(3) Compacted soils can exhibit stress-strain responses varying from that for normally consolidated soil to that for heavily overconsolidated soils. At low confining pressures compacted clays tend to behave like overconsolidated clays, and at high confining pressures, where the effects of compaction no longer dominate their behavior, they behave more like normally consolidated soils.



**Figure D-11. Typical stress-strain curves from CD-S triaxial shear tests on dense and loose sands**

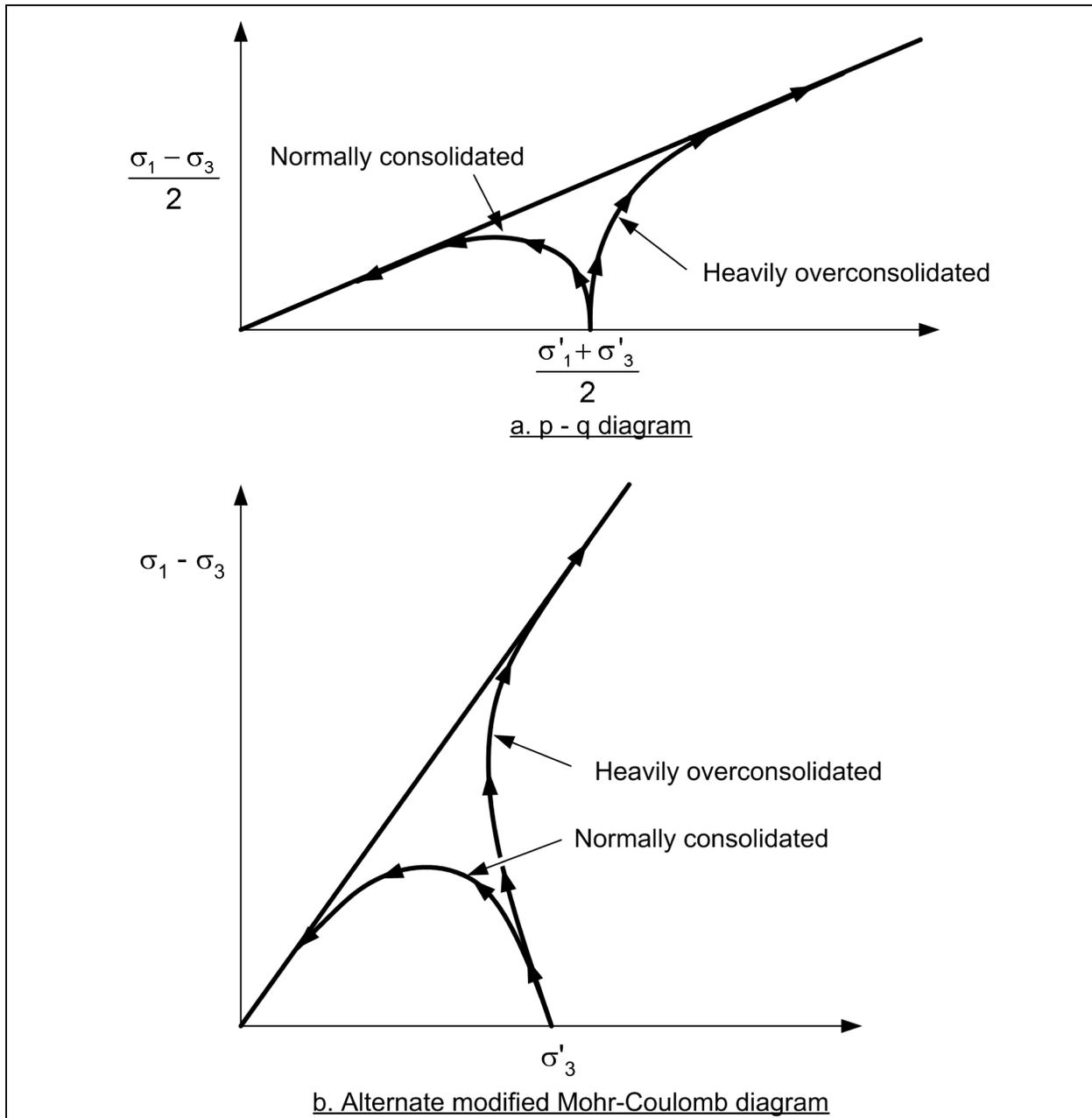
*b. Undrained loading.* Typical effective stress paths for Consolidated-Undrained triaxial shear tests on normally consolidated and overconsolidated clays are illustrated in Figure D-13. Figure D-13a shows the stress paths on a  $p$ - $q$  diagram, while Figure D-13b shows the stress paths on a modified Mohr-Coulomb diagram where  $(\sigma_1 - \sigma_3)$  is plotted versus  $\sigma_3$ . Note how the stress paths for the normally consolidated clay reach a peak value of  $(\sigma_1 - \sigma_3)$  prior to becoming tangent to the effective stress failure envelope. If the specimen were being sheared using controlled load, rather than controlled deformation, the postpeak behavior might either be lost or the sample might deform so rapidly as to make pore water pressure and effective stress measurements meaningless (result of unequalized pore water pressures in the specimen). This is the reason that deformation controlled, rather than load controlled loading is preferred for undrained tests on normally and lightly overconsolidated clays.



**Figure D-12. Critical State line representing combinations of void ratio and confining pressure for soil that has been sheared to large strains – behavior in drained tests**

(1) The concept of a “critical state” line presented earlier is applicable to undrained loading as well as drained loading. Figure D-14 shows a critical state line along with a line representing the line for virgin, isotropic consolidation. Specimens that are normally consolidated prior to undrained shear in the triaxial test have loading paths for shear that start along the virgin isotropic consolidation line. The line labeled a-b represents the loading path for a normally consolidated soil specimen that is sheared to large strains, and eventually reaches the critical state line. The pore water pressure in the specimen continually increases during shear, and the effective confining pressure,  $\sigma'_3$ , continually decreases. Similarly, the line labeled c-d represents a loading path for a heavily overconsolidated soil that is sheared to large strains at the same initial effective confining pressure. The pore water pressures at failure are less than they were at the start of shear and the effective stresses have increased. The lengths of the paths a-b and c-d represent the changes in pore pressure during the tests. Movement to the left indicates increase in pore pressure, and movement to the right indicates decrease in pore pressure.

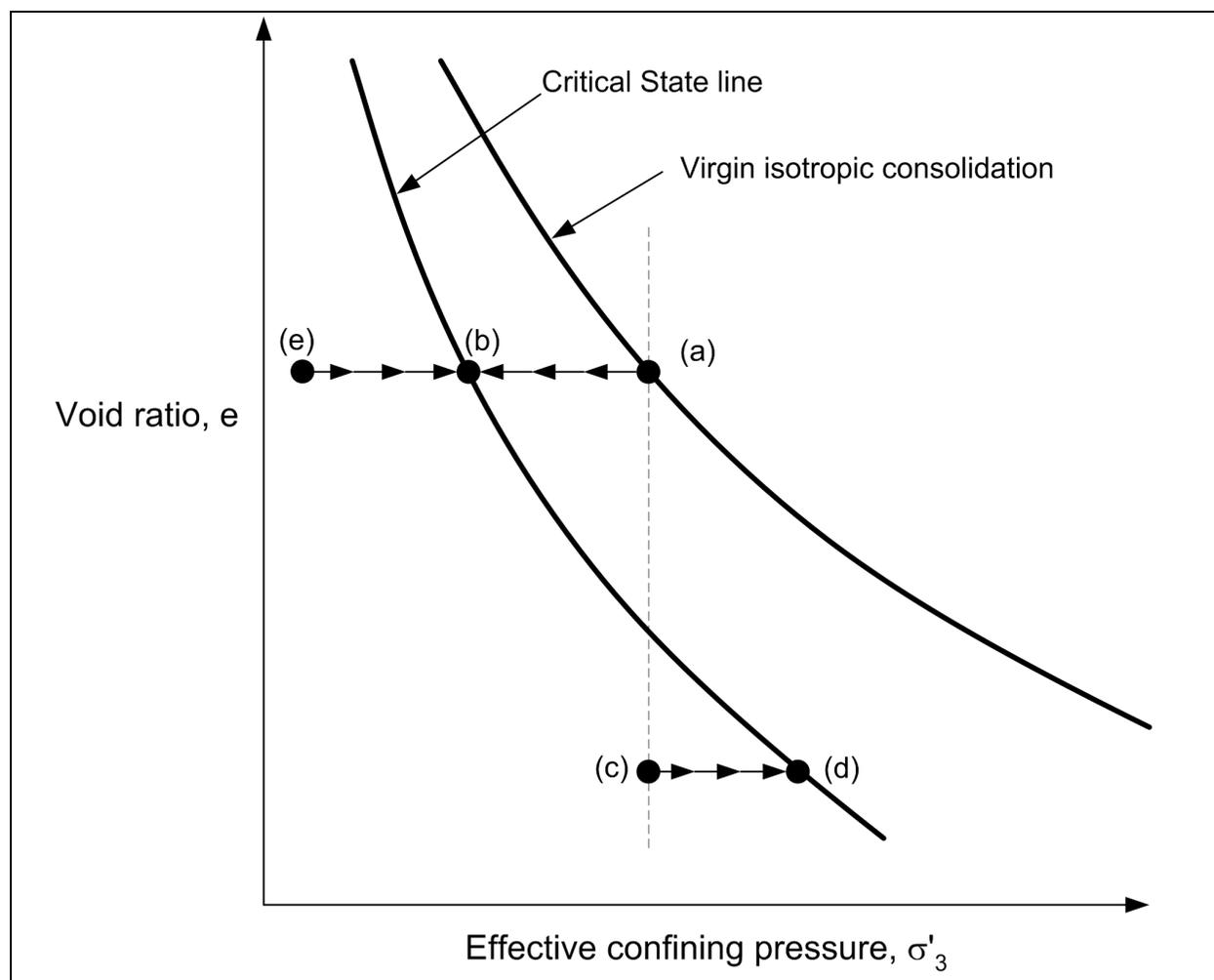
(2) The line labeled e-b in Figure D-14 represents another overconsolidated specimen. However, in this case the specimen has the same void ratio as the normally consolidated specimen. Both specimens have the same void ratio, but prior to shear, the normally consolidated specimen is under a higher confining pressure than the overconsolidated specimen. During shear the pore water pressures in the normally consolidated specimen increase (line a-b) and the pore water pressures in the overconsolidated specimen decrease (line e-b). At large strains (at the critical state line), both specimens have the same void ratio and have reached a



**Figure D-13. Effective stress paths on two types of modified Mohr-Coulomb diagrams for normally consolidated and heavily overconsolidated clays**

“critical confining pressure,”  $\sigma'_{3\text{-critical}}$ . Both specimens also have the same shear strength at the large strains corresponding to the critical state line, although the peak shear strengths of the two specimens would be quite different.

*c. Sample disturbance.* Disturbance of soil specimens caused by sampling and handling affects the stress-strain response of soils. Disturbance can sometimes be detected by simply examining the soil response during shear, especially when the soil is either loose sand or normally consolidated clay. Disturbance densifies loose sands and causes them to behave more like dense sands. Similarly, disturbance of normally

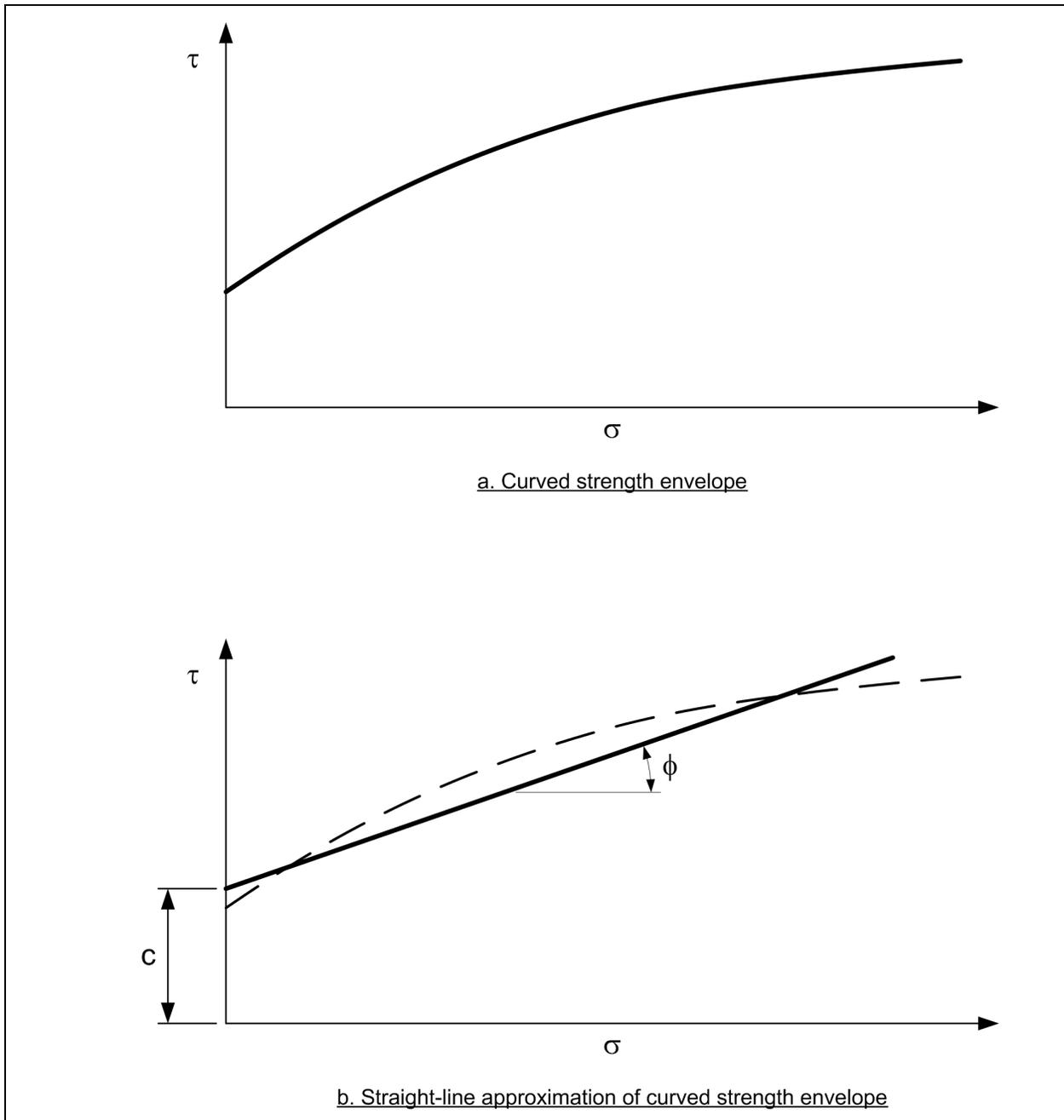


**Figure D-14. Critical State line representing combinations of void ratio and confining pressure for soil that has been sheared to large strains – behavior in undrained tests**

consolidated soil specimens makes them respond more like overconsolidated clays because disturbance causes the pore water pressure to increase and the effective stress to decrease. The axial strain at which the peak strength is developed in Unconsolidated-Undrained (UU or Q) tests may increase when specimens are disturbed, and instead of being in the typical range of 1 to 6 percent expected for normally consolidated clays, may increase to 10 percent or more if the specimens are disturbed. Pore water pressures generated during shear will also be lower if the specimens are disturbed. However, in the case of Unconsolidated-Undrained shear tests, the pore water pressures that exist prior to shear will be higher because of disturbance effects. The higher pore pressures at the start of shear will offset the lower pore water pressures developed during shear, such that sample disturbance will reduce the undrained shear strength measured in Unconsolidated-Undrained tests.

### D-8. Curved Strength Envelopes

*a.* The strength envelopes for many soils are curved, rather than linear, as shown in Figure D-15. If the curvature is small and the range of stresses of interest is small, a curved failure envelope can be approximated by a straight line for purposes of analysis, as shown in Figure D-15b. However, if the envelope is distinctly curved over the range of stresses of interest, use of a straight line failure envelope may significantly



**Figure D-15. Curved strength envelope and straight-line approximation**

overestimate the shear strength at low and at high stresses. Extrapolation to higher or lower stresses can result in an overestimate of shear strength, and a factor of safety with regard to slope stability that is too high.

*b.* It is especially important that laboratory tests be conducted using a range in confining pressures that represents the range of stresses expected along potential sliding surfaces. Once tests have been conducted using a suitable range in stresses, an appropriate decision can be made regarding how the strength envelope will be represented. It is relatively easy to use curved strength envelopes in slope stability computations with software that is currently available, and there is little advantage to using a linear strength envelope when the data suggest otherwise.

c. Frequently, when the strength envelope is curved, the results of strength tests are reported in terms of a secant friction angle,  $\phi_{\text{secant}}$  (Figure D-16). For example, Duncan, Horz, and Yang (1989) present useful correlations for the friction angle of a number of soils in terms of the secant friction angle at one atmosphere confining pressure, and the reduction in friction angle  $\Delta\phi$ , with each ten-fold increase in confining pressure. Stark and Eid (1994, 1997) present correlations for residual and fully softened shear strengths in terms of the secant friction angle at selected values of effective normal stress. Correlations such as those by Duncan, Horz, and Yang (1989) and Stark and Eid (1994, 1997), are useful for estimating shear strength values for preliminary stability analyses and to supplement data from laboratory tests. However, for slope stability computations it is usually preferable to express the strength envelope in terms of a continuous function of shear strength versus normal stress, rather than as a series of discrete values of secant friction angle. This can be done by selecting suitable values of normal stress,  $\sigma_j$ , and determining the corresponding friction angle,  $\phi_{\text{secant}}$  for each value of  $\sigma_j$ . Values of the respective shear strength are then computed as:

$$s_j = \sigma_j \tan \phi_{\text{secant-j}} \quad (\text{D-17})$$

where

$\sigma_j$  = one of a number of values of normal stress

$s_j$  and  $\phi_{\text{secant-j}}$  = corresponding values of secant friction angle and shear strength

The values of shear strength  $s_j$  are plotted against the corresponding values of  $\sigma_j$  to develop a nonlinear strength envelope like the one shown in Figure D-17, which is then used in the slope stability computations.

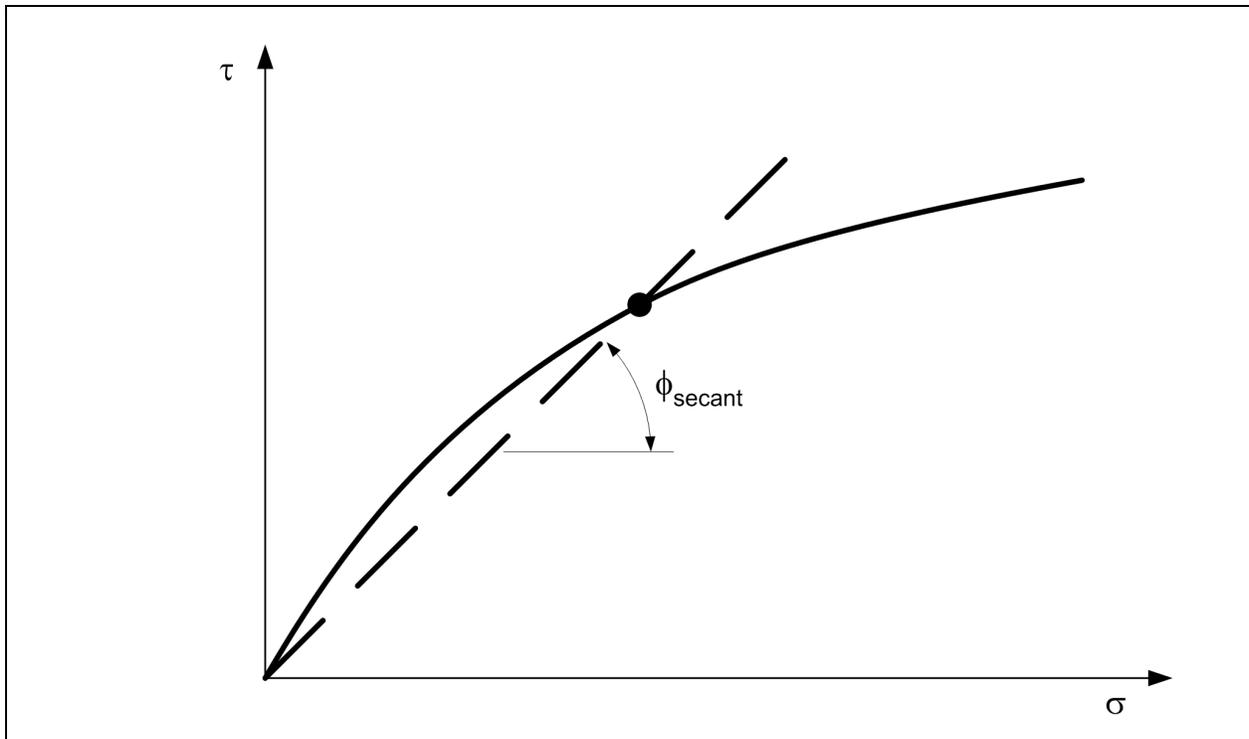


Figure D-16. Curved strength envelope and equivalent secant friction angle

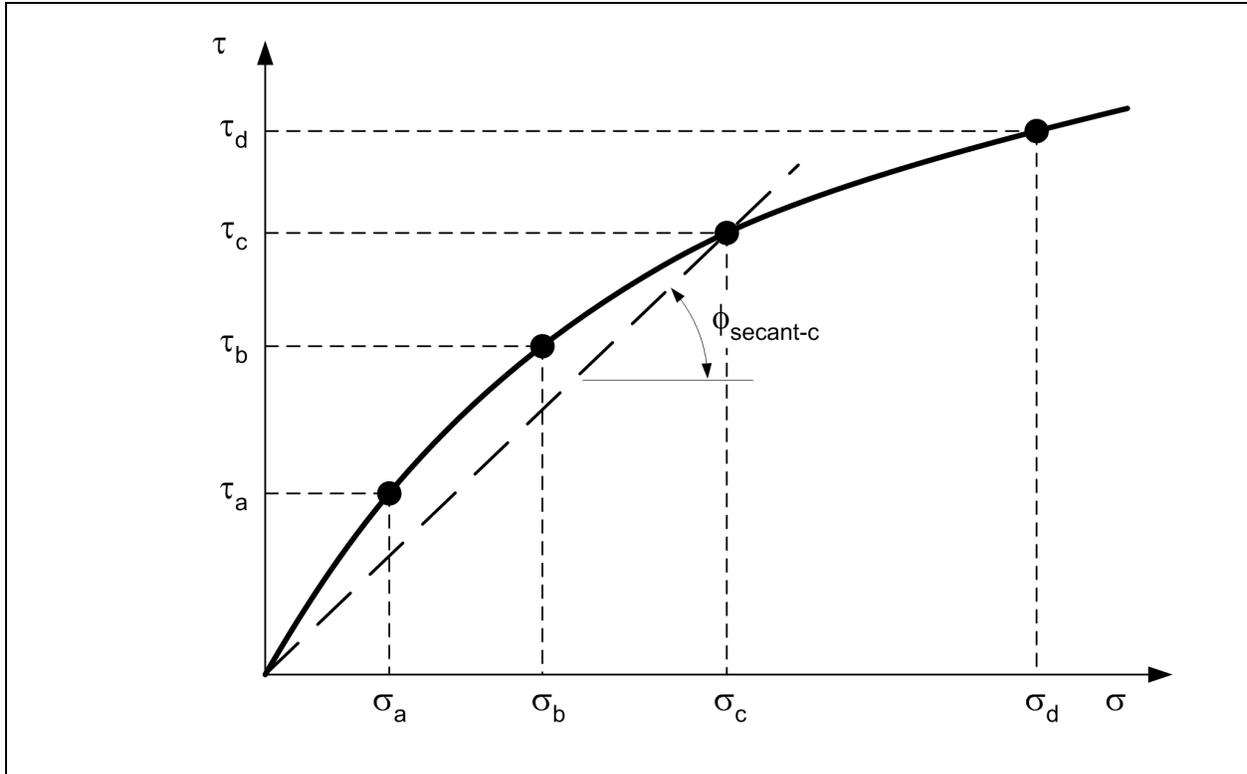


Figure D-17. Construction of curved strength envelope using secant friction angles

### D-9. Strain Compatibility

*a.* “Strain Compatibility” is a term used to refer to the variation in the stress-strain properties of soils along a slip surface. In an actual slope, both the strains that are developed and the stress-strain properties will vary such that it is unlikely that the peak shear strength will be developed simultaneously along the full length of a slip surface. This is important if the soils within the slope exhibit significant strain softening. In such cases, it is appropriate to adopt lower shear strengths or to require higher factors of safety.

*b.* Several approaches have been suggested for handling issues of strain incompatibility. Koutsoftas and Ladd (1985) have suggested a procedure for determining an “equivalent” strain that is used as the failure criterion for determining shear strengths from laboratory test data. Also, Chirapuntu and Duncan (1975) have developed procedures for reducing shear strengths when highly dissimilar soils exist along a potential slip surface. For many soils and slope conditions, no special provisions are necessary. For example, Wright, Kulhawy, and Duncan (1973) showed that for overall factors of safety of at least 1.5, the peak strength of the soil is not fully reached at any point along the potential slip surface, even though the strains and factor of safety may vary significantly. When the possibility of reduced strengths exists, it is helpful to compute the factor of safety using the ultimate (residual) shear strength. This represents a lower bound for shear strength and should indicate if there is any possibility of failure due to strain incompatibility problems. When such a conservative estimate for shear strengths is used, lower than conventional values of factor of safety are acceptable.

## D-10. Staged Construction

*a.* In staged construction, an embankment is built in increments and the foundation soil is allowed to consolidate fully or partially under each stage so that the increased strength will increase stability during subsequent states. Staged construction is generally used when the foundation soils are so weak that the entire embankment cannot be built in a single increment. Analyses for stage construction require special consideration in developing shear strength parameters. One approach is to use Consolidated-Undrained tests to measure the strengths, taking into account increases in strength resulting from increases in effective consolidation pressure. This involves the following steps:

- Step 1: Initial effective stresses and maximum past pressures are determined. Initial stresses are computed from unit weights and the groundwater levels prior to embankment construction. Maximum past effective stresses are determined from oedometer tests on high quality, undisturbed samples.
- Step 2: Normalized shear strengths,  $S_u/\sigma'_{vc}$ , are determined for various overconsolidation ratios by performing Unconsolidated-Undrained shear tests using the SHANSEP procedure.
- Step 3: Pore water pressures and effective stresses in the field are estimated using an appropriate consolidation analysis. The consolidation analysis should take into account the initial stresses, the increase in stress because of added embankment loads, and the subsequent consolidation because of dissipation of excess pore water pressures.
- Step 4: Undrained shear strengths are estimated using the information from Steps 1, 2, and 3. Undrained shear strengths are calculated by multiplying the appropriate values of normalized shear strength by the effective vertical stress, thus accounting for consolidation.
- Step 5: Stability analyses are performed using undrained shear strengths. The undrained shear strengths are assigned as values of cohesion,  $c$ , and  $\phi$  is equal to zero.

*b.* The above procedure requires assumptions about how the initial excess pore water pressures are generated by the embankment loads, especially regarding the pore water pressures beneath and beyond the toe of the slope. Also, a relatively complex analysis of consolidation is required to account for the variation in stresses and excess pore water pressures in the vertical and horizontal directions. Finally, the shear strength is usually related to the vertical effective stress in the field, which, unlike the stresses in the laboratory, is seldom the major principal stress during consolidation. More uncertainty exists in analyses of staged construction than for other cases, and this should be taken into consideration when selecting appropriate shear strength values and factors of safety for design.

*c.* An alternative approach to the undrained strength approach described above is to perform an effective stress analysis using effective stress shear strength parameters ( $c'$  and  $\phi'$ ) and estimated pore water pressures. This approach requires the same relatively complex consolidation analysis used in the first approach and, thus, the second approach is also subject to the same errors. The effective stress approach will also give different values for the factor of safety as the result of fundamental differences between total and effective stress factors of safety: The effective stress approach is based on pore water pressures at working stress levels, rather than values at failure, while the undrained strength approach is based on pore water pressures generated at failure. Because there is no experience to guide selection of safety factors for the effective stress approach, it should not be used.

## D-11. Partially Saturated Soils

*a.* Partially saturated soils present special problems when treated using effective stresses. Significant progress has been made in understanding how partially saturated soils behave and the role of effective stresses (Fredlund and Rahardjo 1993; Fredlund 2000). This work indicates that the simple expression for effective stress where the pore water pressure is subtracted from the total stress to evaluate the effective stress is not valid and that the Mohr-Coulomb shear strength equation for effective stresses in saturated soils is not valid for unsaturated soils. Rigorous treatment of effective stresses in partially saturated soils is beyond the scope of this manual.

*b.* Fortunately, consideration of effective stresses in unsaturated soils can be avoided for many practical slope stability problems. To evaluate strength and stability at the end of construction, Unconsolidated-Undrained shear tests are performed to measure the shear strength. In this case the shear strengths are expressed as a function of total stresses, and the approach is valid for both saturated and unsaturated soils. For long-term stability and stability during rapid drawdown, the soil may be fully or only partially saturated. However, if the soil is below the groundwater table or beneath the phreatic surface, the pore water pressures are positive and the soil is assumed to be saturated for design purposes. If the soil is above the water table or in a zone of capillarity and where pore pressures are negative, the beneficial effects of negative pore water pressures are conservatively neglected by assuming that the pore water pressures are zero. Conventional effective stress shear strength parameters are used for both the saturated (positive pressure) and partially saturated (zero pressure) zones. The effective stress shear strength parameters are measured on specimens that are fully saturated prior to laboratory testing, regardless of the saturation that may exist in the field.

*c.* For cases where substantial portions of the slope are partially saturated and long-term stability is being evaluated, neglecting negative pore water pressures can be very conservative. In such cases, some account of negative pore water pressures may be appropriate (Fredlund 1989, 1995). Beneficial effects of negative pore water pressure can easily be destroyed by rainfall and infiltration of surface water, as well as a rise in ground water table. Thus, negative pore water pressures should be included in stability computations with great caution. Use of negative pore water pressures is not recommended for design of dams and similar critical structures where the consequences of failure are great.