APPENDIX C
ANALYSIS OF PRESSURE INJECTION TESTS (Ziegler 1976)

C-1. Water Pressure Tests

a. Water pressure tests are conducted by pumping water into a borehole at a constant pressure and measuring the flow rate. Water enters the rock mass along the entire length of borehole or along a test section (i.e., borehole interval) sealed off by one or more packers as shown in figure C-1. The test is rapid and simple to conduct and by conducting tests within intervals along the entire length of borehole, a permeability profile can be obtained.

b. In most pressure tests the water injection pressure is limited to a value which is not expected to produce an increase in the fracture width. An increase in the fracture width will cause erroneously high flow rates resulting in higher permeabilities than actually exist. A common criterion is to limit the water injection pressure to 1 psi/ft of borehole depth above the water table and 0.57 psi/ft of borehole depth below the water table. This criterion results in a maximum injection pressure less than the effective overburden pressure if the overburden has a unit weight greater than 144 lb/ft^3.

c. The coefficient of permeability, based upon laminar flow, is computed for a vertical test section with the following assumptions:

(1) Medium is homogeneous and isotropic.

(2) Laminar flow governed by Darcy's law.

(3) Radial flow from a cylindrical and vertical borehole test section length, ℓ . Radial flow implies that the equipotential surfaces form cylinders symmetrical about the axis of the borehole test section.

(4) No inertia effects.

(5) Boundary conditions (fig. C-2).

(a) At $r = r_o$. $H = H_o$.

(b) At $r=R$, $H=0$.

Where

r = radial distance from the test section (L)

r_o = radius of borehole (L)

H = excess pressure head (L)

H_o = excess pressure head at the center of the test section (L) (either measured within the test section, or computed allowing for frictional head losses within the drill pipe)

R = radius of influence: radial distance from the test section corresponding to a 100 percent loss in excess head, H_0 (L)

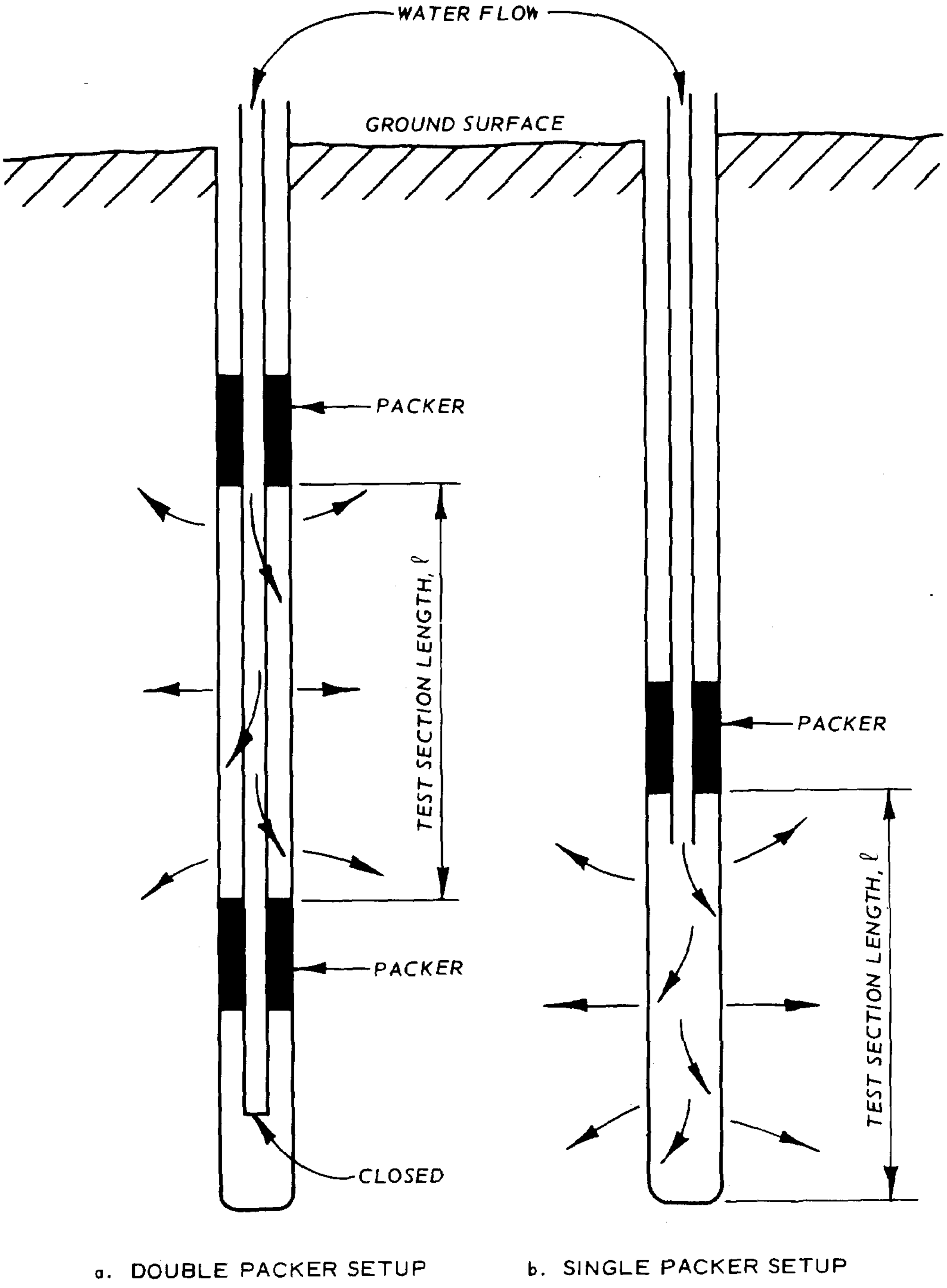

Figure C-1. Typical water pressure test setups (prepared by WES)

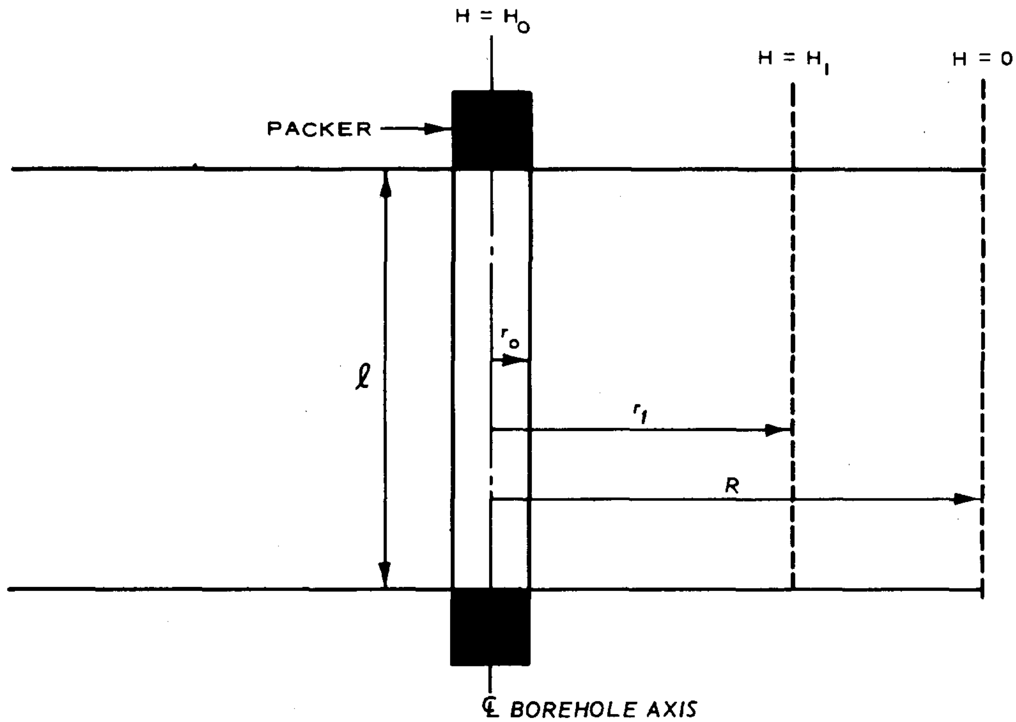

Figure C-2. Homogeneous isotropic material--radial flow (prepared by WES)

d. Darcy's equation may be written

$$v = k_e \frac{dh}{dr}$$

where

v = flow velocity (L/T)

k_e = laminar equivalent permeability (L/T)

dh/dr = hydraulic gradient in the radial flow system (L/L)

e. The volume flow rate from the borehole cavity is

$$Q = k_e \frac{dh}{dr} A$$

where

Q = volume flow rate (L^3/T)

A = area of an equipotential surface (L^2)

The area A in the radial system is

$$A = 2\pi r \ell$$

Thus

$$Q = k_e 2\pi r \ell \frac{dh}{dr}$$

which can be written

$$\frac{dr}{r} = \frac{2\pi \ell}{Q} k_e \, dh$$

and integrated

$$\int_{r_o}^{R} \frac{dr}{r} = \frac{2\pi \ell k_e}{Q} \int_{H_o}^{Q} dh \qquad \text{(C-1)}$$

$$\ln \frac{R}{r_o} = \frac{-2\pi \ell k_e}{Q} H_o$$

$$-Q = \frac{2\pi \ell k_e H_o}{\ln (R/r_o)}$$

The negative sign can be dropped by choosing flow away from the borehole as positive, thus

$$Q = \frac{2\pi \ell k_e H_o}{\ln (R/r_o)}$$

and the permeability is

$$k_e = \frac{Q \ln (R/r_o)}{2\pi \ell H_o} \qquad \text{(C-2)}$$

f. Before equation C-2 can be applied, however, the radius of influence, R , must be determined or estimated. A rough estimate of R is normally adequate since large variations in R generally produce only small variations in the computed permeability. In equation C-2 the permeability is directly related to the term $\ln (R/r_0)$. The rate of change of 1n (R/ro) , and thus permeability, decreases rapidly as R increases. This is illustrated by considering the analysis of a pressure test conducted in an NX-size borehole. The increase in permeability, k_e (based on equation C-2), as R is increased from 1 ft to 10^6 ft is shown below:

Radius of Influence, R , ft	Equivalent Permeability, k_e , ft/sec
1	k_e
10	2.1 k_e
100	3.2 k_e
1,000	4.3 k_e
10,000	5.4 k_e
100,000	6.5 k_e
1,000,000	7.5 k_e

Thus as R is increased from 1 ft to 10^6 ft, the computed permeability increases by less than one order of magnitude. To eliminate the arbitrary choice of R as a source of error in the permeability calculation, it has been suggested that during a pressure test, pressure in the surrounding mass be observed in a nearby borehole. By changing the upper limits of integration in equation C-1 to correspond to an arbitrary distance r , within the boundaries r_o to R (fig. C-2) the following equations are developed:

$$\int_{r_o}^{r_1} \frac{dr}{r} = \frac{2\pi \ell k_e}{Q} \int_{H_o}^{H_1} dh$$

Integration yields

$$H_1 = H_o - \frac{Q}{2\pi \ell k_e} \ln (r_1/r_o)$$

where H_1 = excess pressure head at an arbitrary distance, r_1, from the test section (L) which can be written as

$$k_e = \frac{Q \ln(r_1/r_o)}{2\pi\ell(H_o - H_1)} \qquad \text{(C-3)}$$

g. The coefficient of permeability, based upon nonlinear or turbulent flow, is computed for a vertical test section in a homogeneous, isotropic medium with the following assumptions:

(1) Medium is homogeneous and isotropic.

(2) Nonlinear or turbulent flow governed by the Missbach law $v^m = k'_e i$)

(3) Radial flow from cylindrical and vertical borehole test section of length, ℓ .

(4) No inertia effects.

(5) Boundary conditions (fig. C-2).

(a) At $r = r_o$, $H = H_o$.

(b) At $r = R$, $H = 0$.

h. The Missbach law may be written

$$v^m = k'_e \frac{dh}{dr}$$

where

m = degree of nonlinearity (generally between 1 and 2)

k'_e = turbulent equivalent permeability $(L/T)^m$

Volume flow rate for radial flow from the cavity is

$$Q = 2\pi r \ell v$$

and it follows that:

$$Q^m = (2\pi r \ell)^m v^m$$

$$Q^m = (2\pi r \ell)^m k_e' \frac{dh}{dr}$$

$$\frac{Q^m}{(2\pi \ell)^m} = r^m k_e' \frac{dh}{dr}$$

$$\frac{dr}{r^m} = k_e' \left(\frac{2\pi \ell}{Q}\right)^m dh$$

$$\int_{r_o}^{R} r^{-m}\, dr = k_e' \frac{(2\pi \ell)^m}{Q^m} \int_{H_o}^{Q} dh$$

$$\left(\frac{1}{1-m}\right)\left(R^{1-m} - r_o^{1-m}\right) = -k_e' \frac{(2\pi \ell)^m}{Q^m} H_o$$

$$-Q^m = k_e'(2\pi \ell)^m H_o (1-m)\left(\frac{1}{R^{1-m} - r_o^{1-m}}\right)$$

Drop the negative sign by choosing flow away from the borehole as positive, yielding

$$k_e' = \frac{Q^m \left(R^{1-m} - r_o^{1-m}\right)}{(2\pi \ell)^m (1-m) H_o} \qquad \text{(C-4)}$$

Equation C-4 can be rewritten as

$$H_o = E_1 Q^m \qquad \text{(C-5)}$$

where

$$E_1 = \left(\frac{1}{k_e'}\right)\left(\frac{1}{2\pi\ell}\right)^m \left(\frac{1}{1-m}\right)\left(R^{1-m} - r_o^{1-m}\right)$$

The logarithms of terms in equation C-5 yield the equation of a straight line:

$$\log H_o = \log E_1 + m \log Q$$

A plot of $\log H_o$ versus $\log Q$ will be a straight line with an arithmetic slope equal to the degree of nonlinearity m .

i. The coefficient of permeability for laminar flow in fissures is computed for a vertical test section with the following assumptions:

(1) Vertical borehole test section of length ℓ is intersected by an arbitrary number of horizontal fissures.

(2) Fissures are constant aperture openings between smooth parallel plates.

(3) Radial flow occurs within each fissure and is governed by Darcy's law. No flow occurs in material between fissures.

(4) Each fissure has the same equivalent parallel plate aperture, e .

(5) No inertia effects.

(6) Boundary conditions (fig. C-3).

(a) At $r = r_o$, $H=H_o$.

(b) At $r = R$, $H=0$.

j. The derivation of equations C-6 and C-7 proceeds in the same fashion as that given above for laminar flow through a homogeneous isotropic medium (equations C-2 and C-3). For the fissured medium the test section length, ℓ , is replaced by the quantity (ne) where n is the number of fissures intersecting the test section and e is the equivalent parallel plate aperture of each fissure. The resulting expression for the flow rate Q is

$$Q = \frac{2\pi n e k_j H_o}{\ln (R/r_o)}$$

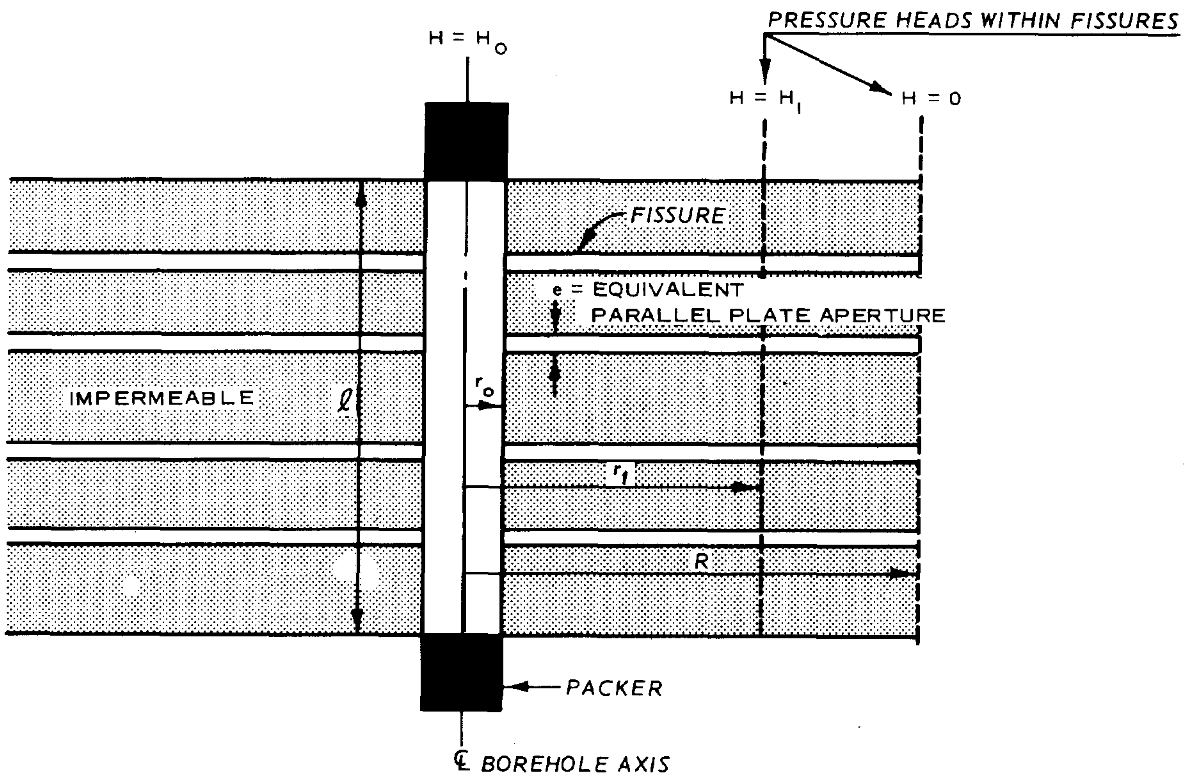

Figure C-3. Fissured-medium--radial flow (prepared by WES)

and the permeability of each fissure is

$$k_j = \frac{Q \ln (R/r_o)}{2\pi n e H_o} \qquad \text{(C-6)}$$

where k_j = laminar fissure permeability (L/T). The general expression for excess pressure head H_1 at an arbitrary distance r_1 between the boundaries r_o and R is given by

$$H_1 = H_o - \frac{Q}{2\pi n e k_j} \ln \frac{r_1}{r_o}$$

which can be written as

$$k_j = \frac{Q \ln (r_1/r_o)}{2\pi n e (H_o - H_1)} \qquad \text{(C-7)}$$

From the theory of viscous flow between smooth parallel plates

$$k_j = \frac{e^2 \gamma_w}{12\mu_w} \tag{C-8}$$

where

γ_w = unit weight of water (F/L^3)

μ_w = dynamic viscosity of water $\left(\frac{FT}{L^2}\right)$

Equation C-8 is substituted into equation C-6 to solve for the equivalent parallel plate aperture, e :

$$e = \left[\frac{Q \ln (R/r_o)}{2\pi n H_o} \left(\frac{12\mu_w}{\gamma_w}\right)\right]^{1}$$

k. The coefficient of permeability for nonlinear or turbulent flow in fissures is computed for a vertical test section with the following assumptions:

(1) Vertical borehole test section of length ℓ intersected by an arbitrary number of horizontal fissures.

(2) Fissures are constant aperture openings between smooth parallel plates.

(3) Radial flow occurs within each fissure and is governed by the Missbach law ($v^m = k'_j i$). No flows occur in material between fissures.

(4) Each fissure has the same equivalent parallel plate aperture, e .

(5) No inertial effects.

(6) Boundary conditions (fig. C-3).

(a) At $r = r_o$, $H = H_o$.

(b) At $r=R$, $H=0$.

1. The derivation of equation C-9 proceeds in the same fashion as that given above for nonlinear flow through a homogeneous isotropic medium (equation C-4). For the fissured medium, the test section length, ℓ , is replaced

by the quantity ne where n is the number of fissures intersecting the test section and e is the equivalent parallel plate aperture of each fissure. The resulting equation for the turbulent fissure permeability is

$$k_j' = \frac{Q^m\left(R^{1-m} - r_o^{1-m}\right)}{(2\pi ne)^m H_o (1 - m)} \tag{C-9}$$

where k_j' = turbulent fissure permeability $(L/T)^m$. Equation C-9 can be rewritten as

$$H_o = E_2 Q^m \tag{C-10}$$

where

$$E_2 = \left(\frac{1}{k_j'}\right)\left(\frac{1}{2\pi ne}\right)^m \left(\frac{1}{1 - m}\right)\left(R^{1-m} - r_o^{1-m}\right)$$

The logarithms of terms in equation C-10 yielded the equation of a straight line:

$$\log H_o = \log E_2 + m \log Q$$

A plot of log H_o versus log Q will be a straight line with an arithmetic slope equal to the degree of nonlinearity, m .

C-2. Air Pressure Tests.

a. Air pressure tests are conducted similarly to water pressure tests, with the essential difference being the replacement of water with air. The use of air, however, requires that permeability equations be modified for application to a compressible fluid and a conversion must be made from air to water permeability. To compute the permeability, flow is assumed to be laminar and governed by Darcy's law. The material tested is assumed to be a homogeneous isotropic porous medium. Darcy's law can be written

$$v = ki$$

where

v = flow velocity (L/T)

k = coefficient of permeability (L/T)

i = hydraulic gradient (L/L)

b. The coefficient of permeability, k , is dependent on properties of the medium and fluid or gas. Since the test involves the flow of air and permeability relating to the flow of water is desired, it is convenient to solve for the intrinsic permeability, k , which is characteristic of the medium alone. The coefficient of permeability and intrinsic permeability are interrelated (Davis and Dewiest 1966, and Muskat 1946). For the flow of water

$$k_e = \frac{K\gamma_w}{\mu_w} \tag{C-11}$$

where

k_e = laminar equivalent permeability (L/T)

γ_w = unit weight of water (F/L^3)

μ_w = dynamic viscosity of water $\left(\frac{FT}{L^2}\right)$

c. The conversion of intrinsic permeability measured by an air test to the water permeability can be in error due to differences in gas and fluid flow phenomena i.e., the Klinkenberg Effect (Weeks 1978), or alteration of the material's physical properties caused by a chemical reaction with the fluid or gas (Davis and Dewiest 1966, and Lynch 1962). In sediments rich in clay minerals, water permeability calculated from air measurements may be overestimated by a factor of 100 (Davis and Dewiest 1966). This overestimation is caused by a hydration of clays during waterflow which does not occur during airflow. Test results should be applied with caution as more experience and studies are needed to determine the limitations of equation C-11.

d. Muskat (1946, p. 679) observed that the flow of incompressible liquids is easily modified for application to the flow of a compressible gas. Steady-state solutions for analyzing constant water pressure test results are generally written in the form

$$k_e = \frac{Q}{\ell H_o} [C]$$

where

Q = volume flow rate (L^3/T)

ℓ = length of test section (L)

H_o = excess pressure head at the center of the test section (L)

The term [C] is dependent on assumed flow and boundary conditions. The following equation is commonly used in analyzing water pressure test results.

$$k_e = \frac{Q}{\ell H_o} \frac{\ln (R/r_o)}{2\pi} \tag{C-12}$$

where

R = radius of influence (L)

r_o = radius of borehole (L)

After solving for Q and converting to intrinsic permeability, k, equation C-12 becomes

$$Q = \left(\frac{2\pi \ell K}{\mu_w}\right)\left[\frac{P_o}{\ln (R/r_o)}\right] \tag{C-13}$$

where $P_o = \gamma_w H_o$ = excess pressure at the center of test section (F/L^2) the equation (C-13) is based-on the assumption that:

(1) The medium is a homogeneous and isotropic porous continuum.

(2) The flow emitting from a cylindrical section of borehole is laminar, and governed by Darcy's equation.

(3) The flow pattern is ellipsoidal and symmetrical about the axis of the borehole test section.

e. Equation C-13 can be modified for analyzing air pressure test results by making similar assumptions and considering the fluid to be compressible. Before modification, equation C-13 must be written in terms of absolute pressures. The pressure, P_o, equals the absolute pressure in the test section, $\bar{P}_b$, minus atmospheric pressure, $\bar{P}_a$. Equation C-13 becomes

$$Q = \left(\frac{2\pi \ell K}{\mu_w}\right)\left[\frac{\bar{P}_b - \bar{P}_a}{\ln (R/r_o)}\right] \qquad \text{(C-14)}$$

f. Equation C-14 is modified to apply to the flow of gas by replacing the pressure term $(\bar{P}_b - \bar{P}_a)$ by the expression

$$\frac{\gamma_o\left(\bar{P}_b^{1+M} - \bar{P}_a^{1+M}\right)}{(1 + M)} \qquad \text{(C-15)}$$

where γ_o is a constant at any point in the flow and defined as

$$\gamma_o = \frac{\rho g}{\bar{P}^M} = \frac{\gamma}{\bar{P}^M} = \text{a constant} \qquad \text{(C-16)}$$

where

ρ = mass density $(F\text{-}T^2/L)/L^3$

g = acceleration due to gravity (L/T^2)

$\bar{P}$ = absolute pressure (F/L^2)

γ = unit weight of gas (F/L^3)

and the exponent, M , determines the thermodynamic nature of the expansion of a gas as it moves from high- to low-pressure regions. The resulting equation (equation C-18 below) is in terms of weight flow rate rather than volume flow rate. Since gases are highly compressible, the volume flow rate will vary with pressure and temperature along the flow path. However, the weight flow rate can be assumed to remain constant. Weight flow rate, Q_{WF} , and volume flow rate, Q , at any point in the flow are related by

$$Q = \frac{Q_{WF}}{\gamma} \qquad \text{(C-17)}$$

Substitution of expression C-15 into equation C-14 (replace μ_w with μ_a) yields the following expression for the weight flow rate of air:

$$Q_{WF} = \frac{2\pi \ell K \gamma_o \left(\bar{P}_b^{1+M} - \bar{P}_a^{1+M}\right)}{\mu_a (1 + M) \ln (R/r_o)} \qquad \text{(C-18)}$$

where

Q_{WF} = weight flow rate (F/T)

μ_a = dynamic viscosity of air (F T/L^2)

g. For convenience, the value of M is assumed equal to 1 which corresponds to isothermal expansion of an ideal gas as it moves from the borehole through the medium (for adiabatic expansion, $M < 1$). Since equation C-16 is valid at any point in the flow and $M + 1$, γ_o can be replaced by $\gamma_b/\bar{P}_b$ where γ_b and $\bar{P}_b$ are the unit weight of air and absolute pressure in the test section, respectively. By substituting $M = 1$ and $\gamma_b/\bar{P}_b$ in equation C-18, the intrinsic permeability, k, can be expressed as

$$k = \left(\frac{Q_{WF}}{\gamma_b}\right) \frac{\mu_a \ln (R/r_o)}{\pi \ell} \left(\frac{\bar{P}_b}{\bar{P}_b^2 - \bar{P}_a^2}\right) \qquad \text{(C-19)}$$

h. Parameters used in equation C-19 are measured or computed from the test data. The test section length, ℓ, and radius of test section, r_o, are determined from the test setup. Standard atmospheric pressure, $\bar{P}_a$, equals 14.7 psi or 2,120 lb_F/ft^2. The absolute pressure in the test section, P_b, is equal to the gage pressure measured in the test section, P_o, plus atmospheric pressure, $\bar{P}_a$ The dynamic viscosity of air, μ_a, depends only on the temperature, and its variation is small over a large range of temperatures. For example, air viscosity increases from approximately 0.035 x 10^{-5} to 0.045 x 10^{-5} lb_F-sec/ft^2 as temperature increases from 0 to 250° F. In general, the dynamic viscosity of air can be assumed to equal 0.38 x 10^{-5} lb_F-sec/ft^2 which is the viscosity of air at 68° F.

i. The weight flow rate from the test section, Q_{WF}, is constant along the flow path and can be determined at the manifold by applying equation C-17:

$$Q_{WF} = Q_m \gamma_m$$

where

Q_{WF} = weight flow rate (F/T)

Q_m = volume flow rate at the manifold (L^3/T)

γ_m = unit weight of air at the manifold (F/L^3)

The value of γ_m is related to the pressure and temperature by the equation of state (Vennard 1965):

$$\gamma_m = \frac{\bar{P}_m}{R_g \bar{T}_m}$$

where

γ_m = unit weight of air at the manifold (lb_F/ft^3)

$\bar{P}_m$ = absolute pressure at the manifold (lb_F/ft^2)

R_g = engineering gas constant for air (53.3 ft-lb_F/lb_F-degrees Rankine)

$\bar{T}_m$ = absolute temperature at the manifold (degrees Rankine)

j. The unit weight of air in the test section, γ_b, is also computed from the equation of state:

$$\gamma_b = \frac{\bar{P}_m}{R_g \bar{T}_b}$$

where $\bar{T}_b$ = absolute temperature in test section (degrees Rankine). All the parameters needed to compute the intrinsic permeability, k , by equation C-19 have now been determined. The equivalent permeability applicable to the flow of water, k_e , is computed by substitution of k into equation C-11 yielding the following general equation:

$$k_e = \left(\frac{Q_m}{\pi \ell}\right)\left(\frac{\gamma_w \mu_a}{\mu_w}\right)\left(\frac{\bar{T}_b \bar{P}_m}{\bar{T}_m \bar{P}_b^2 - \bar{P}_a^2}\right) \ln \left(\frac{R}{r_o}\right)$$

where

k_e = laminar equivalent permeability (L/T)

Q_m = volume flow rate at the manifold (L^3/T)

ℓ = length of the test section (L)

γ_w = unit weight of water (F/L^3)

μa = dynamic viscosity of air $\left(\frac{FT}{L^2}\right)$

μ_w = dynamic viscosity of water $\left(\frac{FT}{L^2}\right)$

$\bar{T}_b$ = absolute temperature in test section (degrees Rankins)

$\bar{P}_m$ = absolute pressure at the manifold (F/L^2)

$\bar{T}_m$ = absolute temperature at the manifold (degrees Rankins)

$\bar{P}_b$ = absolute pressure in test section (F/L^2)

$\bar{P}_a$ = atmospheric pressure (F/L^2)

R = radius of influence (L)

r_o = radius of the borehole (L)

C-3. Pressure Holding Tests.

a. The pressure holding test, sometimes called a pressure duration or pressure drop test, is conducted by pressurizing the test section to a known value, then stopping the water supply and observing the rate of pressure decay. A pressure holding test is usually conducted in conjunction with a water pressure test. For the conduct of the test, a pressure transducer mounted in the test section is used to provide a continuous record of pressure versus time.

b. Tests in fissured rock can be analyzed through application of the parallel plate analogy. The laminar fissure permeability, k_j , is related to the pressure drop data by the expression

$$k_j = \frac{r_o^2 \ln\left(H_{o_1} / H_{o_2}\right) \ln\left(R/r_o\right)}{2ne\left(t_{drop}\right)} \qquad \text{(C-20)}$$

where

r_o = radius of borehole (L)

H_{o_1} = excess pressure head at the center of the test section at the initiation of a pressure drop test (L)

H_{o_2} = excess pressure head at the center of the test section at the completion of a pressure drop test (L)

R = radius of influence (estimated) (L)

n = number of fissures intersecting the test section

t_{drop} = duration of the pressure drop test (T)

c. Equation C-20 was derived by Maini (1971) based on the following assumptions.

(1) Radial flow occurs from a vertical test section and is governed by Darcy's law.

(2) Fissure system intersecting the test section is represented by horizontal parallel plates of equal aperture and spacing.

(3) Test zone is saturated. Maini (1971) suggests that before conducting tests in zones above the ground-water table, water be pumped into the borehole test section to saturate the fissure system in the immediate vicinity of the borehole.

d. The equivalent parallel aperture, e , in equation C-20 is unknown; however, by parallel plate analogy

$$k_j = \frac{e^2 \gamma_w}{12\mu_w} \qquad \text{(C-21)}$$

which when substituted into equation C-20 yields

$$e^3 = \frac{r_o^2 \ln\left(H_{o_1} / H_{o_2}\right) \ln\left(R/r_o\right) 12\mu_w}{2n\gamma_w\left(t_{drop}\right)} \qquad \text{(C-22)}$$

The aperture, e , is computed by equation C-22 and substituted into equation C-21 to obtain the permeability k_j . If the test zone is modeled as a homogeneous isotropic porous medium and assumptions (1) and (3) above are valid, an expression for the laminar equivalent permeability, k_e , is obtained by replacing the quantity ne in equation C-20 with the length of the test section, ℓ , to yield

$$k_e = \frac{r_o^2 \ln\left(H_{o_1} / H_{o_2}\right) \ln\left(R/r_o\right)}{2\ell\left(t_{drop}\right)} \tag{C-23}$$

e. The pressure drop test is a suitable supplement to the water pressure test and possesses certain advantages. The pressure drop test is likely to require a significantly smaller volume of water than that needed in a constant pressure test. The savings in water can be important when conducting tests in regions with a limited water supply. The measurement of pressure in the test section by an electric transducer allows the pressure drop test to be conducted with low initial pressures regardless of the depth of the test section, since the water level in the flow pipe only needs to be above the top of the test section to initiate a test. The use of low initial pressures has the additional advantage of reducing friction pressure losses during the conduct of a test.