APPENDIX D

CALCULATIONS FOR FLOW-DURATION METHOD EXAMPLE

D-1. General. This appendix includes the backup calculations used in deriving the figures which illustrate the example described in Section 5-7 (computing energy using the flow-duration method). Data is presented only for a sufficient number of points to define the curves.

D-2. Total Energy Potential.

a. Table D-1 summarizes the calculations used to derive the total energy potential curves shown as dashed lines on Figures 5-20 and 5-21 and described in Section 5-7i. Generation was computed for 100 percent exceedance (60 cfs), minimum discharge (155 cfs), discharge at rated head (400 cfs), discharge at minimum head (1450 cfs), and several additional points. Power output at each discharge level was computed using the water power equation, as described in Section 5-7i. Net head values were obtained from Figures 5-16 and 5-17, and percent exceedance values were taken from Figure 5-15, with both values based on total discharge. The net discharge value is equal to the total discharge minus the 20 cfs loss (Section 5-7e). A fixed overall efficiency of 85 percent was assumed for all discharge levels. It should be noted that the total energy curves on Figures 5-20 and 5-21 do not represent gross theoretical energy potential, but the total developable potential, which reflects friction head losses, flow losses due to leakage, and turbine-generator efficiency losses.

b. The dashed line on Figure D-1 (and Figure 5-21) is a plot of the data shown on Table D-1. It should be noted that this figure is not a true generation-duration curve, because the generation drops off at exceedance levels greater than eight percent. This is because of the low heads that occur at high discharge levels. In plotting Figure 5-20, the data shown on Figure D-1 was rearranged in true duration curve format.

D-3. Usable Generation. Table D-2 summarizes the calculations used for describing the usable generation curve, which is the curve enclosing the shaded area on Figure D-1. Figure 5-20 shows the same data plotted in true duration curve format (see also Section 5-7i). These curves describe that portion of the total energy that could be developed by a single tubular turbine with a rated head of 31.0 feet and a rated discharge of 380 cfs. The calculations are identical to

TABLE D-1
Total Energy Potential

Total Discharge (cfs)	Net Head (feet)	Net Discharge (cfs)	Efficiency (percent)	Power Output (kW)	Percent Exceedance
60	35.0	40	85	100	100
155	34.0	135	85	330	77
250	33.0	230	85	550	49
400	31.0	380	85	850	32
600	28.0	580	85	1170	22
1000	21.0	980	85	1480	11
1200	16.7	1180	85	1420	8
1450	11.0	1430	85	1130	5
1750	5.2	1730	85	650	4
2000	1.7	1980	85	240	3
2100	0.8	2080	85	120	2

TABLE D-2
Usable Generation Using Approximate Method

Total Discharge (cfs)	Net Head (feet)	Net Discharge (cfs)	Efficiency (percent)	Power (kW)	Percent Exceedance
60	35.0	40 1/	85	0 1/	100
155	34.0	135	85	330	77
250	33.0	230	85	550	49
400	31.0	380	85	850	32
600	28.0	380 2/	85	760	22
1000	21.0	380 2/	85	570	11
1200	16.7	380 2/	85	460	8
1450	11.0	380 2/	85	300	5
1500	10.0 3/	380 2/	85	0 3/	5

1/ Net discharge is less than 135 cfs minimum discharge.
2/ Limited by 380 cfs full gate turbine discharge (see Section D-4).
3/ Net head is less than 11.0 ft. minimum.

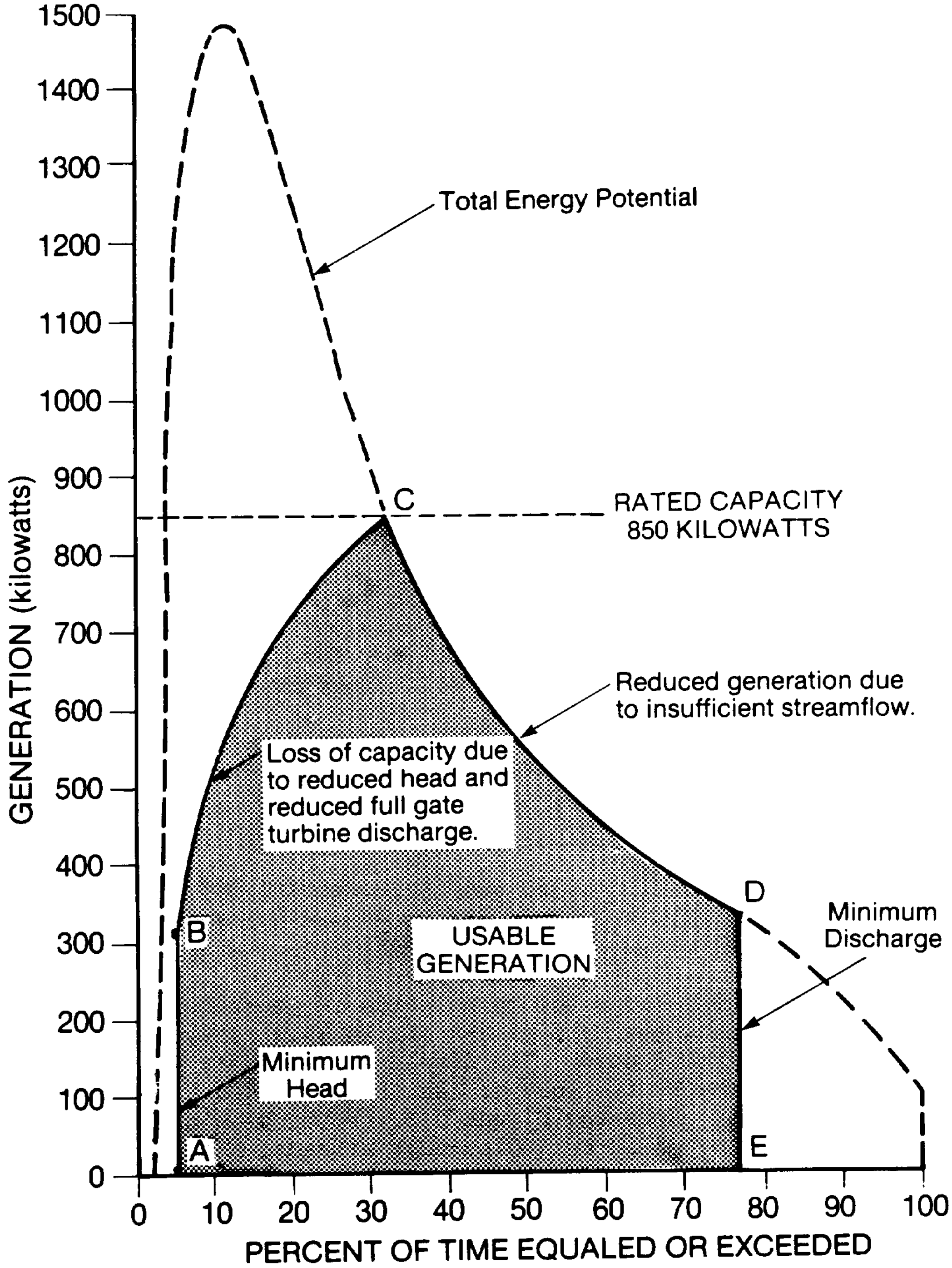

Figure D-1. Usable generation

those shown in Table D-1 except that discharge is limited by the 135 cfs minimum turbine discharge (to the right of line D-E on Figure D-1), the 380 cfs turbine full gate discharge (above line B-C), and the 11.0 foot minimum head (to the left of line A-B).

D-4. Effect of Fixed Overall Efficiency and Fixed Full Gate Discharge Assumptions.

a. The calculations described in Sections D-2 and D-3 are based on a fixed overall efficiency of 85 percent and the assumption that the full gate discharge at heads below rated head is equal to the rated discharge (380 cfs). In reality, turbine efficiency may vary considerably over the unit's operating range, and full gate discharge is always less than rated discharge at heads less than rated head. These factors can be accounted for by using a turbine performance curve in making power calculations.

b. In this section, the example project will be reevaluated using a sample performance curve for an adjustable blade turbine (Figure 39) from Bureau of Reclamation Engineering Monograph No. 20 (64), included here as Figure D-2. This curve shows only the turbine efficiency. The overall unit efficiency for each condition will be computed by applying a generator efficiency of 98 percent.

c. Figure D-2 shows a turbine efficiency of just over 88 percent when operating at rated head and rated discharge, for an overall efficiency of 86 percent. Applying the water power equation, the unit's rated output would then be

$$\text{Rated Capacity} = \frac{(380\text{ cfs})(31.0\text{ feet})(0.86)}{11.81} = 858\text{ kW.}$$

d. Table D-3 shows the computation of generation using Figure D-2. For example, the head at 250 cfs is 33.0 feet, which is 106 percent of the rated head. The discharge available for generation is 250 cfs minus the 20 cfs loss or 230 cfs, which is 60 percent of the rated discharge. Entering Figure D-2, the turbine efficiency corresponding to a head of 106 percent of rated head and a discharge of 60 percent of rated discharge would be about 92.0 percent. The overall efficiency would be (0.92)(0.98) = 90.2 percent. The generation would be

$$\text{Generation} = \frac{(230\text{ cfs})(33.0\text{ feet})(0.902)}{11.81} = 580\text{ kW.}$$

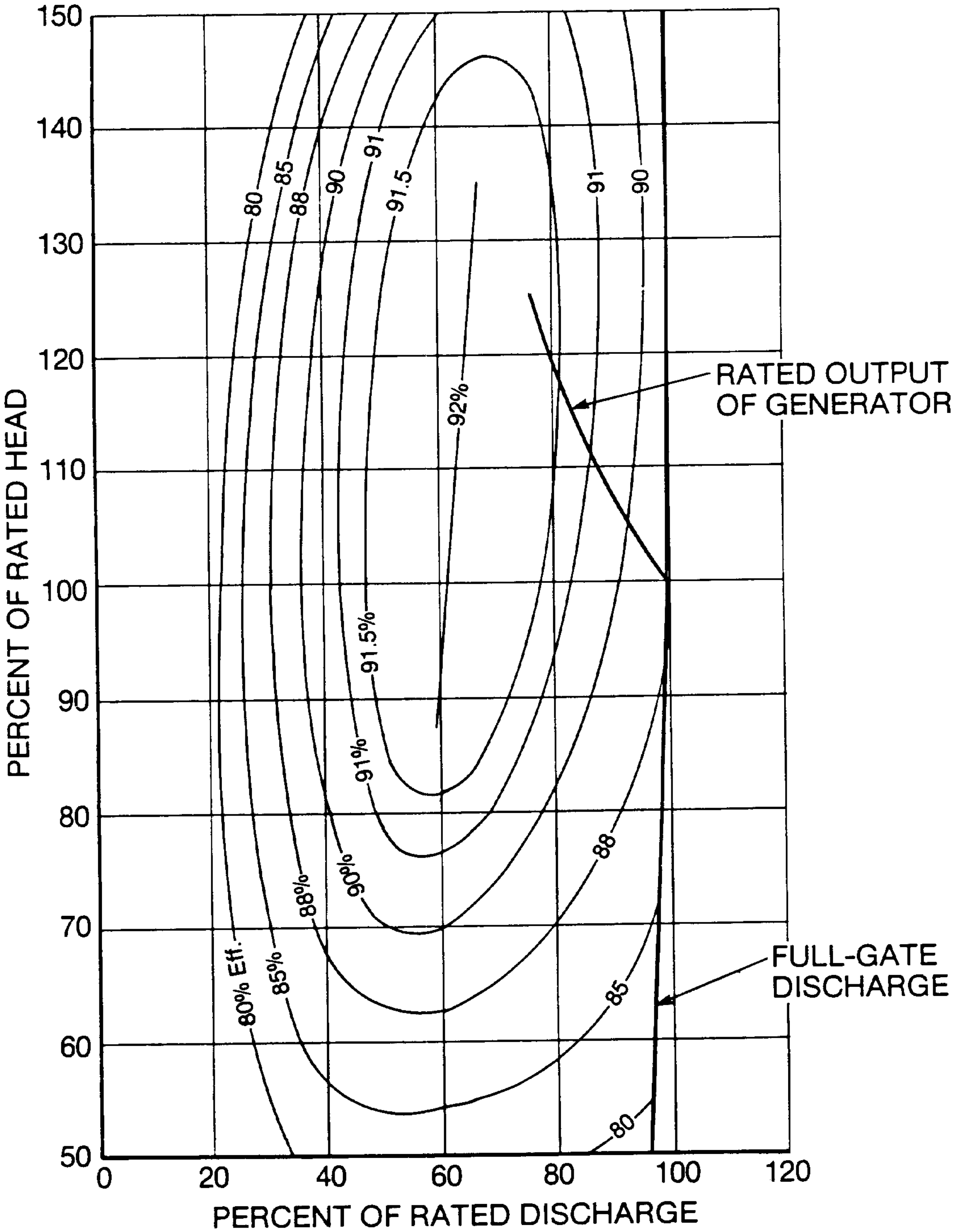

Figure D-2. Turbine performance curve-adjustable blade propeller turbine (Courtesy of U.S. Bureau of Reclamation)

TABLE D-3
Calculation of Usable Generation Using Turbine Performance Curve

Total Discharge (cfs)	Net Head (feet)	Percent Rated Head	Power Discharge (cfs)	Percent Rated Discharge	Overall Efficiency (percent) 1/	Power (kW)
60	35.0	113	40	10 2/	-	0
155	34.0	110	135	35	87.8	341
250	33.0	106	230	60	90.2	580
400	31.0	100	380	100	86.1	858
600	28.0	90	376 3/	99 3/	85.3	760
1000	21.0	68	367 3/	97 3/	82.8	540
1200	16.7	54	365 3/	96 3/	78.4	404
1450	11.0	35	357 3/	94 3/	68.9	229
1500	10.0	32 4/	-	-	-	0

1/ The product of the turbine efficiency from Figure D-2 and an assumed generator efficiency of 98 percent.
2/ Discharge below minimum discharge of 35 percent of rated discharge (135 cfs).
3/ Unit operating at full gate discharge below rated head (see paragraph D-4e).
4/ Head below minimum head (33 percent of maximum head, or 11.0 feet).

e. Similar computations would be made at other discharges. At heads of less than rated head, the full gate discharge curve would limit output. For example, the head corresponding to a discharge of 1200 cfs would be 16.7 feet, or 54 percent of rated head. Entering Figure D-2, the full gate discharge corresponding to 54 percent of rated head would be 96 percent of rated discharge, or (0.96)(380 cfs) = 365 cfs. The turbine efficiency at that point is 80.0 percent, giving an overall efficiency of 78.4 percent. The power output at that discharge would be

$$\text{Generation} = \frac{(365\ \text{cfs})(16.7\ \text{feet})(0.784)}{11.81} = 404\ \text{kW.}$$

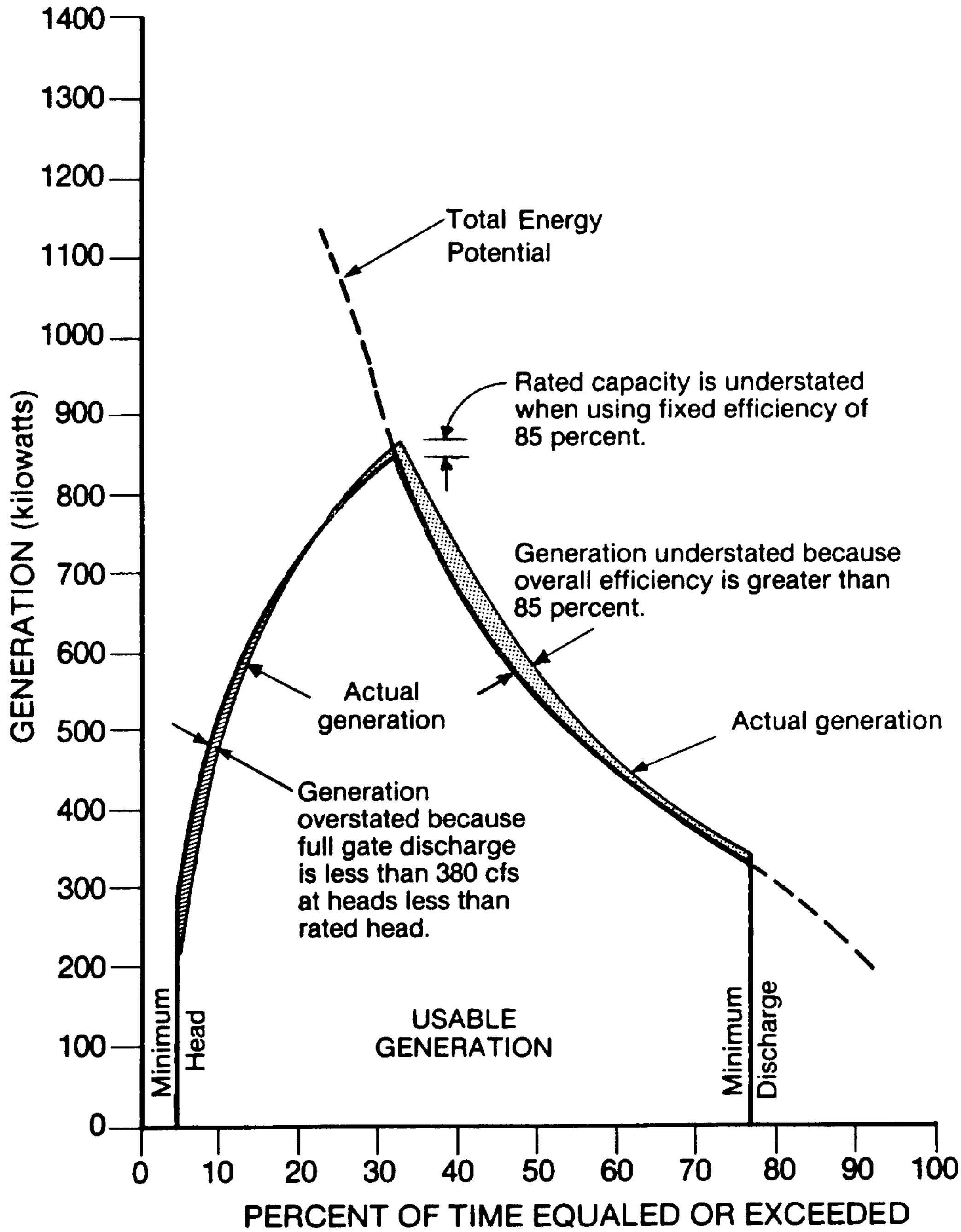

Figure D-3. Effect of using fixed efficiency of 85 percent instead of using turbine rating curve -- tubular turbine with movable blades

f. Figure D-3 shows a comparison of the generation using the performance curve (solid line) compared to that obtained using the simplified assumptions (dotted line). Note that in this example, using the simplified assumptions understates generation at discharges of less than 22 percent exceedance (600 cfs) because the actual efficiency in this range is greater than the assumed fixed efficiency of 85 percent and because the actual rated output is somewhat greater when the efficiency from the performance curve is used. At higher discharges, the simplified assumptions overestimated the generation, because the analysis fails to recognize that full gate discharge is less than rated discharge at heads less than rated head, and because the actual efficiency is less than 85 percent over most of this range.

g. In this example, the use of the simplified assumptions underestimates the average annual generation of the project by about two percent. However, this illustrates only one type of installation. Figure D-4 illustrates a similar analysis for a single Francis unit. In this case, the generation is overestimated by about two percent using the simplified assumptions. In other situations, the discrepancy could be less or it could be even greater. However, it is obvious that using the simplified assumptions is satisfactory for reconnaissance and preliminary feasibility study analyses. Note that the Francis turbine was selected for comparsion only to illustrate that the characteristics of different turbines vary. In reality, the operating head range of 11.0 to 33.9 feet is below the head range where Francis units are normally applied.

h. It should be noted that the above analysis is applicable only to the evaluation of a project where discharge is proportional to head. Refer to Sections 5-5e and 5-6k for a discussion of how to analyze projects where head is independent of discharge.

D-5. Peaking Flow-Duration Curve.

a. Sections D-5 and D-6 provide the backup for Section 5-7l and Figures 5-24 and 5-25. The peaking flow duration curve shown on Figure 5-24 was derived using the usable flow duration curve shown on the same figure and the peaking discharge pattern shown on Figure 5-23. A required minimum continuous discharge of 150 cfs is assumed, part of which will be met by the 20 cfs leakage loss. Any remaining flow above the 150 cfs minimum will be available for peaking.

b. Figure 5-23 shows that the peaking discharge is to be provided for a minimum of eight hours per day. To define the peaking flow-duration curve, a series of calculations were done at various

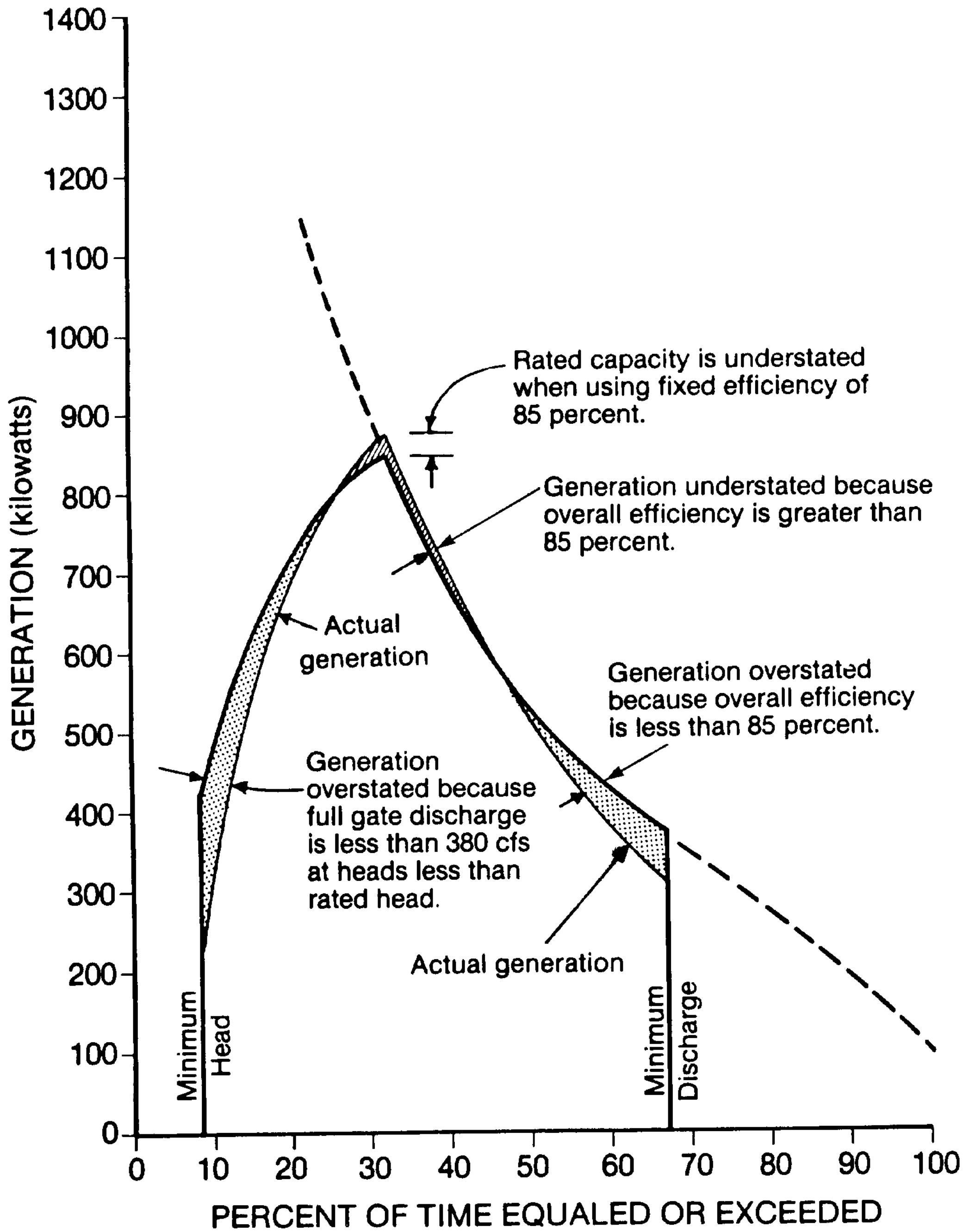

Figure D-4. Effect of using fixed efficiency of 85 percent instead of using turbine rating curve -- Francis turbine average discharge levels. For example, for an average daily discharge of 180 cfs, the peaking discharge would be computed as follows.

Total average daily discharge = 180 cfs

Average net discharge available for generation =
(180 cfs - 20 cfs) = 160 cfs

The minimum discharge that must be maintained at all times is 150 cfs, of which 20 cfs would be supplied from the leakage losses. This leaves 130 cfs which must be met from the 160 cfs average net discharge available for power generation. If 130 cfs is allocated to maintaining the minimum discharge, the remaining (160 cfs - 130 cfs) = 30 cfs daily average discharge is available to be used for peaking, and this is to be released if possible in the 8-hour peak demand period. The 30 cfs daily average discharge, when concentrated in the peak demand period, would equate to a peaking discharge of

(30 cfs)(24 hours)/(8 hours) = 90 cfs.

The total discharge available for generation would then be (130 cfs + 90 cfs) = 220 cfs during the eight peak demand hours and 130 cfs during the remainder of the day. Adding in the 20 cfs loss, the total project discharge would then be 240 cfs in the peak demand hours and 150 cfs during the remainder of the day.

c. At a total discharge of 233 cfs, the plant will be capable of operating at the total rated capacity of 380 cfs for eight hours per day, while maintaining the minimum discharge the remainder of the time. At higher discharges, the number of hours the plant can operate at rated capacity will increase, up to the maximum of 24 hours per day at 400 cfs (380 cfs rated discharge plus 20 cfs loss). At flows greater than 400 cfs, the peaking flow-duration curve would be identical to the average daily flow-duration curve.

d. Table D-4 summarizes these calculations.

D-6. Peaking Capacity-Duration Curve.

a. For pure run-of-river projects, the peaking capacity-duration curve would be identical to the generation-duration curve for the peak demand months, and dependable capacity would be computed as described in Section 5-7k.

b. If pondage were added to the example project, the capacity-duration curve would be modified to reflect the regulation of the project for peaking. Section D-5 describes the computation of

TABLE D-4
Total Discharge When Peaking

Percent Exceedance	Average Daily Discharge Total (cfs) 6/	Average Daily Discharge Avail. for Generation (cfs) 1/	Avail. for Peaking (cfs) 2/	Hours on Peak	Discharge in Peak Hours Peaking (cfs) 3/	Discharge in Peak Hours Total (cfs) 4/
70	152	132	2	8.0	6	155
65	155	135	5	8.0	15	165
60	160	140	10	8.0	30	180
50	180	160	30	8.0	90	240
40	225	205	75	8.0	225	375
38.5	233	213	83	8.0	250	400
30	300	280	150	14.4	250 5/	400
22	400	380	250	24.0	250 5/	400

1/ Total average daily discharge minus 20 cfs loss.
2/ Total average daily discharge minus 150 cfs minimum discharge.
3/ (Average daily discharge available for peaking x 24 hours) divided by number of hours on peak.
4/ Peaking discharge plus 150 cfs minimum discharge.
5/ Limited to 250 cfs by the 380 cfs hydraulic capacity.
6/ From Figure 5-24 (the average daily flow-duration curve).

discharge in the peak load hours, based on the daily operating pattern shown on Figure 5-23. Figure 5-24 (incorporating the solid line between 22 and 70 percent plant factors) shows the resulting peaking flow-duration curve. Using the data from this curve, the peaking capacity would be computed for a series of exceedance levels in the same manner as was described in Sections D-2 and D-3. The calculations for the example problem are shown in Table D-5, and the resulting curve is plotted as Figure 5-25. In order to simplify the example, a constant efficiency of 85 percent was assumed for all discharge levels and no adjustment was made for reduced full gate discharge at heads less than rated head (see Sections 5-7n and D-4).

c. When pondage is used for peaking, there is a loss of head when the pondage is drafted. It is assumed that two feet of pondage is available at the example project between El. 266.0 feet and El. 268.0 feet (normal full pool). When the pondage is being used, the amount of drawdown varies over the course of the day. Referring to Figure 5-23, the reservoir would be full when peaking starts at 8 am, and there would be no loss of head. At 4 pm, when the peaking cycle is complete, the reservoir would be at its minimum level. Between 4 pm and 8 am the next morning, the reservoir would fill again. Precise estimates of the amount of head loss due to reservoir drawdown could be made for each average daily discharge level by doing hourly reservoir routings (see Section 6-9). However, an approximate estimate can be made by assuming an average drawdown of 30 percent over the discharge range where the pondage would be used (between 22 and 70 percent exceedance in the case of the example problem (see Figure 5-24)). The 30 percent average drawdown accounts for the fact that the average daily drawdown would vary from zero at 22 percent exceedance (because the plant is operating at full hydraulic capacity 24 hours per day) to one foot at 40 percent exceedance (when the plant is using the full two feet of pondage) and back to zero at 70 percent (when the plant is receiving the 150 cfs minimum discharge for 24 hours per day). The computations shown on Table D-5 reflect an average drawdown of 30 percent, or (0.30 x 2.0 ft.) = 0.6 ft.

d. Note that peaking capacity drops off at total discharges greater than 400 cfs (22.0 percent exceedance) due to falling head. As a result, plotting peaking capacity versus the percent exceedance values from Table D-5 would not produce a true duration curve. In plotting Figure 5-25, however, the data was converted to true duration curve format (see Section D-2b).

D-7. Turbine Efficiency.

a. This section provides the backup for Section 5-7n. Table D-6 summarizes the calculations required to derive the turbine efficiency-discharge curve shown in Figure 5-27. Turbine discharges and corresponding heads are obtained from the flow-duration curve (Figure 5-15) and the head-discharge curve (Figure 5-16). These figures are converted to percent of rated discharge (Q_R) and percent of rated head (H_R) values. In this example, a corresponding value of turbine efficiency is taken from the movable blade propeller turbine performance curve (Figure D-2). The overall efficiency is computed by applying a generator efficiency of 98 percent. The resulting efficiencies are plotted as Figure 5-27 (see Sections 5-7n(4) and (5)).

b. At heads less than the rated head of 31.0 feet, the net turbine discharge is limited by the full gate discharge (see Section D-4). The turbine efficiencies in this range can be determined from Figure D-2 by reading the efficiency values on the full gate discharge line corresponding to the respective percent of rated head values.

TABLE D-5
Peaking Capacity

Percent Exceedance	Total Discharge in Peak Hours (cfs) 6/	Net Head 1/ (feet)	Net Peak Discharge (cfs) 2/	Efficiency (%)	Peaking Capacity (kW)
100.0	110	34.2	90 3/	85	0 3/
70.0	155	33.4	135	85	320
65.0	165	33.2	145	85	350
60.0	180	33.1	160	85	380
50.0	240	32.6	220	85	520
40.0	375	30.9	355	85	790
38.5	400	30.4	380 4/	85	830
30.0	400	30.4	380 4/	85	830
22.0	400	31.0	380 4/	85	850
14.0	600	28.0	380 4/	85	770
9.0	800	24.7	380 4/	85	680
1.5	1450	11.0	380 4/	85	300
1.0	1600	9.0 5/	380 4/	85	0 5/

1/ Head between 22 and 70 percent exceedance incorporates an average head loss of 0.6 feet to account for pondage drawdown (see Section D-6c).
2/ Total discharge in peak hours minus 20 cfs losses.
3/ Net discharge is less than the 135 cfs minimum turbine discharge.
4/ Output limited by the 380 cfs turbine full gate discharge.
5/ Net head is less than the 11.0 foot minimum head.
6/ From Figure 5-24 (peaking flow-duration curve).

For example, for a total discharge of 1000 cfs, the net head is equal to 21.0 feet, or 0.68 H_R. From Figure D-2, turbine efficiency at 0.68 H_R would be 84.5 percent, and the overall efficiency would be (0.845)(0.98) = 82.8 percent.

TABLE D-6
Turbine Efficiency Curve Calculations

Total Discharge (cfs)	Net Head (feet)	Percent of Rated Head	Net Turbine Discharge (cfs) 1/	Percent of Rated Discharge	Turbine Efficiency (percent)	Overall Efficiency (percent) 3/
60	35.0	113	40	- 2/	-	-
155	34.0	110	135	35	89.6	87.8
250	33.0	106	230	61	92.0	90.2
400	31.0	100	380	100	88.0	86.2
500	29.2	94	376	99	88.0	86.2
600	28.0	90	376	99	87.6	85.8
800	24.7	80	372	98	86.8	85.1
1000	21.0	68	367	97	84.5	82.8
1200	16.7	54	365	96	80.0	78.4
1450	11.0	35	361	95	70.3	68.9
1600	8.1	26 4/	-	-	-	-

1/ Total discharge minus 20 cfs loss; limited by full gate turbine discharge (see Section D-7b).
2/ Net flow less than the 135 cfs minimum turbine discharge.
3/ (Turbine efficiency) x (98 percent generator efficiency).
4/ Head is less than the minimum turbine operating head of 11.0 feet.