Chapter 5
Uncertainty of Stage-Discharge Function

5-1. Overview of Stage-Discharge Uncertainty

a. The determination of stage-discharge uncertainty requires accounting for the uncertainty associated with factors affecting the stage-discharge relationship. These factors include bed forms, water temperature, debris or other obstructions, unsteady flow effects, variation in hydraulic roughness with season, sediment transport, channel scour or deposition, changes in channel shape during or as a result of flood events, as well as other factors. In some instances, uncertainty might be introduced into the stage-discharge curve due to measurement errors from instrumentation or method of flow measurement, waves, and other factors in the actual measurement of stage and discharge.

b. Numerical models are commonly issued in project studies. While most studies use one-dimensional models, a number of studies now use multi-dimensional modeling to simulate flows in both the without- and with-project conditions. Models are limited by the inherent inability of the theory to model exactly the complex nature of the hydraulic processes. Data used in the models are also not exact, introducing errors in the model geometry and coefficients used to describe the physical setting. Many of the factors which determine stage-discharge uncertainty and which are estimated for modeling purposes are time-dependent, both seasonally as well as during a flow event. Many of the factors are also spatially variable both laterally and longitudinally in the channel and associated floodplain. In general, the more complex the flow conditions, the greater the need to use models that replicate the significant physical processes.

c. Several different methods can be used to estimate the stage-discharge uncertainty for a stream reach. Where possible, each should be applied to provide a check on uncertainty estimates derived from the other methods. The most applicable method will depend on the data available and the method used in project studies. Stage-discharge uncertainty can be evaluated for contributing factors, or for each factor individually. When the factors are analyzed separately, care must be taken to ensure that the resulting uncertainty from combining the factors is reasonable. An example would be a stream where floods always occur significantly after ice melt but where the ice creates significant stage increases when present. In this case the uncertainty for ice should not be imposed in addition to the uncertainty due to increased resistance from early summer vegetation. Any correlation of separate factors should also be considered in the analysis and accounted for in the combination of individual uncertainties.

5-2. Development of the Stage-Discharge Function

a. Stage-discharge rating curves are developed by several methods. The most common and precise practice is to measure stream flow and stage simultaneously and to plot discharge versus stage. U.S. Geological Survey (USGS) (1977) provides a technical procedure for measuring stage and velocity at a given channel section and the development of stage-discharge ratings curves. The stage-discharge function is developed as the best fit curve through the observed stage-discharge measurements. Where these gauge ratings are available, analysis of the measured data versus the rating curve can provide insight into the natural variability at the gauged location.

b. Gauged records may be used to directly estimate stage-discharge uncertainty. The gauged data are assembled, adjusted to remove non-stationary effects of datum changes, gauge location changes, and stream aggradation or degradation. Statistical outlier tests may be used to examine data anomalies. Engineering judgement is needed to identify and handle correctly occurrences of coincidental effects such as ice jams, debris blockages, etc.

c. Figure 5-1 is a plot of stage discharge data for a stream with more than 70 years of record where non-stationary effects have been removed from the record. The record is broken into sections to represent three zones of flow. The first zone is the within-bank flow zone; the second is measured-out-of-bank flow zone (or bank full to the highest measured flow), and the third the rare event zone where occasionally an event may have been measured. A minimum of 8 to 10 measurements out of banks is normally required for meaningful results. Unfortunately, it is not common to have measured events in the range of interest for flood damage reduction studies.

d. The method described in USGS (1977) uses an equation of the form:

$$Q = C\,(G - e)^b \tag{5-1}$$

to describe the stage discharge relationship where Q is discharge, G is the stage reading, and C, e, and b are coefficients used to match the curve to the data. It should

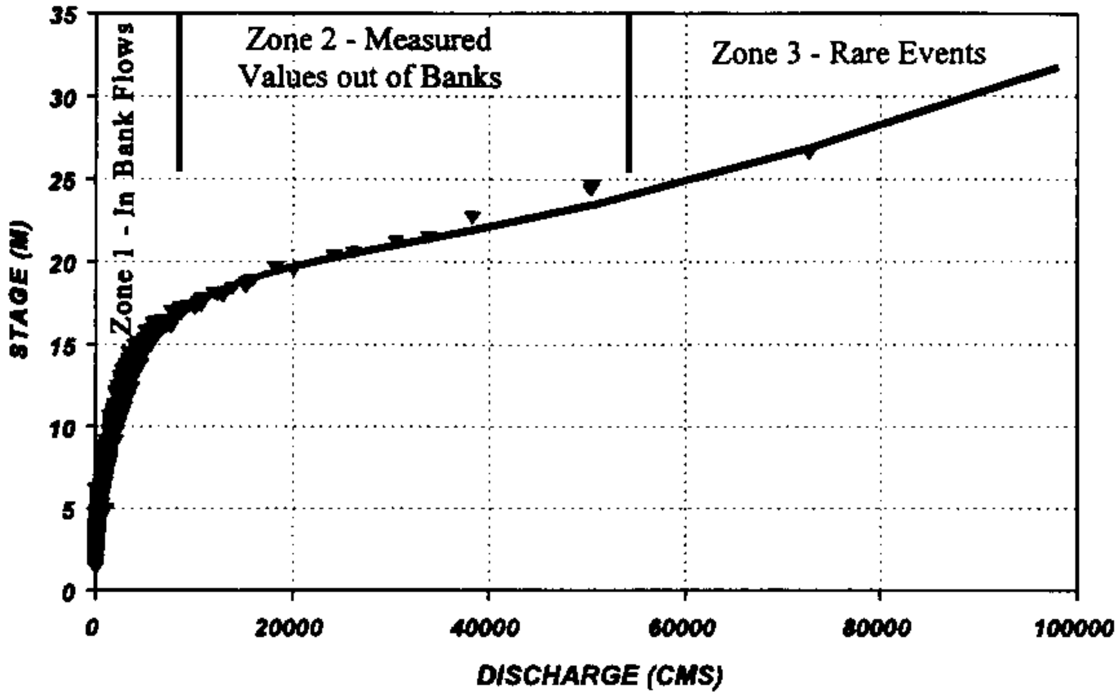

Figure 5-1. Stage-discharge plot showing uncertainty zones, observed data, and best-fit curve

be noted that the value of *b* is usually between 1.3 and 1.8.

e. An alternate equation reported by Freeman, Copeland, and Cowan (1996) is an exponential curve with decreasing exponents:

$$STAGE = a + bQ^{1/2} + cQ^{1/3} + dQ^{1/4} + eQ^{1/5} + fQ^{1/6} \quad (5\text{-}2)$$

where STAGE is in feet, Q is flow in cfs, and *a* through *f* are coefficients determined by a best fit algorithm to fit the equation to the data. This equation yielded an R^2 better than 0.80 for 115 rivers and streams out of 116 analyzed. Additionally, for 75 percent of the streams the R^2 was better than 0.97. Equation 5-2 does not accurately predict very low flows but these are not generally of concern in flood damage reduction studies.

5-3. Determination of Stage-Discharge Uncertainty for Gauged Reaches

a. The measure used to define the uncertainty of the stage-discharge relationship is the standard deviation. The stage residuals (difference between observed and rating function values) provide the data needed to compute uncertainty. It is recommended that only data values for flows above bank-full be used, since low flows are generally not of interest in flood studies. Note that the objective is to calculate uncertainty in stage, not discharge. These residuals characterize the uncertainty in the stage-discharge function and can be described with a probability distribution. The standard deviation of error (or square root of the variance) within a zone (or for the whole record) *S* can be estimated as:

$$S = \sqrt{\frac{\sum_{i=1}^{N} (X_i - M)^2}{N - 1}} \quad (5\text{-}3)$$

where X_i = stage for observation *I* which corresponds with discharge Q_i; *M* = best-fit curve estimation of stage corresponding with Q_i; and *N* = number of stage-discharge observations in the range being analyzed.

b. The distribution of error from the best-fit lines can vary significantly from stream to stream. The

Gaussian (normal) distribution can be used for the description of many rivers but not all. Freeman, Copeland, and Cowan (1996) found that for many streams, the data were much more concentrated near the mean value and the central portion of the distribution was much narrower than is the case for a normal distribution. On other streams, the distribution was markedly skewed. The gamma distribution can represent a wide range of stream conditions from normal to highly skewed and is suggested for use in describing stage uncertainty.

c. The gamma distribution is defined by a scale parameter and a shape parameter curve. Once the scale and shape parameters are known, the skew is fixed (McCuen and Snyder 1986). The values for the shape and scale parameters may be computed from the sample estimates of mean and variance.

d. For the gamma distribution, the standard deviation of the uncertainty is defined as:

$$S = \sqrt{\frac{\kappa}{\lambda^2}} \tag{5-4}$$

where κ = the shape parameter and λ = the scale parameter for the distribution and are simple functions of the sample parameters.

e. Where bank-full elevations and discharges are not available, 20 percent of the daily mean discharge exceedance value may be used instead. Leopold (1994) recommends the 1.5-year recurrence interval in the annual flood series for the approximate location of bank-full. For the streams reported by Freeman, Copeland, and Cowan (1996), there was at times a significant difference in uncertainty between the total record and the flows greater than the 20-percent exceedance flow, as shown in Figure 5-2.

f. If the gauging station is representative of the study reach, then the gauge results are representative. If the gauged results are not representative, other reaches must be analyzed separately.

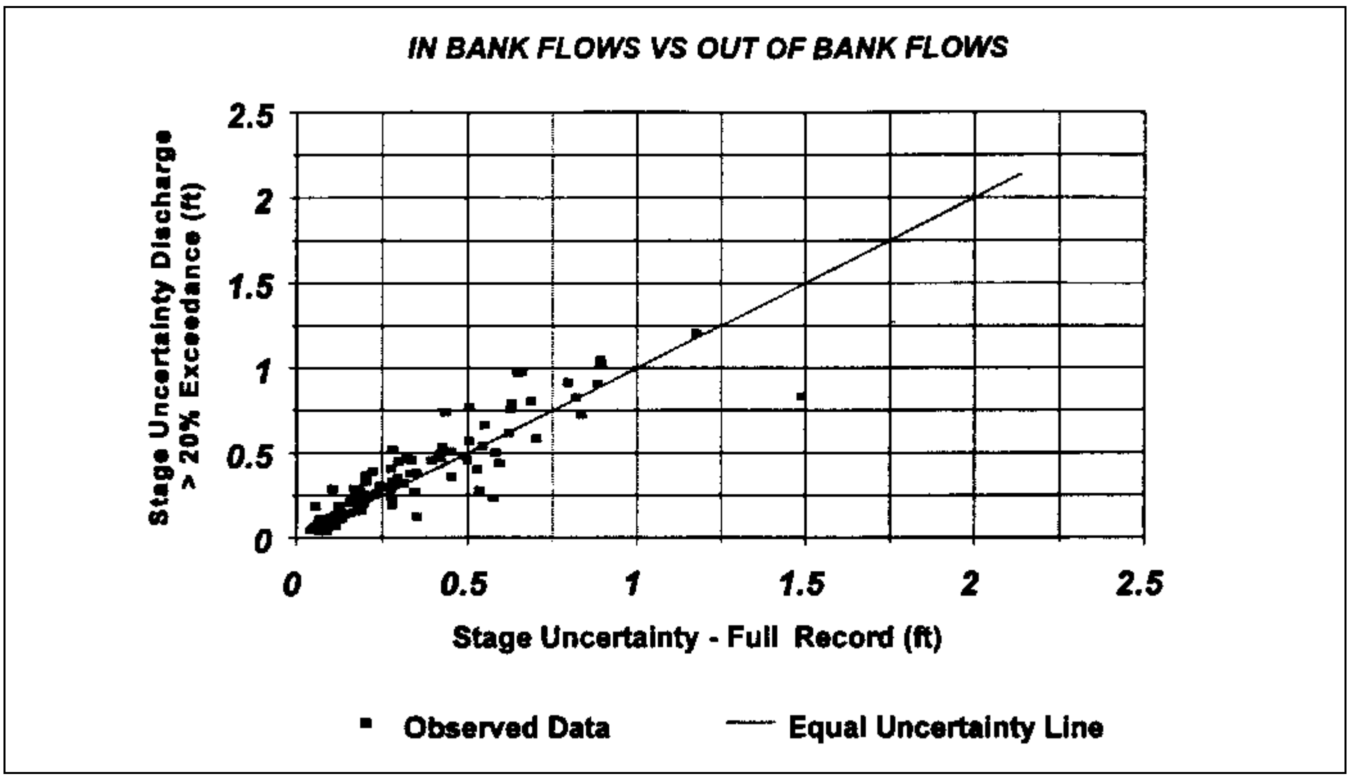

Figure 5-2. Stage-discharge uncertainty for flows greater than 20 percent exceedance compared with full record uncertainty

5-4. Uncertainty in Stage for Ungauged Stream Reaches

Efforts to develop correlations between stage uncertainty and measurable stream parameters have met with modest success (Freeman, Copeland, and Cowan 1996). The correlation between slope and uncertainty can be used as an upper bound estimate in the absence of other data. Figure 5-3 shows the standard deviation of uncertainty based on the Gamma distribution for U.S. streams studied. Using this same data, Equation 5-5 can predict the uncertainty in river stages with R^2 of 0.65.

$$S = [0.07208 + 0.04936\ I_{Bed} - 2.2626x10^{-7}A_{Basin} + 0.02164\ H_{Range} + 1.4194 \times 10^{-5}Q_{100}]^2 \qquad (5\text{-}5)$$

where S = the standard deviation of uncertainty in meters, H_{Range} = the maximum expected or observed stage range, A_{Basin} = basin area in square kilometers, Q_{100} = 100-year estimated discharge in centimeters, and I_{Bed} is a stream bed identifier for the size bed material which controls flow in the reach of interest from Table 5-1. Equation 5-5 is not physically based but can give reasonable results for ungauged reaches using data that can be obtained from topographic maps at site reconnaissance, an estimate of the expected 100-year flow.

Table 5-1
Bed Identifiers

Material	Identifier
Rock/Resistant Clay	0
Boulders	1
Cobbles	2
Gravels	3
Sands	4

5-5. Uncertainty in Stages for Computed Water Surface Profiles

a. Computed water surface profiles provide the basis for nearly all stage-discharge ratings needed for the “with-project” conditions of Corps flood damage

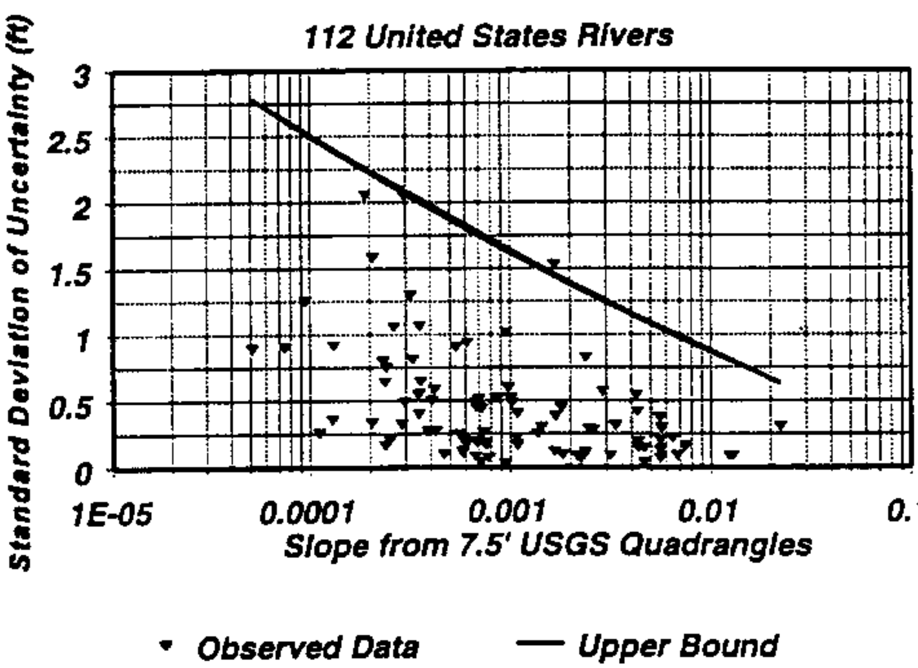

Figure 5-3. Stage-discharge uncertainty compared with channel slope from USGS 7.5-in. quadrangles, with upper bound for uncertainty

reduction studies. Published methods and guidelines for interpreting the accuracy, and thus uncertainty in computed stages, are few. For now, estimated uncertainties must be based on analytical studies of gauged ratings (where they are available), on methods described in Paragraph 5-4 for ungauged reaches, interpretation of the success (or lack thereof) of model adjustment/validation studies, and sensitivity studies designed to determine the stability/ robustness of computed profiles. Professional judgement is required to validate the reasonable limits for uncertainty. Uncertainty in stage-discharge ratings will be the synthesized result of several analyses.

b. The uncertainty in stage due to model and data limitations is best minimized by selecting the most appropriate model for the situation under study. Subsequent to model selection, model adjustment and calibration studies using observed flood data are performed to further minimize uncertainty in results from model applications for study conditions.

c. Research at the Hydrologic Engineering Center (HEC) and the U.S. Army Engineer Waterways Experiment Station (WES) (USACE 1986; Freeman, Copeland, and Cowan 1996) provides information for estimating uncertainty in water surface profiles obtained when using a gradually varied flow model. The standard deviation of the normally distributed errors in the estimated stages are based on topographic information and confidence in estimated Manning's *n* value as shown in Table 5-2.

d. Uncertainty due to natural variations as determined from gauged data, from Figure 5-3, or from Equation 5-5 should be combined with the values from Table 5-2 or values obtained from methods described later in this chapter to obtain an estimate of total uncertainty in a modeled reach of river as follows:

$$S_t = \sqrt{S_{natural}^2 + S_{model}^2 +} \tag{5-6}$$

where S_t is standard deviation of the total uncertainty, $S_{natural}$ is natural uncertainty, and S_{model} is modeling uncertainty. In general, the standard deviation of stage uncertainty could be expected to increase with decrease in data availability, accuracy, and model adjustment/ validation results. Stage uncertainty may also increase with increased complexity of analysis.

5-6. Analysis Complexity

While the majority of water surface profile analyses are within the capabilities of such programs as HEC-2 (USACE 1985), there is need, at times, for more complex analysis. For streams that have rapidly varying flows, or are subject to tides, an unsteady flow analysis may be needed. Sand bed streams may require mobile boundary modeling. Complex flow fields in unusual floodplains or estuaries may require multi-dimensional (and in a few cases, unsteady) flow analysis. In such cases a stage discharge rating for the highest stages commensurate with flow conditions of interest should be developed. The uncertainty associated with the rating is interpreted from the analysis results. Often, sensitivity analysis as discussed below is an appropriate approach to such determination. If it is not possible to develop a rating from the results, then analysis dealing directly with stage-frequency is likely to be necessary.

5-7. Sensitivity Analysis and Professional Judgement

a. One approach to estimating stage uncertainty that can always be used is to estimate the upper and lower bounds on stage for a given discharge and convert the

Table 5-2
***Minimum* Standard Deviation of Error in Stage**

	Standard Deviation (in feet)	
Manning's n Value Reliability[1]	**Cross Section Based on Field Survey or Aerial Spot Elevation**	**Cross Section Based on Topographic Map with 2-5' Contours**
Good	0.3	0.6
Fair	0.7	0.9
Poor	1.3	1.5

[1] Where good reliability of Manning's *n* value equates to excellent to very good model adjustment/validation to a stream gauge, a set of high water marks in the project effective size range, and other data. Fair reliability relates to fair to good model adjustment/ validation for which some, but limited, high-water mark data are available. Poor reliability equates to poor model adjustment/validation or essentially no data for model adjustment/validation.

stage range to the needed uncertainty statistic. For example, 95 percent of the error range would be encompassed by stages two standard deviations above and below the mean. Professional judgement could thus be applied to estimate the "reasonable" upper and lower bounds of stage, and the standard deviation estimated as the total range divided by 4. Sensitivity analysis in which reasonable likely combinations of upper and lower bound estimates of model parameter values are used to obtain a range of predicted stages for a given discharge could augment or serve as an alternative to the range determined from professional judgement. Figure 5-4, derived by WES as an extension of the HEC analysis, can be used as a guide to estimating the reasonable bounds to the Manning's *n* value model parameter in sensitivity studies. Figure 5-5 is an example that shows high-water marks and upper and lower limits from sensitivity analysis.

b. The range between the upper and lower limit water stages is used to estimate the standard deviation of stage uncertainty. The mean reach profile differences may be estimated by inspection or determined from cross-sectional profile elevation differences, weighted by distances between cross sections, and averaged over the entire study reach. If the stage difference between the upper and lower limits is taken to be the "reasonable bounds," e.g., 95 percent of the stage uncertainty range, then the standard deviation may be estimated by the following equation:

$$S = \frac{E_{mean}}{4} \tag{5-7}$$

where E_{mean} = mean stage difference between upper and lower limit water surface profiles as shown in Figure 5-5.

c. It would be possible to sketch or estimate the profile range that encompasses the "majority" of the high water marks, compute the difference, and calculate the standard deviation using Equation 5-6. If the "majority" means accounting for two thirds of the marks, Equation 5-6 is used with a divisor of 2 instead of 4. The high-water marks should also be used as a check on the reasonableness of model parameters used in a sensitivity analysis.

5-8. Stage Uncertainty for With-Project Conditions

The discussion has focused on estimating stage uncertainty for the "without-project" condition. The stage

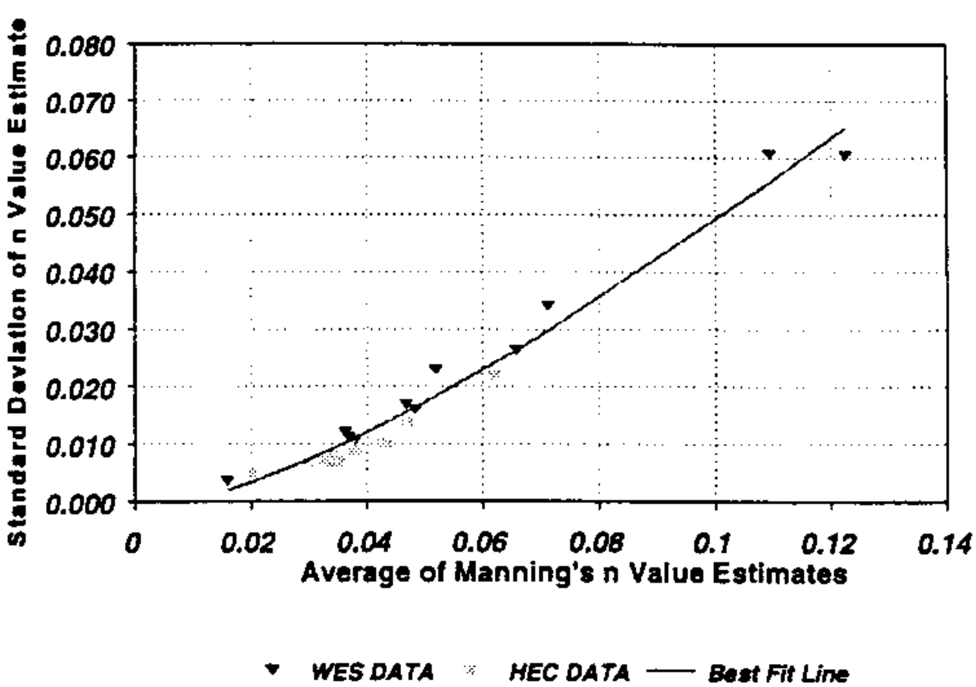

Figure 5-4. Uncertainty of Manning's *n* value estimates based on estimated mean values

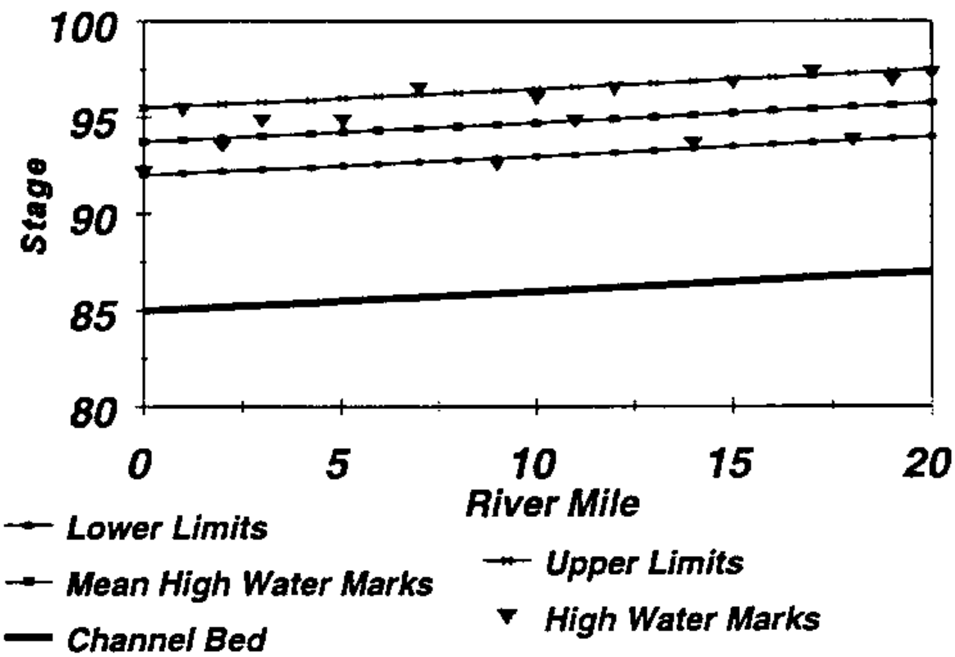

Figure 5-5. Water surface profiles from sensitivity analysis compared with high-water marks from field data

uncertainty for the with-project condition must also be estimated. If the flow conditions and conveyance are expected to be markedly different from the without-project condition, analysis as suggested previously in this chapter is appropriate to estimate stage uncertainty. If flow conditions and conveyance are expected to remain similar, then stage uncertainty may be taken to be the same or similar to the without-project condition.