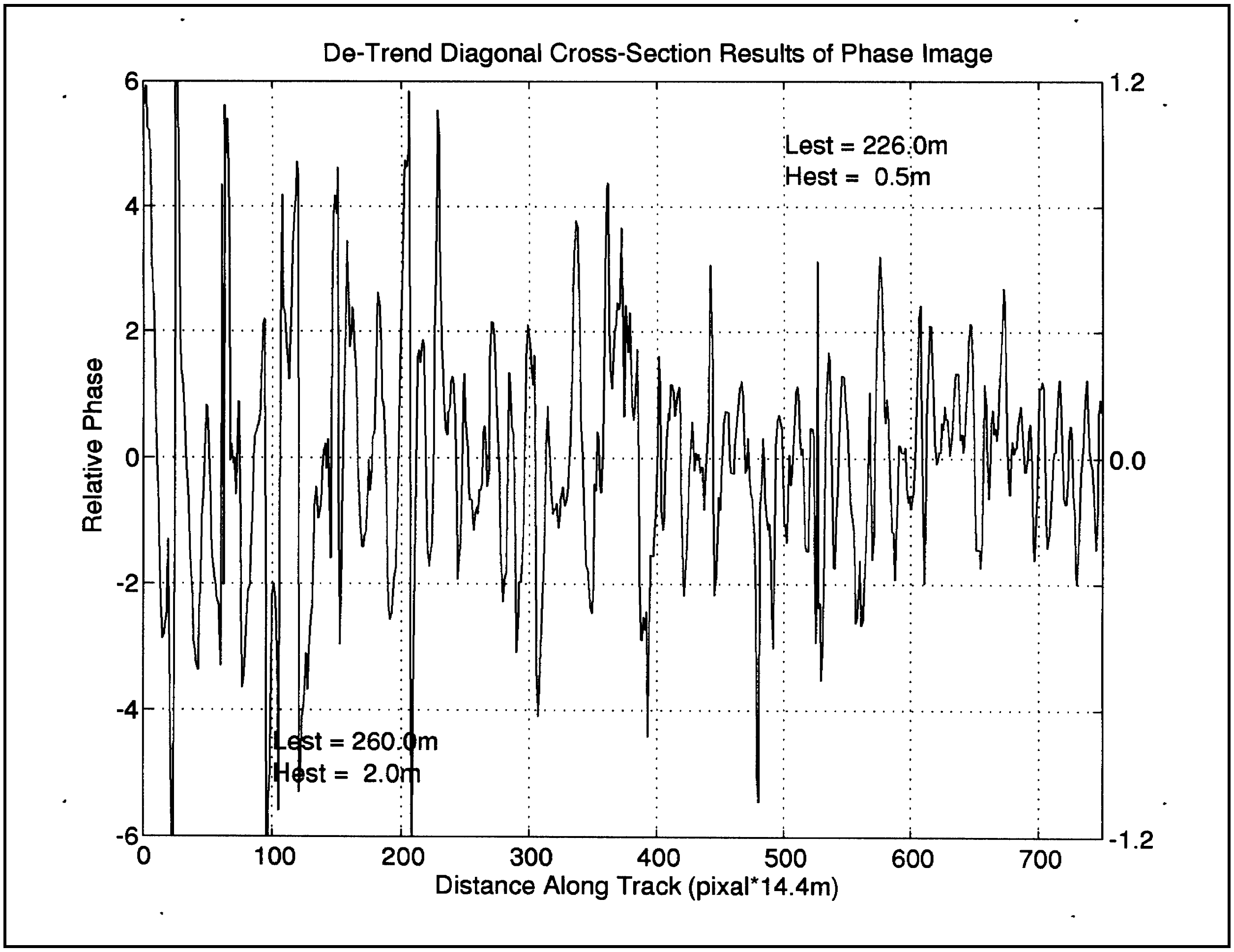

Figure II-1-23. Measured sea surface velocity in the entrance to San Francisco Bay

If a recorder were to measure waves at a fixed location on the ocean, a non-repeating wave profile would be seen and the wave surface record would be rather irregular and random (Figure II-1-23). Although individual waves can be identified, there is significant variability in height and period from wave to wave. Consequently, definitions of wave height, period, and duration must be statistical and simply indicate the severity of wave conditions.

(4) Wave profiles are depicted in Figure II-1-24 for different sea conditions. Figure II-1-25 shows a typical wave surface elevation time series measured for an irregular sea state. Important features of the field-recorded waves and wave parameters to be used in describing irregular waves later in this section are defined in Figures II-1-26 and II-1-27. We note that the sea state in nature during a storm is always short-crested and irregular. Waves that have traveled far from the region of generation are called *swells*. These waves have much more limited range of variability sometimes appearing monochromatic and long-crested.

(5) This part of Part II-1 will develop methods for describing and analyzing natural sea states. The concept of *significant wave height*, which has been found to be a very useful index to characterize the heights of the waves on the sea surface, will be introduced. *Peak period* and *mean wave direction* which characterize the dominant periodicity and direction of the waves, will be defined. However, these parameterizations of the sea surface in some sense only index how big some of the waves are. When using irregular wave heights in engineering, the engineer must always recognize that larger and smaller (also longer and shorter) waves

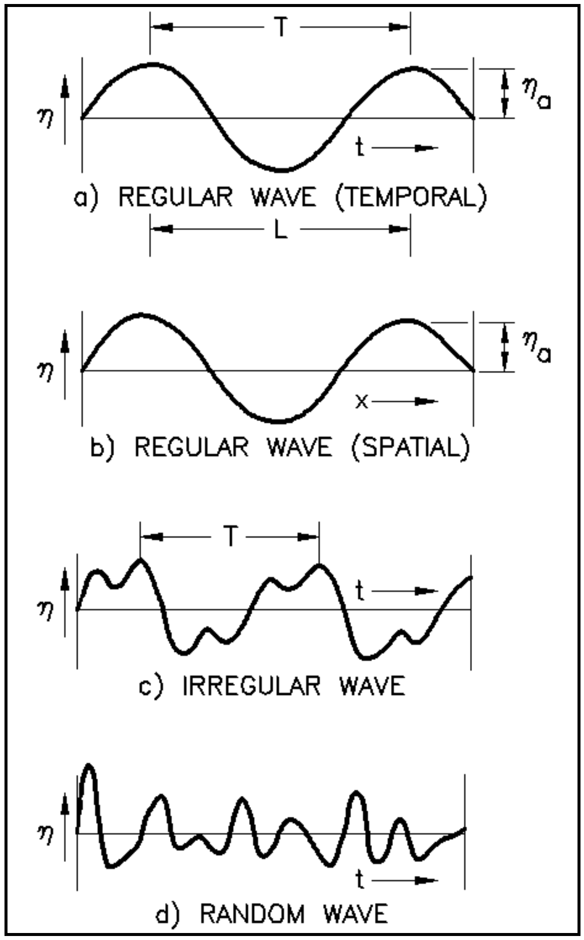

Figure II-1-24. Representations of an ocean wave

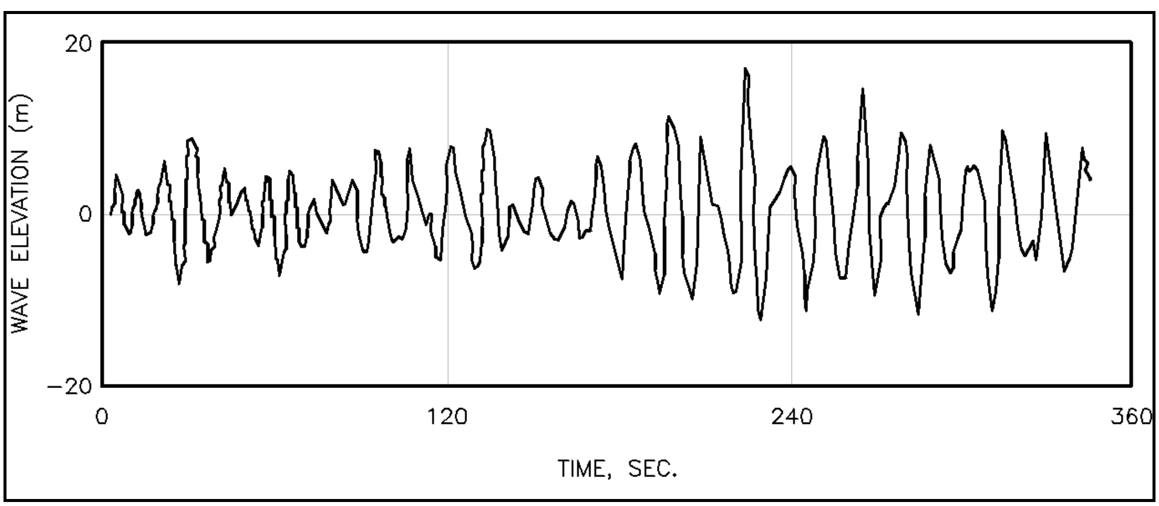

Figure II-1-25. Wave profile of irregular sea state from site measurements

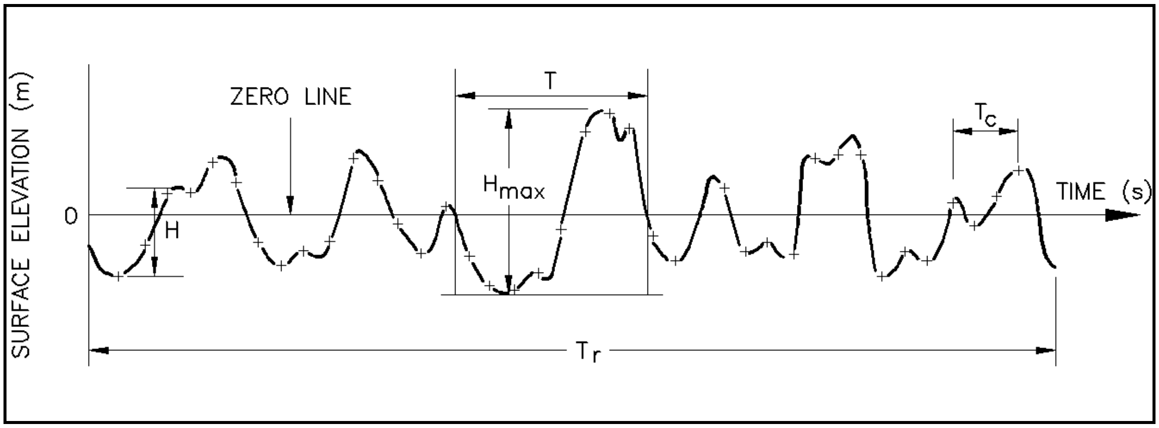

Figure II-1-26. Definition of wave parameters for a random sea state

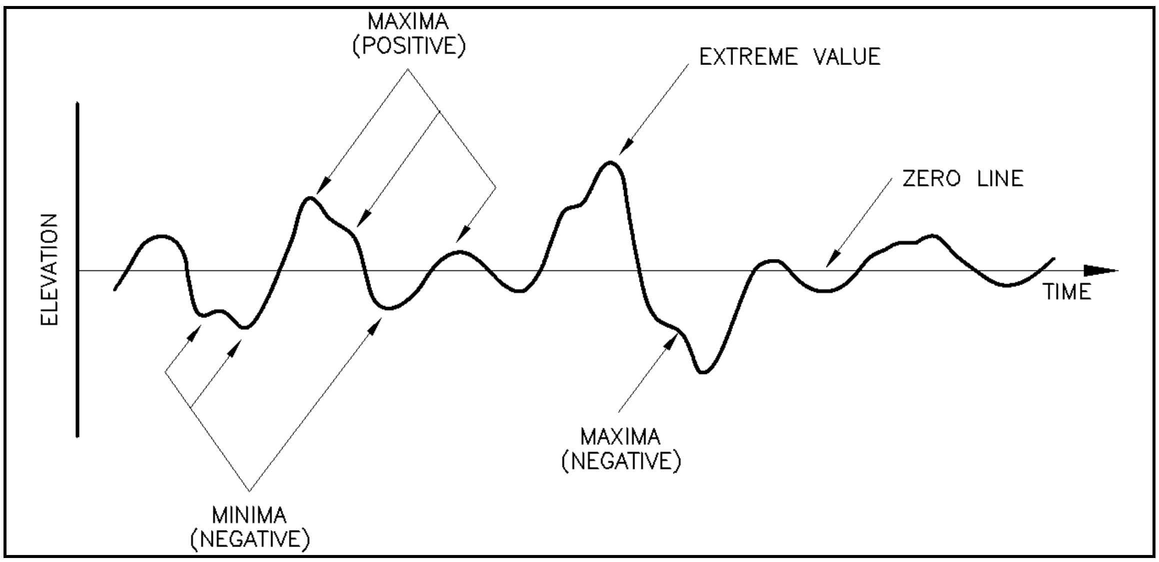

Figure II-1-27. Definition sketch of a random wave process (Ochi 1973)

are present. The monochromatic wave theories described in the first part of this chapter will be seen to have two major uses. One use is to estimate the kinematics and dynamics associated with a wave with the significant wave height, peak period, and direction. The other is when an individual wave has been isolated in a wave record to estimate the velocities, accelerations, forces, etc., associated with that individual wave event. The engineer must recognize that the implication of the statistical nature of irregular waves implies that the kinematics and dynamics likewise require statistical treatment. IAHR (1986) provides a detailed description of parameters and terminology used with irregular waves.

(6) Two approaches exist for treating irregular waves: *spectral methods* and *wave-by-wave (wave train) analysis*. Spectral approaches are based on the Fourier Transform of the sea surface. Indeed this is currently the most mathematically appropriate approach for analyzing a time-dependent, three-dimensional sea surface record. Unfortunately, it is exceedingly complex and at present few measurements are available that could fully tap the potential of this method. However, simplified forms of this approach have been proven to be very useful. The other approach used is wave-by-wave analysis. In this analysis method, a time-history of

the sea surface at a point is used, the undulations are identified as waves, and statistics of the record are developed. This is a very natural introduction to irregular waves and will be presented first before the more complicated spectral approach is presented. The primary drawback to the wave-by-wave analysis is that it cannot tell anything about the direction of the waves. Indeed, what appears to be a single wave at a point may actually be the local superposition of two smaller waves from different directions that happen to be intersecting at that time. Disadvantages of the spectral approach are the fact that it is linear and can distort the representation of nonlinear waves.

b. Wave train (wave-by-wave) analysis.

(1) Introduction.

(a) Wave train analysis requires direct measurements of irregular seas. A typical irregular wave record obtained from a wave-measuring device is shown in Figure II-1-25. The recorded wave traces have to be of finite length with the sea surface sampled at a set interval (typically every second). The time-history of sea surface elevation at a point is a random-appearing signal exhibiting many maxima and minima (Figures II-1-26 and II-1-27). It is necessary to develop a criterion for identifying individual waves in the record.

(b) In a wave-by-wave analysis, undulation in the time-history of the surface must be divided into a series of segments, which will then be considered as individual waves. The height and period of each wave will be measured. Once this is done for every segment of the record, statistical characteristics of the record can be estimated, and the statistics of the record are compiled.

(c) Knowing the statistics of one record can be useful in itself, particularly if the record is important (such as the observation of waves at a site when a structure failed). However, it would be helpful to know whether the statistical characteristics of individual wave records followed any consistent pattern. Statistics of the sea state could be predicted knowing only a little about the wave conditions. It would be very useful if the distribution of wave characteristics in a wave record followed a known statistical distribution. After defining characteristics of individual records, the larger statistical question will be addressed.

(d) In the time-domain analysis of irregular or random seas, wave height and period, wavelength, wave crest, and trough have to be carefully defined for the analysis to be performed. The definitions provided earlier in the regular wave section of this chapter assumed that the crest of a wave is any maximum in the wave record, while the trough can be any minimum. However, these definitions may fail when two crests occur within an intervening trough lying below the mean water line. Also, there is not a unique definition for wave period, since it can be taken as the time interval between either two neighboring wave troughs or two crests. Other more common definitions of wave period are the time interval between successive crossings of the mean water level by the water surface in a downward direction called *zero down-crossing period* or *zero up-crossing period* for the period deduced from successive up-crossings.

(2) Zero-crossing method.

(a) The adopted engineering procedure is the zero-crossing technique, where a wave is defined when the surface elevation crosses the zero-line or the mean water level (MWL) upward and continues until the next crossing point. This is the *zero-upcrossing* method. When a wave is defined by the downward crossing of the zero-line by the surface elevation, the method is the *zero-downcrossing*.

(b) The *zero-crossing wave height* is the difference in water surface elevation of the highest crest and lowest trough between successive zero-crossings. The definition of wave height depends on the choice of trough occurring before or after the crest. Here, a wave will be identified as an event between two successive zero-upcrossings and wave periods and heights are defined accordingly. Note that there can be differences between the definitions of wave parameters obtained by the zero up- and down-crossing methods for description of irregular sea states.

(c) Both methods usually yield statistically similar mean values of wave parameters. There seems to be some preference for the zero-downcrossing method (IAHR 1986). The downcrossing method may be preferred due to the definition of wave height used in this method (the vertical distance from a wave trough to the following crest). It has been suggested that this definition of wave height may be better suited for extreme waves (IAHR 1986).

(d) Using these definitions of wave parameters for an irregular sea state, it is seen in Figures II-1-26 and II-1-27 that, unlike the regular (monochromatic) sinusoidal waves, the periods and heights of irregular waves are not constant with time, changing from wave to wave. Wave-by-wave analysis determines wave properties by finding average statistical quantities (i.e., heights and periods) of the individual wave components present in the wave record. Wave records must be of sufficient length to contain several hundred waves for the calculated statistics to be reliable.

(e) Wave train analysis is essentially a manual process of identifying the heights and periods of the individual wave components followed by a simple counting of zero-crossings and wave crests in the wave record. The process begins by dissecting the entire record into a series of subsets for which individual wave heights and periods are then noted for every zero down-crossing or up-crossing, depending on the method selected. In the interest of reducing manual effort, it is customary to define wave height as the vertical distance between the highest and lowest points, while wave period is defined as the horizontal distance between two successive zero-crossing points (Figures II-1-26 and II-1-27). In this analysis, all local maxima and minima not crossing the zero-line have to be discarded. From this information, several wave statistical parameters are subsequently calculated. Computer programs are available to do this (IAHR 1986).

(3) Definition of wave parameters.

(a) Determination of wave statistics involves the actual processing of wave information using the principles of statistical theory. A highly desirable goal is to produce some statistical estimates from the analyzed time-series data to describe an irregular sea state in a simple parametric form. For engineering, it is necessary to have a few simple parameters that in some sense tell us how severe the sea state is and a way to estimate or predict what the statistical characteristics of a wave record might be had it been measured and saved. Fortunately, millions of wave records have been observed and a theoretical/empirical basis has evolved to describe the behavior of the statistics of individual records.

(b) For parameterization, there are many short-term candidate parameters which may be used to define statistics of irregular sea states. Two of the most important parameters necessary for adequately quantifying a given sea state are characteristic height H and characteristic period T. Other parameters related to the combined characteristics of H and T, may also be used in the parametric representation of irregular seas.

(c) Characteristic wave height for an irregular sea state may be defined in several ways. These include the *mean height*, the *root-mean-square height*, and the mean height of the highest one-third of all waves known as the *significant height*. Among these, the most commonly used is the significant height, denoted as H_s or $H_{1/3}$. Significant wave height has been found to be very similar to the estimated visual height by an experienced observer (Kinsman 1965). The characteristic period could be the *mean period*, or *average zero-crossing period*, etc.

(d) Other statistical quantities are commonly ascribed to sea states in the related literature and practice. For example, the mean of all the measured wave heights in the entire record analyzed is called the *mean wave height* $\bar{H}$. The largest wave height in the record is the *maximum wave height* H_{max}. The root-mean-square of all the measured wave heights is the *rms wave height* H_{rms}. The average height of the largest $1/n$ of all waves in the record is the $H_{1/n}$ where n = 10, 11, 12, 13,..., 99, 100 are common values. For instance, $H_{1/10}$ is the mean height of the highest one-tenth waves. In coastal projects, engineers are faced with designing for the maximum expected, the highest possible waves, or some other equivalent wave height. From one wave record measured at a point, these heights may be estimated by ordering waves from the largest to the smallest and assigning to them a number from 1 to N. The significant wave height $H_{1/3}$ or H_s will be the average of the first (highest) $N/3$ waves.

(e) The probability that a wave height is greater (less) than or equal to a design wave height H_d may be found from

$$P(H > H_d) = \frac{m}{N}$$
$$P(H \le H_d) = 1 - \frac{m}{N} \qquad \text{(II-1-114)}$$

where m is the number of waves higher than H_d. For an individual observed wave record the probability distribution $P(H > H_d)$ can be formulated in tabular form and possibly fitted by some well-known distribution. The root-mean-square wave height H_{rms} may be computed as

$$H_{rms} = \sqrt{\frac{1}{N} \sum_{j=1}^{N} H_j^2} \qquad \text{(II-1-115)}$$

in which H_j denote the ordered individual wave heights in the record.

(f) Probability distributions discussed in the irregular wave section of the CEM refer to <u>short term wave statistics</u>. This subject concerns the probability that a wave of a given height will occur given that we know the statistics of the sea surface over a 16- to 60-min period. A short-term wave statistics question might be, for example, "If we have measured the waves for 15 min and found that H_s is 2m, what is the chance that a wave of 4 m may occur?" This must be contrasted to <u>long-term wave statistics</u>. To obtain long-term wave statistics, a 15-min record may have been recorded (and statistics of each record computed) every 3 hr for 10 years (about 29,000 records) and the statistics of the set of 29,000 significant wave heights compiled. A long-term wave statistics question might be, "If the mean <u>significant</u> wave height may be 2m with a standard deviation of 0.75m, what is the chance that once in 10 years the significant wave height will exceed 4 m?" These are two entirely different statistical questions and must be treated differently.

(g) A similar approach can be used for the wave period. The mean zero-crossing period is called the *zero-crossing period* T_z. The average wave period between two neighboring wave crests is the wave crest period T_c. Therefore, in the time domain wave record analysis, the average wave period may also be obtained from the total length of *record length* T_r either using T_z or T_c (Tucker 1963). These periods are related to T_r by

$$T_z = \frac{T_r}{N_z}$$
$$T_c = \frac{T_r}{N_c} \qquad \text{(II-1-116)}$$

where N_z and N_c are the number of zero-upcrossings and crests in the wave record, respectively. We emphasize that in Tucker's method of wave train analysis, crests are defined by zero-crossing. Note also by definition of these periods that $T_c \le T_z$.

(h) The list of definitions stated above is not all-inclusive, and several other statistical quantities may be obtained from a wave train analysis (Ochi 1973; IAHR 1986). For example, the rms surface elevation η_{rms} (described later in the short-term sea states section) (σ in IAHR list) defines the standard deviation of the surface elevation, and the significant wave height H_s is related to η_{rms} by

$$H_s = 3.8\ \eta_{rms} \approx 4\ \eta_{rms} \tag{II-1-117}$$

(4) Significant wave height.

(a) The *significant wave height* H_s (or $H_{1/3}$) is the most important quantity used describing a sea state and thus, is discussed further here for completeness. The concept of significant wave height was first introduced by Sverdrup and Munk (1947). It may be determined directly from a wave record in a number of ways. The most frequently used approach in wave-by-wave analysis is to rank waves in a wave record and then choose the highest one-third waves. The average of the chosen waves defines the significant wave height as

$$H_s = \frac{1}{\frac{N}{3}} \sum_{i=1}^{N/3} H_i \tag{II-1-118}$$

where N is the number of individual wave heights H_i in a record ranked highest to lowest.

(b) Sverdrup and Munk (1947) defined significant wave height in this fashion because they were attempting to correlate what sailors reported to what was measured. Hence, this is an empirically driven definition. Today, when wave measuring is generally automated, some other parameter might be appropriate, but significant wave height remains in recognition of its historical precedence and because it has a fairly tangible connection to what observers report when they try to reduce the complexity of the sea surface to one number. It is important to recognize that it is a statistical construct based only on the height distribution. Knowing the significant height from a record tells us nothing about period or direction.

(5) Short-term random sea state parameters.

(a) It is well-known that any periodic signal $\eta(t)$ with a zero mean value can be separated into its frequency components using the standard *Fourier analysis*. Periodic wave records may generally be treated as random processes governed by laws of the probability theory. If the wave record is a random signal, the term used is *random waves*. For a great many purposes, ocean wave records may be considered random (Rice 1944-1945, Kinsman 1965, Phillips 1977, Price and Bishop 1974).

(b) The statistical properties of a random signal like the wave surface profile may be obtained from a set of many simultaneous observations called an *ensemble* or set of signals $\{\eta_1(t), \eta_2(t), \eta_3(t),...\}$, but not from a single observation. A single observation even infinitely long may not be sufficient for determining the spatial variability of wave statistics. An ensemble consists of different realizations or measurements of the process $\eta(t)$ at some known locations. To determine wave properties from the process $\eta(t)$, certain assumptions related to its time and spatial variation must be made.

(c) First, it would be necessary to assume that the process described by the wave record (i.e., a sea state), say $\eta(t)$, is *stationary*, which means that the statistical properties of $\eta(t)$ are independent of the origin of time measurement. Since the statistics of stationary processes are time-invariant, there is no drift with time in the statistical behavior of $\eta(t)$. The stationarity requirement is necessary as we shall see later for developing a *probability distribution* for waves, which is the fraction or percentage of time an *event* or *process* (say, the sea state depicted in time series of the wave surface profile) is not exceeded. The probability distribution may be obtained by taking $\eta_1(t_1)$, $\eta_2(t_1)$, $\eta_3(t_1)$,..., as variables, independent of the instant t_1. If in addition, $\eta(t)$ can be measured at different locations and the properties of $\eta(t)$ are invariant or do not depend on location of measurements, the process may then be assumed *homogenous*. In reality, $\eta(t)$ may be assumed stationary and homogenous only for a limited duration at the location data are gathered. Wind waves may be considered approximately stationary for only a few hours (3 hr or less), beyond which their properties are expected to change.

(d) Second, the process $\eta(t)$ is assumed to be *ergodic*, which means that any measured record of the process say $\eta_1(t)$ is typical of all other possible realizations, and therefore, the average of a single record in an ensemble is the same as the average across the ensemble. For an ergodic process, the sample mean from the ensemble approaches the real mean μ, and the sample variance approaches the variance σ of the process (sea state). The ergodicity of $\eta(t)$ implies that the measured realization of $\eta(t)$, say $\eta_1(t_1)$ is typical of all other possible realizations $\eta_2(t_1)$, $\eta_3(t_1)$, ..., all measured at one instant t_1. The concept of ergodicity permits derivation of various useful statistical information from a single record, eliminating the need for multiple recordings at different sites. The assumptions of stationarity and ergodicity are the backbones of developing wave statistics from wave measurements. It is implicitly assumed that such hypotheses exist in reality, and are valid, particularly for the sea state.

(e) To apply these concepts to ocean waves, consider an ensemble of records representing the sea state by $\eta(t)$ over a finite time T. The *mean* or *expected value* of the sea state, denoted by $\bar{\eta}$, or μ_η, or $E[\eta]$, is defined as

$$\mu_\eta = E[\eta(t)] = \frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \eta(t)\; dt \tag{II-1-119}$$

where the symbol E denotes the expected value of $\eta(t)$. Similarly, the mean-square of η corresponds to the second moment of η, denoted by $E[\eta^2]$. The standard deviation σ_η or the root-mean-square value of the process is the square root of this. The *variance* of η, represented by σ_η^2 may be expressed in terms of the variance of the process V as

$$\sigma_\eta^2 = V[\eta(t)] = E[\eta^2] - \mu_\eta^2 \tag{II-1-120}$$

(f) The *standard deviation* σ_η is the square root of the variance, also called the second central moment of $\eta(t)$. The standard deviation characterizes the spread in the values of $\eta(t)$ about its mean.

(g) The *autocorrelation* or *autocovariance function* of the sea state is denoted by R_η, relating the value of η at time t to its value at a later time $t+\tau$. This is defined as

$$R_\eta(t,\ t+\tau) = E[\eta(t)\ \eta(t+\tau)] \tag{II-1-121}$$

(h) The value of R_η gives an indication of the correlation of the signal with itself for various time lags τ, and so it is a measure of the temporal variation of $\eta(t)$ with time. If the signal is perfectly correlated with itself for zero lag τ, its autocorrelation coefficient, defined as

$$\rho_\eta = \frac{E[\eta(t)\ \eta(t+\tau)]}{E[\eta^2]} = \frac{R_\eta}{E[\eta^2]} \qquad \text{(II-1-122)}$$

will be equal to 1.

(i) For two different random signals η_1 and η_2, the *cross-correlation coefficient R* may be defined as

$$R = E[\eta_1(t)\ \eta_2(t+\delta t)] = \frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \eta_1(t)\ \eta_2(t+\delta t)\ dt \qquad \text{(II-1-123)}$$

which measures the degree of correlation between two signals. This concept is useful for example in relating wave velocities and pressures obtained at two separate locations during wave gauge measurements in coastal projects. Note that the process $\eta(t)$ is stationary if μ_η and σ_η are constant for all values of t, and that R is a function only of $\tau = t_2 - t_1$.

(j) Assuming that the water surface elevation $\eta(t)$ is a stationary random process, representing a sea state over the duration of several hours, we will next focus our attention on defining the probabilistic properties of ocean waves. The probabilistic representation of sea state is useful in practice for two reasons. First, it allows the designer to choose wave parameters within a limit that will yield an acceptable level of risk. Second, a probabilistic-based design criterion may result in substantial cost savings by considering uncertainties in the wave estimates. Therefore, an overview of the probability laws and distributions for ocean waves follows.

(6) Probability distributions for a sea state.

(a) As noted earlier, irregular sea states are random signals. For engineers to effectively use irregular waves in design, properties of the individual wave records must follow some probability laws so that wave statistics can readily be obtained analytically. Rice (1944-1945) developed the statistical theory of random signals for electrical noise analysis. Longuet-Higgins (1952) applied this theory to the random water surface elevation of ocean waves to describe their statistics using certain simplified assumptions. Longuet-Higgins found that the parameters of a random wave signal follow known probability laws.

(b) The *probability distribution P(x)* is the fraction of events that a particular event is not exceeded. It can be obtained directly from a plot of the proportion of values less than a particular value versus the particular value of the variable x_0, and is given by

$$P(x) = prob\{x \le x_0\} \qquad \text{(II-1-124)}$$

(c) The *probability density p(x)* is the fraction of events that a particular event is expected to occur and thus, it represents the rate of change of a distribution and may be obtained by simply differentiating $P(x)$ with respect to its argument x.

(d) The two most commonly used probability distributions in the study of random ocean waves are the *Gaussian* (Figure II-1-28) and *Rayleigh* distributions (Figure II-1-29). The Gaussian distribution is particularly suited for describing the short-term probabilities of the sea surface elevation η. Its probability density is given by

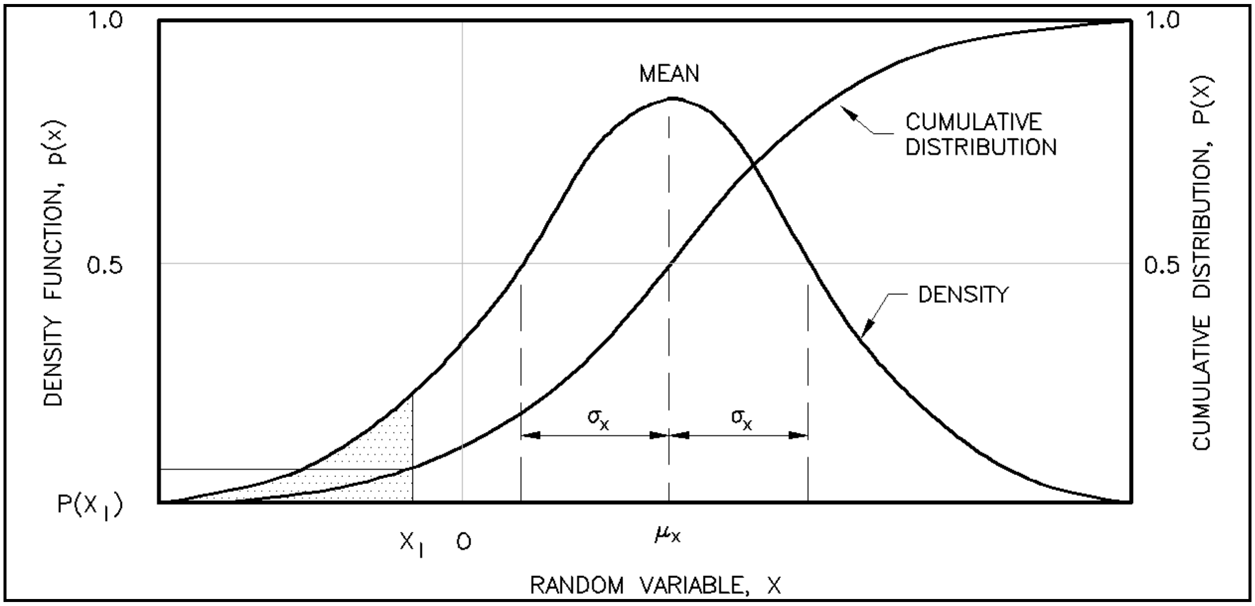

Figure II-1-28. The Gaussian probability density and cumulative probability distribution

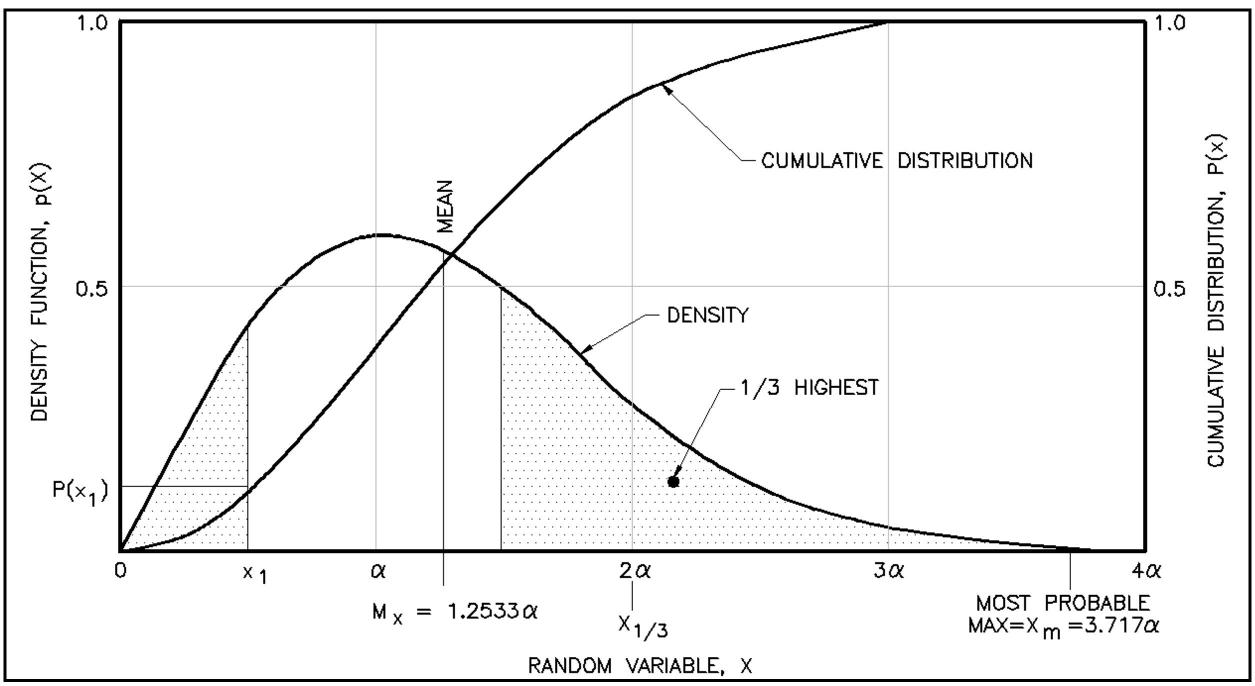

Figure II-1-29. The Rayleigh probability density and cumulative probability distribution (x = α corresponds to the mode)

$$p(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \, e^{-\left(\frac{(x - \mu_x)^2}{2\sigma_x^2}\right)} \tag{II-1-125}$$

where μ_x is the mean of x and σ_x is the standard deviation. The Gaussian cumulative probability or probability distribution denoted by *P(x)* in Figure II-1-28, is the integral of p(x). A closed form of this integral is not possible. Therefore, Gaussian distribution is often tabulated as the normal distribution with the mean μ_x and standard deviation σ_x in handbooks (e.g., Abramowitz and Stegun (1965)), and is written as

$$p(x) = N(\mu_x, \sigma_x) \qquad P(x) = \Phi\left[\frac{x - \mu_x}{\sigma_x}\right] \tag{II-1-126}$$

For zero mean ($\mu_x = 0$) and unit standard deviation ($\sigma_x = 1$), the Gaussian probability density and distributions reduce to

$$\begin{aligned} p(x) &= \frac{1}{\sqrt{2\pi}} \, e^{\left(-\frac{x^2}{2}\right)} \\ \Phi(x) &= \int_0^x p(y) \, dy \end{aligned} \tag{II-1-127}$$

where the last integral is the *error function*.

(f) The probability of exceedence *Q(x)* may be expressed in terms of the probability of non-exceedence P(x) as

$$Q\left[x(t) > x_1\right] = 1 - P\left[x(t) < x_1\right] = 1 - \Phi\left[\frac{x - \mu_x}{\sigma_x}\right] \tag{II-1-128}$$

(g) This is the probability that x will exceed x_1 over the time period t, and is shown as the shaded area in the bottom lower end of Figure II-1-28. The probability of exceedence is an important design parameter in risk-based design.

(h) In engineering practice, we are normally concerned with wave height rather than surface elevation. However, to define wave height distribution, we only need to examine the statistics of the slowly varying envelope of the surface elevation $\eta(t)$. With this approach, Longuet-Higgins (1952) found from statistical theory that both wave amplitudes and heights follow the Rayleigh distribution shown in Figure II-1-29. Note that this distribution can never be negative, it decays asymptotically to zero for large x, but never reaches zero. The probability density function of the Rayleigh distribution and its cumulative probability are given by

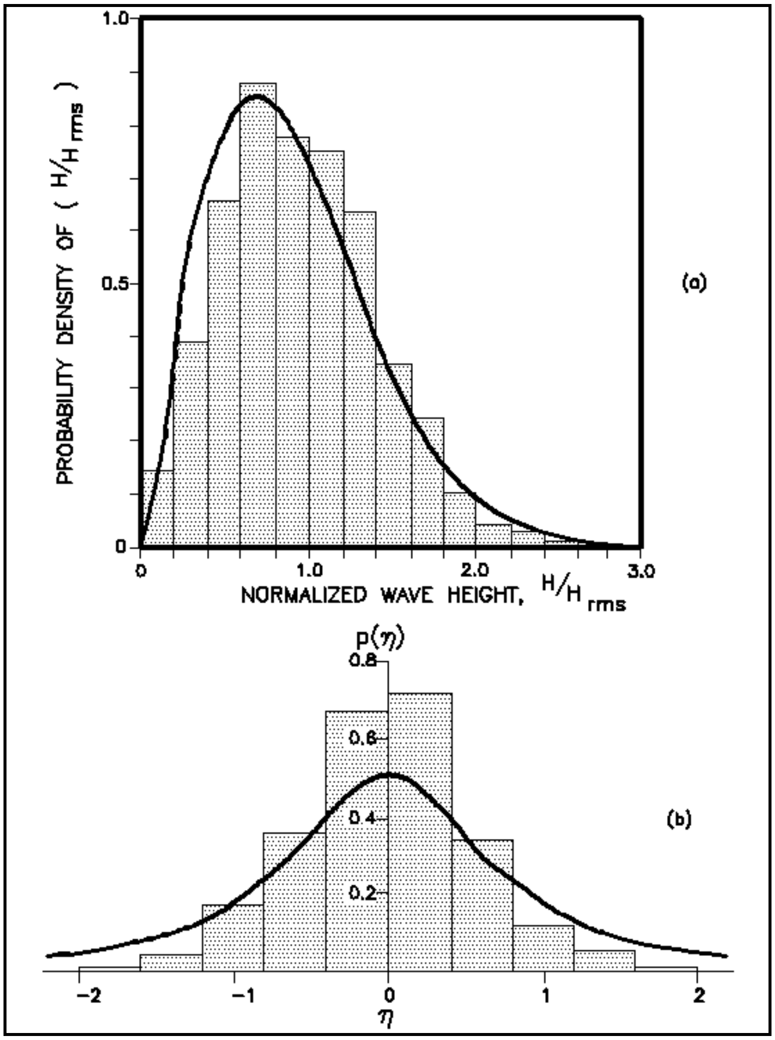

Figure II-1-30. Histograms of the normalized (a) wave heights, and (b) wave periods with theoretical distributions (Chakrabarti 1987)

$$p(x) = \left\{ \frac{\pi x}{2\mu_x^2} \, e^{-\frac{\pi}{4}\left(\frac{x}{\mu_x}\right)^2} \qquad for \;\; x \geq 0 \right\} \tag{II-1-129}$$

$$P(x) = \left\{ 1 - e^{-\frac{\pi}{4}\left(\frac{x}{\mu_x}\right)^2} \quad for \;\; x \geq 0 \right\}$$

where μ_x is the mean. These are displayed in Figure II-1-29 in which the density function is offset to the right and has only positive values. The distributions used for wave heights, wave periods, and their joint relations are described next.

(7) Wave height distribution.

(a) The heights of individual waves may be regarded as a stochastic variable represented by a probability distribution function. From an observed wave record, such a function can be obtained from a histogram of wave heights normalized with the mean heights in several wave records measured at a point (Figure II-1-30). Thompson (1977) indicated how well coastal wave records follow the Rayleigh distribution. If wave energy is concentrated in a very narrow range of wave period, the maxima of the wave profile will coincide with the wave crests and the minima with the troughs. This is termed a *narrow-band condition*. Under the narrow-band condition, wave heights are represented by the following Rayleigh distribution (Longuet-Higgins 1952, 1975b, 1983)

$$p(H) = \frac{2H}{H_{rms}^2} \exp\left[-\frac{H^2}{H_{rms}^2}\right]$$
$$P(H) = 1 - \exp\left[-\frac{H^2}{H_{rms}^2}\right] \qquad \text{(II-1-130)}$$

(b) The significant wave height $H_{1/3}$ is the centroid of the area for $H \geq H_*$ under the density function where $H > H_*$ corresponds to waves in the highest one-third range as shown in Figure II-1-29, that is

$$P(H_*) = 1 - \frac{1}{3} = 1 - e^{\left(-\frac{H_*^2}{H_{rms}^2}\right)} \qquad \text{(II-1-131)}$$

from which we find $H_* = 1.05H_{rms}$. Various estimates of wave heights may then be obtained upon integration of the above equation using certain mathematical properties of the Error function (Abramowitz and Stegun 1965). We find

$$H_{1/3} \approx 4.00\sqrt{m_0} = 1.416\ H_{rms}$$
$$H_{1/10} = 1.27\ H_{1/3} = 1.80\ H_{rms} = 5.091\sqrt{m_0} \qquad \text{(II-1-132)}$$
$$H_{1/100} = 1.67\ H_{1/3} = 2.36\ H_{rms} = 6.672\sqrt{m_0}$$
$$H_{\max} = 1.86\ H_{1/3} \quad (\textit{for}\ 1000\ \textit{wave cycles in the record})$$

(c) The *most probable maximum wave height* in a record containing N waves is related to the rms wave height (Longuet-Higgins 1952) by

$$H_{\max} = \left[\sqrt{\log N} + \frac{0.2886}{\sqrt{\log N}} - \frac{0.247}{(\log N)^{3/2}}\right] H_{rms} \qquad \text{(II-1-133)}$$

(d) The value of H_{max} obtained in this manner can be projected to a longer period of time by adjusting the value of N based on the mean zero-upcrossing period (Tucker 1963).

(e) The fact that the statistics of wave height for wave records in general follows a Rayleigh distribution is of great significance in coastal engineering. For instance, an engineer may have information from a hindcast (see Part II-2) that the significant height for a storm is 10 m. Assuming that the Rayleigh distribution describes the wave record, the engineer can estimate that the 10-percent wave will be 12.7 m and that the H_{max} (assuming 1,000 waves in the record) will be 18.6 m. Often measured ocean wave records are analyzed spectrally (see "Spectral Analysis" section later in this chapter) by the instrument package and only condensed information is reported via satellite to a data bank, with no other information retained. The inherent assumption made is that the Rayleigh distribution is adequate.

(f) Theoretical relationships derived from the Rayleigh distribution generally agree well with the values determined directly from the records. The Rayleigh probability distribution density function is compared with a histogram of the measured deepwater wave heights in Figure II-1-30 (Chakrabarti 1987). Clearly the Rayleigh distribution fits this data well, even though the frequency spectra of ocean waves may not always be narrow-banded as assumed in the Rayleigh distribution. Field measurements sometimes deviate from the Rayleigh distribution, and the deviation appears to increase with increasing wave heights, and decrease as the wave spectrum becomes sharply peaked. The effect of bandwidth on wave height distribution has been accounted for theoretically (Tayfun 1983).

(g) Deepwater wave height measurements from different oceans have been found to closely obey a Rayleigh distribution (Tayfun 1983a,b; Forristall 1984; Myrhaug and Kjeldsen 1986). This is not true for shallow-water waves, which are strongly modulated by the bathymetric effects combined with the amplitude nonlinearities. The wave energy spectrum of the shallow-water waves is not narrow-banded and may substantially deviate from the Rayleigh distribution especially for high frequencies. In general, the Rayleigh distribution tends to overpredict the larger wave heights in all depths.

(h) In summary, the Rayleigh distribution is generally adequate, except for near-coastal wave records in which it may overestimate the number of large waves. Investigations of shallow-water wave records from numerous studies indicate that the distribution deviates from the Rayleigh, and other distributions have been shown to fit individual observations better (SPM 1984). The primary cause for the deviation is that the large waves suggested in the Rayleigh distribution break in shallow water. Unfortunately, there is no universally accepted distribution for waves in shallow water. As a result, the Rayleigh is frequently used with the knowledge that the large waves are not likely.

(8) Wave period distribution.

(a) Longuet-Higgins (1962) and Bretschneider (1969) derived the wave period distribution function assuming the wave period squared follows a Rayleigh distribution. This distribution is very similar to the normal distribution with a mean period given by

$$T_{0,1} = \frac{m_0}{m_1} \qquad \text{(II-1-134)}$$

where the moments are defined in terms of cyclic frequency (i.e., Hertz). The probability density of wave period T is given by (Bretschneider 1969)

$$p(T) = 2.7 \frac{T^3}{\overline{T}} \exp\left[-0.675\tau^4\right]$$

$$\tau = \frac{T}{\overline{T}} \tag{II-1-135}$$

(b) A different probability density distribution of the wave period has been derived by Longuet-Higgins (1962). This is given by

$$p(\tau) = \frac{1}{2(1 + \tau^2)^{3/2}}$$

$$\tau = \frac{T - T_{0,1}}{\upsilon T_{0,1}} \quad ; \quad \nu = \frac{m_0 m_2 - m_1^2}{m_1^2} \tag{II-1-136}$$

where v is the *spectral width parameter* and m_0, m_1, and m_2 are *moments* of the wave spectrum, which will be defined later. This probability density function is symmetric about $\tau = 0$ where it is maximum, and is similar to the normal distribution with a mean equal to $T_{0,1}$. This distribution fits field measurements reasonably well, and is often used in offshore design. In general, probability density for the wave period is narrower than that of wave height, and the spread lies mainly in the range 0.5 to 2.0 times the mean wave period.

(c) Various characteristic wave periods are related. This relationship may be stated in a general way as

$$T_{\max} \approx T_{1/3} \approx C\overline{T} \tag{II-1-137}$$

where the coefficient C varies between 1.1 and 1.3.

(9) Joint distribution of wave heights and periods.

(a) If there were no relation between wave height and wave period, then the joint distribution between wave height and wave period can simply be obtained from the individual probability distributions of the height and period by

$$p(H,T) = p(H)\ p(T) \tag{II-1-138}$$

(b) The distribution p(H,T) so obtained is inappropriate for ocean waves, since their heights and periods are correlated. For the joint distribution of wave height-period pairs, Longuet-Higgins (1975b) considered wave heights and periods also representable by a narrow-band spectrum. He derived the joint distribution assuming wave heights and periods are correlated, a more suitable assumption for real sea states.

(c) The probability density function of wave period may be obtained directly from the joint distribution, provided that a measure of the spectrum width is included in the latter. Under this condition, the distribution of wave period is simply the marginal probability density function of the joint distribution of H and T. This is done by integrating *p(H,T)* for the full range of H from 0 to ∞. Likewise, the distribution for wave heights may be obtained by integrating *p(H,T)* for the full range of periods. The joint distribution derived by Longuet-Higgins (1975b) was later modified (Longuet-Higgins 1983), and is given by

$$p(H,T) = \frac{\pi\, f(\nu)}{4} \left(\frac{H_*}{T_*}\right)^2 \exp\left\{-\frac{\pi H_*^2}{4}\left[1 + \frac{1 - \frac{\sqrt{1+\nu^2}}{T_*}}{\nu^2}\right]\right\}$$

$$H_* = \frac{H}{\overline{H}} \quad ; \quad T_* = \frac{T}{\overline{T}_z} \quad ; \quad f(\nu) = \frac{2\,(1+\nu^2)}{\nu + \frac{\nu}{\sqrt{1+\nu^2}}} \tag{II-1-139}$$

with ν as the spectral width parameter. The period $\overline{T}_z$ is the mean zero-upcrossing period and its relation to the mean wave period $\overline{T}$ and mean crest period $\overline{T}_c$ defined in terms of moments of spectrum is as follows:

$$\overline{T}_z = 2\pi \sqrt{\frac{m_0}{m_2}} \quad ;$$

$$\overline{T} = 2\pi \frac{m_0}{m_1} \quad ; \quad \overline{T}_c = 2\pi \sqrt{\frac{m_2}{m_4}} \tag{II-1-140}$$

(d) The *most probable maximum period* associated with any given H_* is

$$T_*^{\max} = \frac{2\sqrt{1+\nu^2}}{1 + \sqrt{1 + \frac{16\nu^2}{\pi H_*^2}}} \tag{II-1-141}$$

(e) Chakrabarti and Cooley (1977) investigated the applicability of the joint distribution and determined that it fits field data provided the spectrum is narrow-banded and has a single peak. A different theoretical model has been suggested by Cavanie et al. (1978), and it also compares well with the field data.

c. Spectral analysis.

(1) Introduction.

(a) In the period 1950-1960, Rice's (1944-1945) work on signal processing was extended to ocean waves (Kinsman 1965; Phillips 1977). In pinciple, the time-history of surface elevation (such as in Figures II-1-31 and II-1-32) was recognized to be similar to a noise record. By assuming that it is a discrete sample of a continuous process, the principles of Fourier analysis could be extended to describe the record. The power of Fourier representation is such that given a series of time snapshots of measurements of a three-dimensional surface, a full mathematical representation of the surface and its history may be obtained. Unfortunately, this is a lot of information. As an example, the image in Figure II-1-22 of the entrance to San Francisco Bay is one snapshot of the surface current field and represents nearly 1 million sample points. To understand the time variation of the field it would be reasonable to do this every 2 sec or so for an hour. The result is about 1.8 billion sample points that would need to be Fourier transformed. Although, this is computationally feasible such a measurement cannot be made on a routine basis and it is not clear how the information could be condensed into a form for practical engineering. However, the utility of the spectral analysis approach is that it uses a reduced dimensional approach that is powerful and useful. This section will discuss the underlying approach to using spectral representations in engineering, discuss the basic approach for the simplified spectral approaches, and describe how the spectral information can be used. However, the underlying statistical theory and assumptions will only be touched upon and details of the derivations will only be referenced.

(b) The easiest place to begin is with a nonrigorous discussion of what a spectral analysis of a single-point measurement of the surface can produce and then generalize it to the case of a sea surface. The following sections would then describe of the procedure.

(c) Considering a single-point time-history of surface elevation such as in Figures II-1-25, II-1-31, and II-1-32, spectral analysis proceeds from viewing the record as the variation of the surface from the mean and recognizes that this variation consists of several periodicities. In contrast to the wave-by-wave approach, which seeks to define individual waves, the spectral analysis seeks to describe the distribution of the variance with respect to the frequency of the signal. By convention, the distribution of the variance with frequency is written as $E(f)$ or $S(f)$ with the underlying assumption that the function is continuous in frequency space. The reason for this assumption is that all observations are discretely sampled in time, and thus, the analysis should produce estimates as discrete frequencies which are then statistically smoothed to estimate a continuum. Although $E(f)$ is actually a measurement of variance, it is often called the *one-dimensional* or *frequency energy spectrum* because (assuming linear wave theory) the energy of the wave field may be estimated by multiplying $E(f)$ by ρg.

(d) Figures II-1-31 (a regular wave) and II-1-32 (an irregular wave) provide two wave records and their spectrum. One immediate value of the spectral approach is that it tells the engineer what frequencies have significant energy content and thus acts somewhat analogous to the height-period diagram. The primary disadvantage of spectral analysis is that information on individual waves is lost. If a specific record is analyzed, it is possible to retain information about the phases of the record (derived by the analysis), which allows reconstruction of waves. But this is not routinely done.

(e) The surface can be envisioned not as individual waves but as a three-dimensional surface, which represents a displacement from the mean and the variance to be periodic in time and space. The simplest spectral representation is to consider $E(f,\theta)$, which represents how the variance is distributed in frequency f and direction θ (Figure II-1-33). $E(f,\theta)$ is called the 2-D or directional energy spectrum because it can be multiplied by ρg to obtain wave energy. The advantage of this representation is that it tells the engineer about the direction in which the wave energy is moving. A directional spectrum is displayed in Figure II-1-34 with its frequency and direction spectrums.

(f) The power of spectral analysis of waves comes from three major factors. First, the approach is easily implemented on a microchip and packaged with the gauging instrument. Second, the principal successful theories for describing wave generation by the wind and for modelling the evolution of naturalsea states in coastal regions are based on spectral theory. Third, it is currently the only widely used approach for measuring wave direction. A final factor is that Fourier or spectral analysis of wave-like phenomena has an enormous technical literature and statistical basis that can be readily drawn upon.

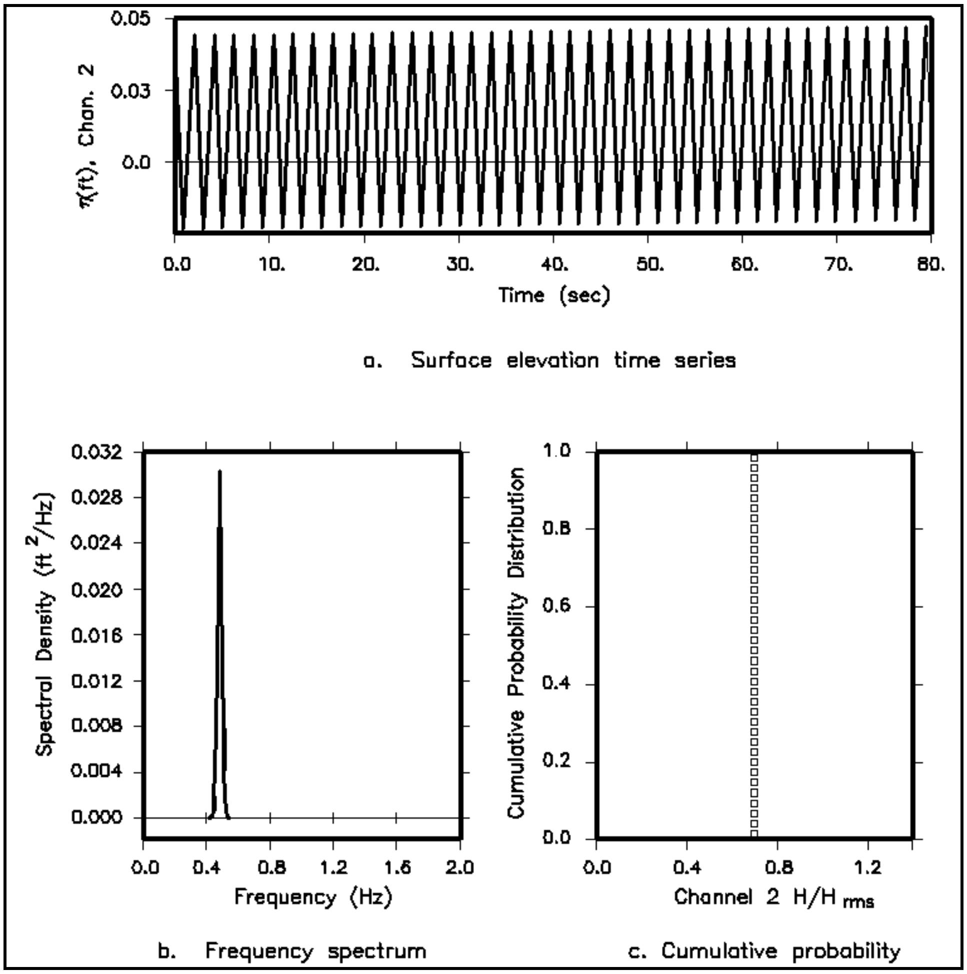

Figure II-1-31. Surface elevation time series of a regular wave and its spectrum (Briggs et al. 1993)

(g) Before proceeding to the details of how a wave spectrum is derived from a record, it is important to touch upon some statistical assumptions that are important in analyzing a wave record spectrally. Many of these assumptions also hold for making a wave-by-wave analysis useful as well. First of all, wave records are finite in length (typically 17-68 min long) and are made up of samples of surface elevation at a discrete sampling interval (typically 0.5-2.0 sec). For the wave records to be of general use, the general characteristics of the record should not be expected to change much if the record was a little shorter or longer, if the sampling was started some fraction of time earlier or later, or if the records were collected a short distance away. In addition, it is desirable that there not be any underlying trend in the data.

(h) If the above assumptions are not reasonably valid, it implies that the underlying process is unstable and may not be characterized by a simple statistical approach. Fortunately, most of the time in ocean and coastal areas, the underlying processes are not changing too fast and these assumptions reasonably hold. In principal the statistical goal is to assume that there is some underlying statistical process for which we have obtained an observation. The observation is processed in such a way that the statistics of the underlying process are obtained.

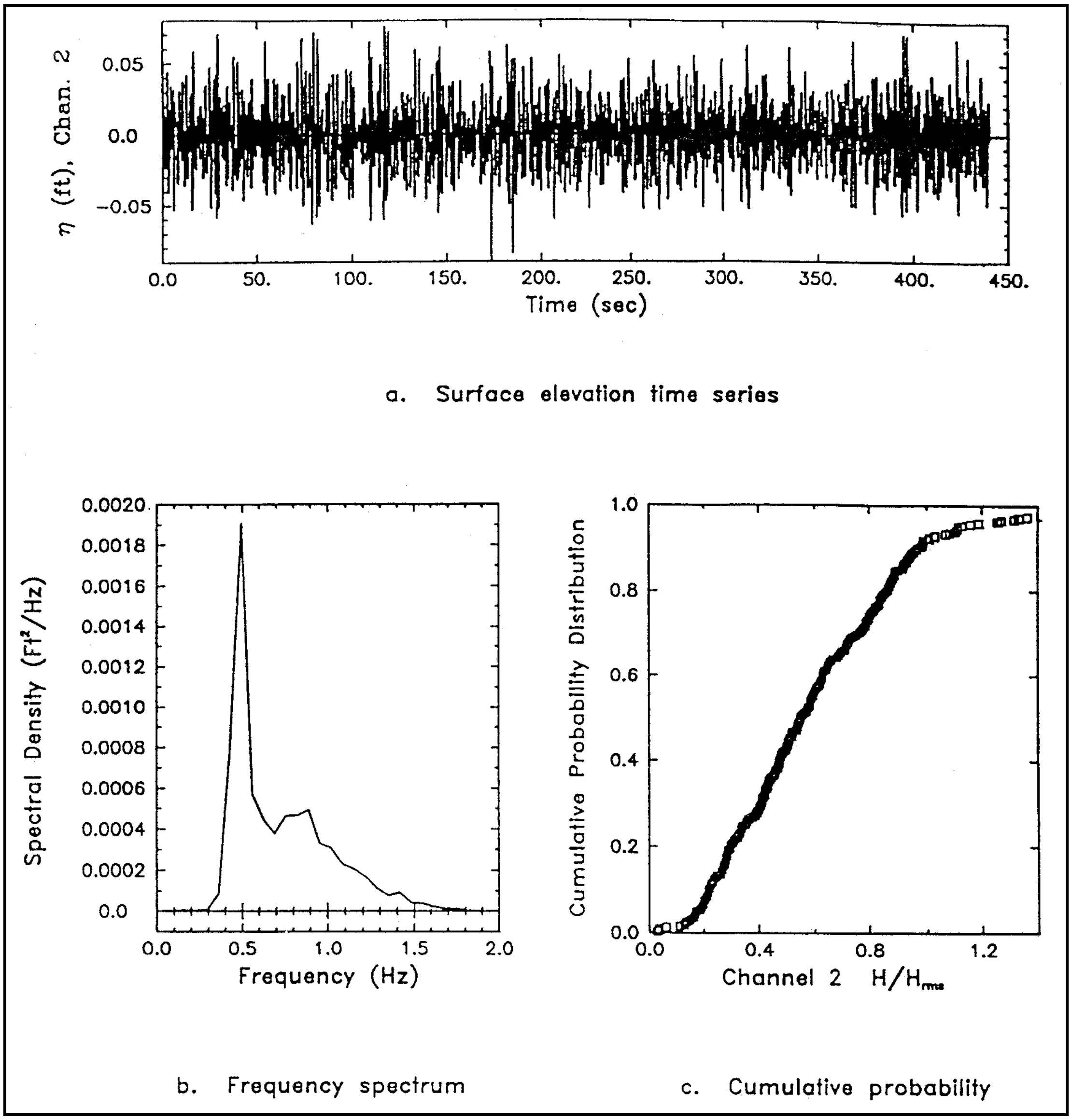

Figure II-1-32. Surface elevation time series of an irregular wave and its spectrum (Briggs et al. 1993)

(2) Description of wave spectral analysis.

(a) Unlike the wave train or wave-by-wave analysis, the spectral analysis method determines the distribution of wave energy and average statistics for each wave frequency by converting time series of the wave record into a wave spectrum. This is essentially a transformation from time-domain to the frequency-domain, and is accomplished most conveniently using a mathematical tool known as the Fast Fourier Transform (FFT) technique (Cooley and Tukey 1965). Here we will treat analysis of the time recording ofthe surface at a point, in order to obtain a frequency spectrum of the record. In a later section, we will describe how to obtain a frequency-directional spectrum.

(b) The *wave energy spectral density* $E(f)$ or simply the *wave spectrum* may be obtained directly from a continuous time series of the surface $\eta(t)$ with the aid of the Fourier analysis. Using a Fourier analysis, the

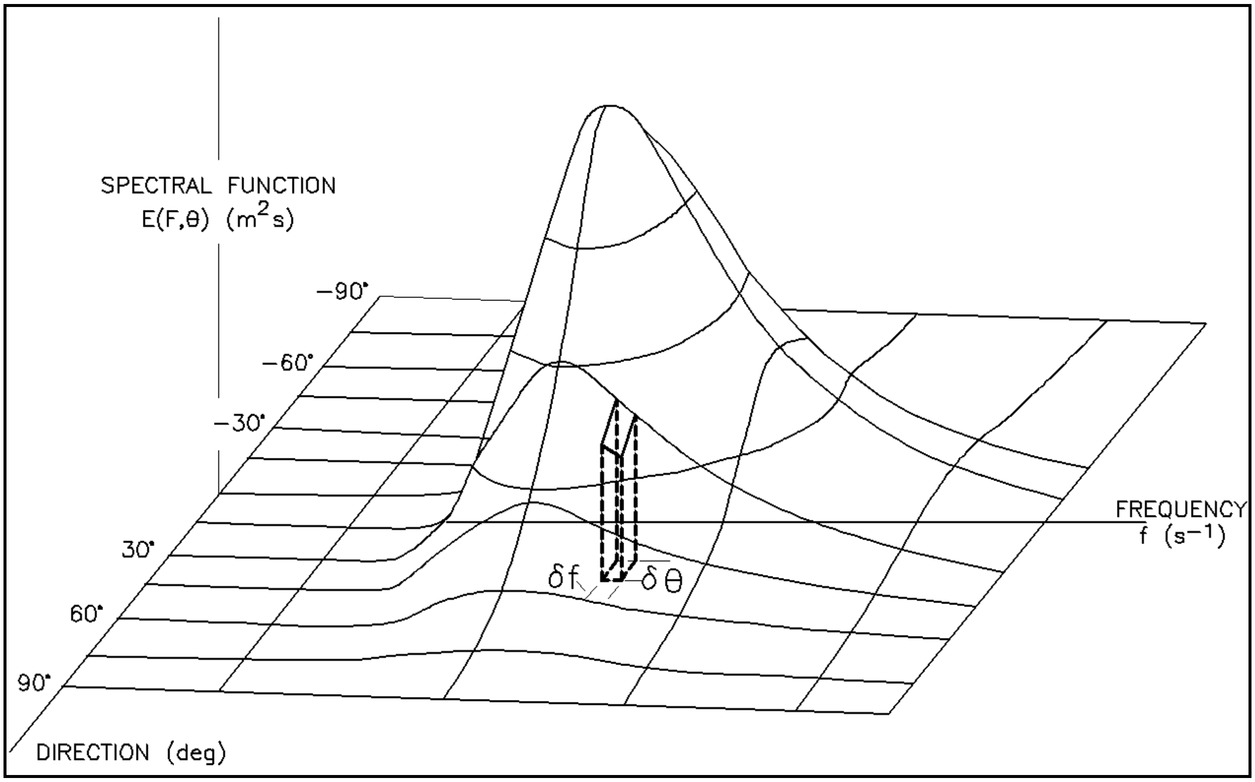

Figure II-1-33. A schematic for a two-dimensional wave spectrum E(f,θ)

wave profile time trace can be written as an infinite sum of sinusoids of amplitude A_n, frequency ω_n, and relative phase ε_n, that is

$$\begin{aligned}\eta(t) &= \sum_{n=0}^{\infty} A_n \cos\,(\omega_n t - \varepsilon_n) \\ &= \sum_{n=0}^{\infty} a_n \cos\, n\omega t + b_n \sin\, n\omega t\end{aligned} \tag{II-1-142}$$

(c) The coefficients a_n and b_n in the above equation may be determined explicitly from the orthogonality properties of circular functions. Note that a_0 is the mean of the record. Because real observations are of finite length, the finite Fourier transform is used and the number of terms in the sum n is a finite value.

(d) The *covariance* of $\eta(t)$ is related to the wave energy spectrum. This is defined in terms of the squares of component amplitudes as

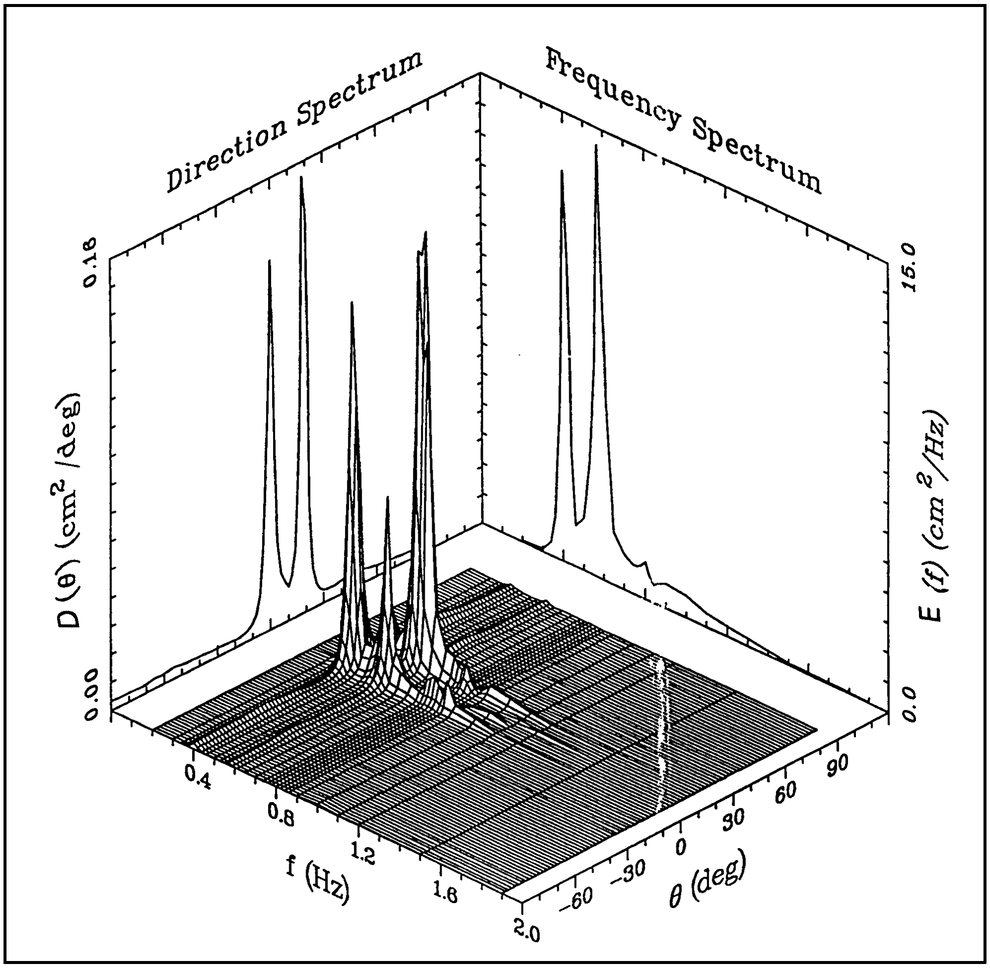

Figure II-1-34. A directional spectrum and its frequency and direction spectrum (Briggs et al 1993)

$$\overline{\eta}^2(t) = \sum_0^\infty A_n^2 \; \Delta f$$

$$A_n^2 = \frac{1}{2}\sqrt{a_n^2 + b_n^2} \qquad \text{(II-1-143)}$$

$$\epsilon_n = \tan^{-1} \frac{b_n}{a_n}$$

(e) By induction, an estimate of the continued energy spectrum of *η(t)* may be obtained by

$$E(f) = \frac{1}{T_r}\left[\sum_{n=0}^{N} \eta(n\Delta t) \; e^{2\pi i f (n \Delta t)} \; \Delta t\right]^2 \qquad \text{(II-1-144)}$$

where T_r is the *record length* and *Δt* is the *sampling interval*.

(f) There are numerous intricacies involved in the application of these discrete formulas, ranging from the length of time series necessary to digitizing frequency and many others. For unfamiliar users, most computer library systems now have *FFT (Finite Fourier Transform) algorithms* available to perform the above computations. Part VII-3 of the CEM provides a discussion of the methods. Some general guidelines are provided next.

(g) In actual practice, the total data length is divided into M smaller segments with equal number of data points N. By letting N be a power of 2 for computational efficiency, the result then is averaged over the M sections. In an FFT, the variables M, N, and Δt have to be independently selected, though T_r and Δt are fixed for a given record so that the total number of data points can be obtained from these values. Therefore, the only choice that has to be made is the number of sections M. Traditionally, the most common values of N used range from *512* to *2,048*, while the value of M is usually *8* or greater. Since T_r is dependent on N, M, and Δt as $T_r = M\,N\,\Delta t$, then higher N and M values in general yield better resolution and high confidence in the estimate of spectra. The larger the N, the more spiky or irregular the spectrum, and the smaller the N, the smoother the spectrum (Cooley and Tukey 1965; Chakrabarti 1987).

(h) To better understand the wave spectrum by the FFT method, consider first the wave surface profile of a single-amplitude and frequency wave given by a sinusoidal function as

$$\eta(t) = a \sin \omega t \tag{II-1-145}$$

where a and ω are the amplitude and frequency of the sine wave. The variance of this wave over the wave period of 2π is

$$\begin{aligned}\sigma^2 &= \overline{[\eta(t)]^2} = \frac{1}{2\pi}\int_0^{2\pi} a^2 \sin^2 2\pi f t \; d(2\pi f t)\\ &= \frac{a^2}{2} = 2\int_0^{\infty} E^1(f)\; df = \int_{-\infty}^{\infty} E^2(f)\; df\end{aligned} \tag{II-1-146}$$

(i) Thus the quantity $a^2/2$ represents the contribution to the variance σ^2 associated with the component frequency $\omega = 2\pi f$ (Figure II-1-35). The connection between the variance, wave energy, and the wave energy spectrum is now more obvious since these all are proportional to the wave amplitude (or height) squared. For consistency of units, an equality between these quantities requires that the wave spectrum not include the ρg term.

(j) The difference between a *two-sided spectrum* E^2 and a *one-sided spectrum* E^1 as illustrated in Figure II-1-36 is quite important. Note that the two-sided spectrum is symmetric about the origin, covering both negative and positive frequencies to account for all wave energy from $-\infty$ to $+\infty$. But, it is customary in ocean engineering to present the spectrum as a one-sided spectrum. This requires that the spectral density ordinates of E^2 be doubled in value if only the positive frequencies are considered. This is the reason for introducing a factor of two in Equation II-1-146. This definition will be used subsequently throughout Part II-1; thus, it is henceforth understood that $E(f)$ refers to E^1 (Figures II-1-35 and II-1-36).

(k) By an intuitive extension of this simple wave, the variance of a random signal with zero mean may be considered to be made up of contributions with all possible frequencies. For a random signal using the above equations, we find

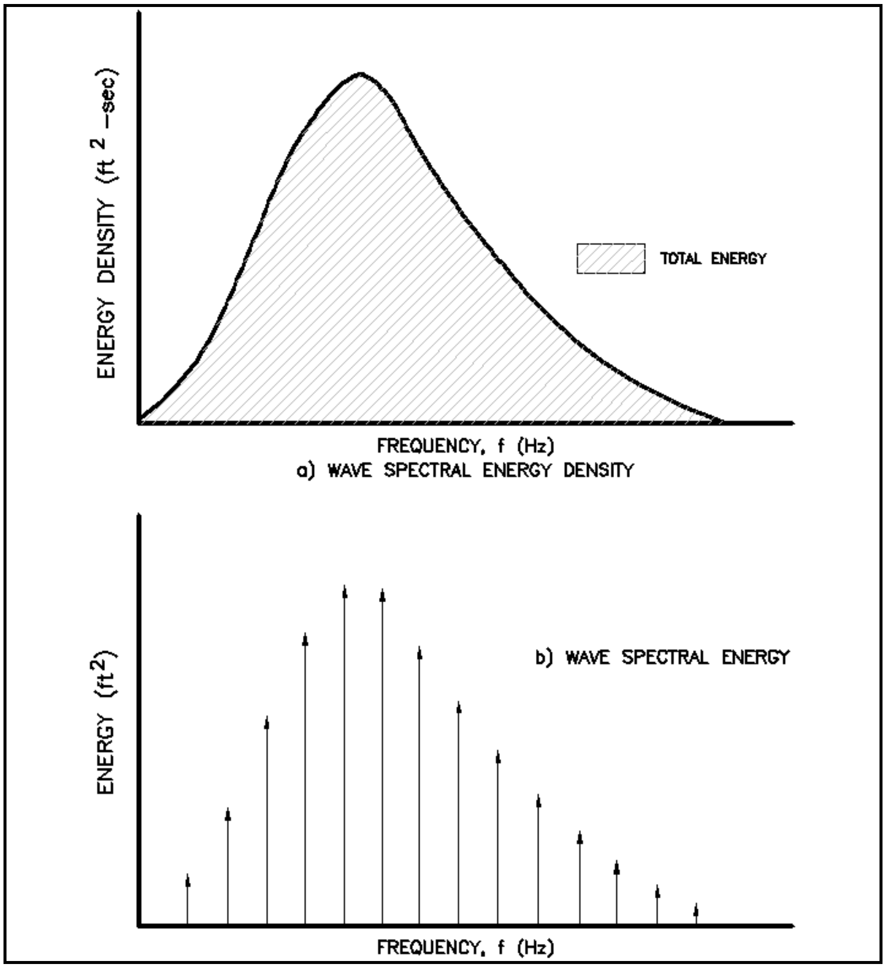

Figure II-1-35. Sketches of wave spectral energy and energy density (Chakrabarti 1987)

$$\sigma_\eta^2 = \sum_{n=1}^{\infty} \frac{a_n^2}{2} = \int_0^\infty E(f)\ df = m_0 \tag{II-1-147}$$

where m_0 is the zero-th moment of the spectrum. Physically, m_0 represents the area under the curve of $E(f)$. The area under the spectral density represents the variance of a random signal whether the one-sided or two-sided spectrum is used.

(l) The moments of a spectrum can be obtained by

$$m_i = \int_0^\infty f^i\ E(t)\ df \qquad \mathrm{i} = 0,1,2,... \tag{II-1-148}$$

(m) We now use the above definition of the variance of a random signal to provide a third definition of the significant wave height. As stated earlier, this gives an estimate of the significant wave height by the wave spectrum. For Rayleigh distributed wave heights, H_s may be approximated (Longuet-Higgins 1952) by

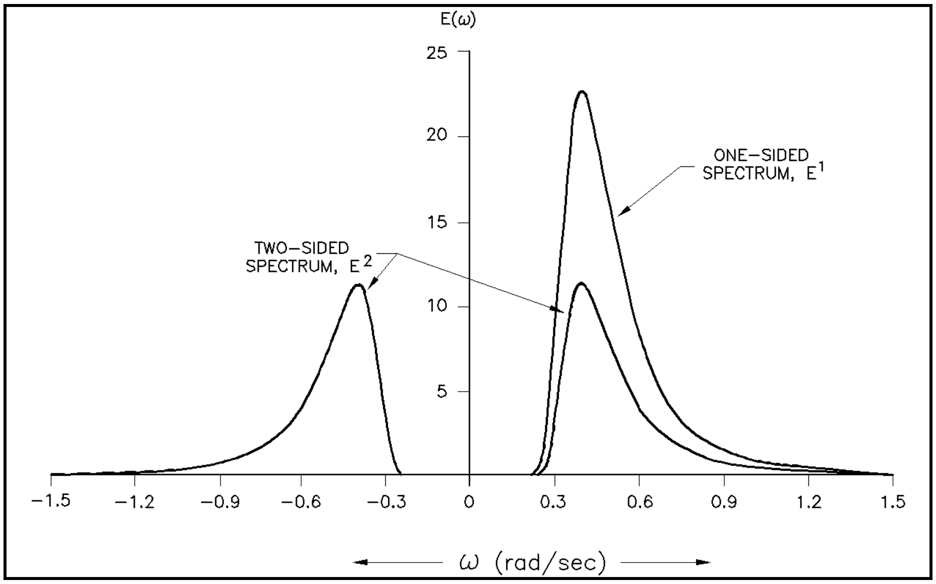

Figure II-1-36. Definition of one- and two-side wave spectrum (Chakrabarti 1987)

$$H_s = 3.8\sqrt{m_0} \approx 4\sqrt{m_0} \qquad \text{(II-1-149)}$$

(n) Therefore, the zero-th moment m_0, which is the total area under the wave energy density spectrum, defines the significant wave height for a given *E(f)* (Figure II-1-37).

(3) Examples of frequency spectra. The frequency spectrum is normally plotted as energy density on the ordinate versus frequency on the abscissa (Figures II-1-31 through II-1-37). In principal, the form of *E(f)* can be quite variable. However, some generalizations are possible. First of all, during strong wind events, the spectrum tends to have a strong central peak and a fairly predictable shape. For swell that has propagated a long distance from the source of generation, waves tend to have a single sharp peak. Waves in shallow water near breaking tend to have a sharp peak at the peak frequency f_p and have a series of smaller peaks at frequencies $2f_p$, $3f_p$, etc., which are harmonics of the main wave. The presence of harmonics indicates that the wave has the sharp crest and flat trough of highly nonsinusoidal waves often found near breaking. To complicate matters, Thompson (1977) has shown that about two-thirds of U.S. coastal wave records have more than one peak, indicating the presence of multiple wave trains. These wave trains most likely originated from different areas and have different directions of propagation. Moreover, it is possible to have a single-peak spectrum, which consists of two trains of waves of about the same frequency but different directions of propagation. In order to sort these issues out, observations of the directional spectrum are required. Figures II-1-31, II-1-32, and II-1-35 include examples of different frequency spectra providing some indication of their range of variability.

(4) Wave spectrum and its parameters.

(a) Two parameters are frequently used in the probability distribution for waves. These are the s*pectral width* ν and the *spectral bandwidth* ε, and are used to determine the narrowness of a wave spectra. These parameters range from 0 to 1, and may be approximated in terms of spectral moments by

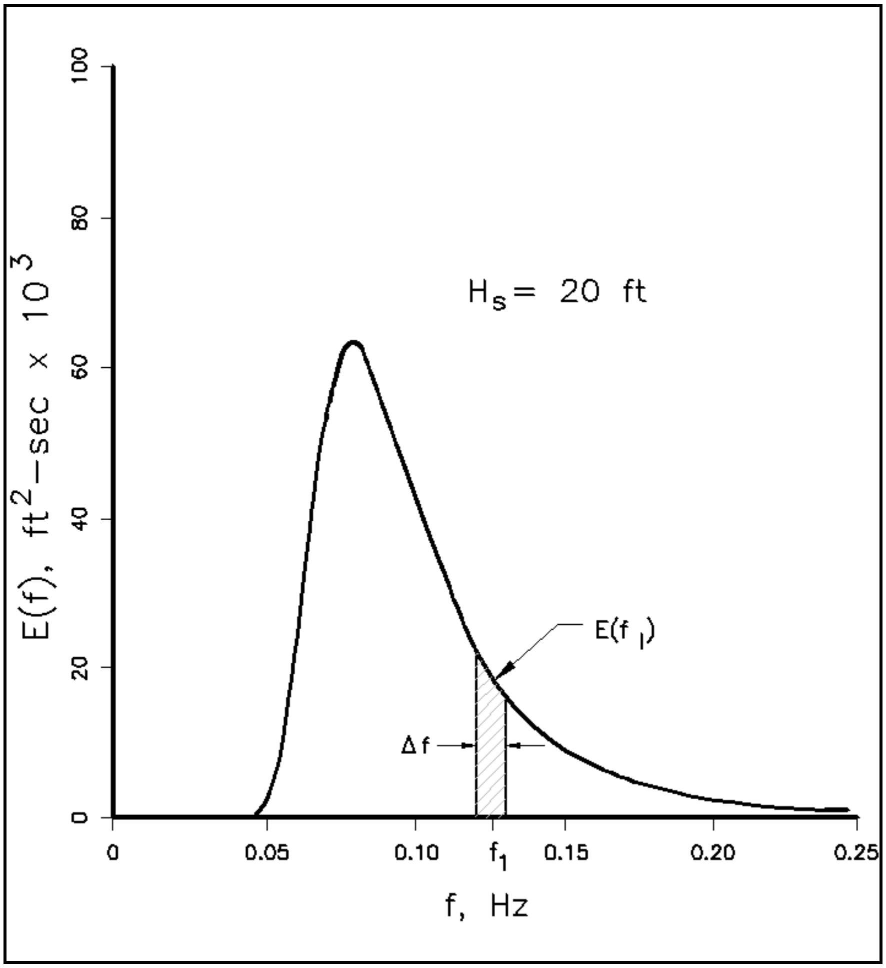

Figure II-1-37. Energy density and frequency relationship (Chakrabarti 1987)

$$\nu = \sqrt{\frac{m_0\, m_2}{m_1^2} - 1}$$

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0\, m_4}} \qquad \text{(II-1-150)}$$

(b) For a narrow-band spectrum, both ν and ε must be close to *0* (Figure II-1-38). For example, for the two most common empirical spectra, the *Pierson-Moskowitz (PM)* spectrum (Pierson and Moskowitz 1964) and the *JONSWAP* spectrum (Hasselmann et al. 1973), which are discussed in the next section, $\nu = 0.425$ and 0.389, respectively, with $\varepsilon = 1$ for both. Natural ocean waves, therefore, have a broad-banded spectrum.

(c) The values of ε obtained from a wave energy spectrum are generally not considered as the sole indication of how broad the spectra are. This is due to the amplification of the noise present in the wave energy spectral density at higher frequencies that enters into the calculation of the higher moments m_2 and m_4 in the above equation for ε. Goda (1974) proposed a *spectral peakedness parameter* called Q_p defined as

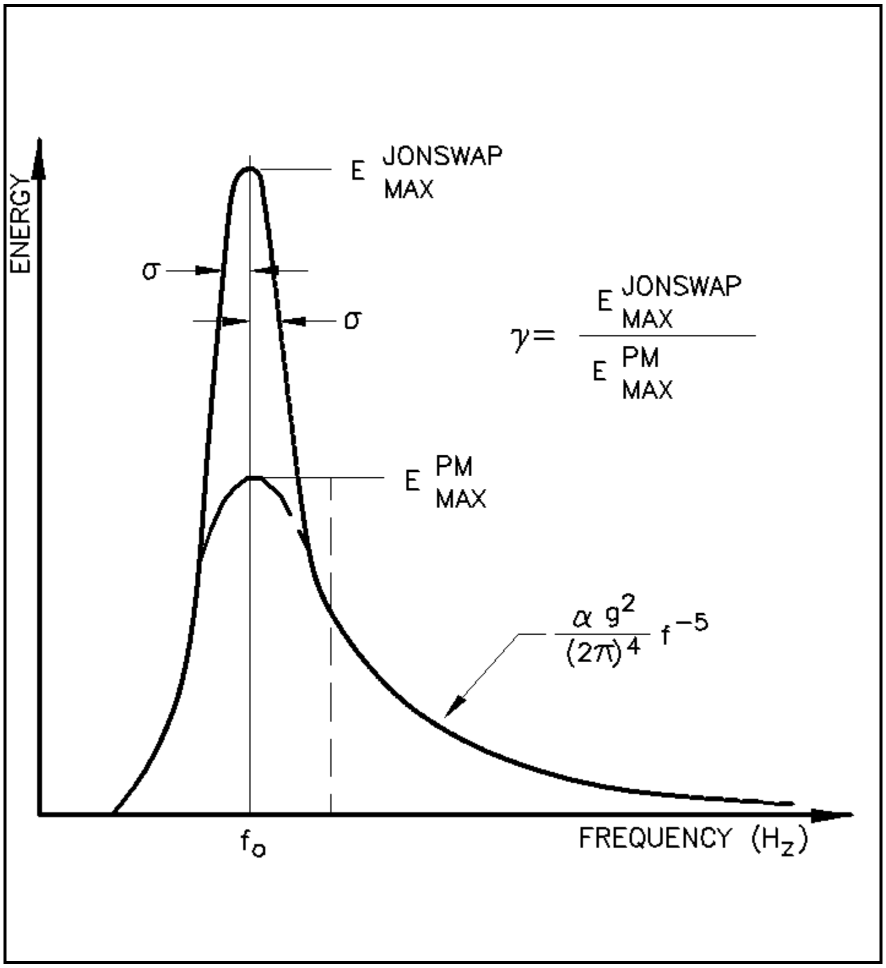

Figure II-1-38. Comparison of the PM and JONSWAP spectra (Chakrabarti 1987)

$$Q_p = \frac{2}{m_0^2} \int_0^{\infty} f\, E^2(f)\; df \tag{II-1-151}$$

which depends only on the first moment of the energy density spectrum, and is not directly related to ε. In general, a small ε implies that Q_p is large, and a large ε means Q_p is small.

(d) Approximate relations for most common wave parameters by the statistical analysis are

$$H_S = 4.0\,\sqrt{m_0} \quad ; \quad H_{1/10} = 5.1\,\sqrt{m_0}$$

$$T_z = \sqrt{\frac{m_0}{m_2}} \quad ; \quad T_c = \sqrt{\frac{m_2}{m_4}} \tag{II-1-152}$$

$$\bar{\eta} = \sqrt{m_0} \quad ; \quad \varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}}$$

(e) In deep and intermediate water depths, the significant wave height obtained by the spectral analysis using the above equation is usually greater than that from the wave train analysis. The zero-crossing period from the spectral method is only an approximation, while the period associated with the largest wave energy known as the *peak period* T_p, can only be obtained via the spectral analysis. In the spectral representation of swell waves, there is a single value of the peak period and wave energy decays at frequencies to either side. The spectra for storm waves is sometimes multi-peaked. One peak (not always the highest) corresponds to the swell occurring at lower frequencies. One and sometimes more peaks are associated with storm waves occurring at comparatively higher frequencies. In a double-peaked spectra for storm waves, the zero-crossing period generally occurs at higher frequencies than the peak period. In a multi-peaked spectrum, the zero-crossing period is not a measure of the frequency where peak energy occurs.

(5) Relationships among $H_{1/3}$, H_s, and H_{m0} in shallow water.

(a) By conception, significant height is the average height of the third-highest waves in a record of time period. By tradition, wave height is defined as the distance from crest to trough. Significant wave height H_s can be estimated from a wave-by-wave analysis in which case it is denoted $H_{1/3}$, but more often is estimated from the variance of the record or the integral of the variance in the spectrum in which case it is denoted H_{m0}. Therefore, H_s in Equation II-1-152 should be replaced with H_{m0} when the latter definition of H_s is implied. While $H_{1/3}$ is a direct measure of H_s, H_{m0} is only an estimate of the significant wave height which under many circumstances is accurate. In general in deep water $H_{1/3}$ and H_{m0} are very close in value and are both considered good estimates of H_s. All modern wave forecast models predict H_{m0} and the standard output of most wave gauge records is H_{m0}. Few routine field gauging programs actually compute and report $H_{1/3}$ and report as H_s with no indication of how it was derived. Where $H_{1/3}$ and H_s are equivalent, this is of little concern.

(b) Thompson and Vincent (1985) investigated how $H_{1/3}$ and H_{m0} vary in very shallow water near breaking. They found that the ratio $H_{1/3}/H_{m0}$ varied systematically across the surf zone, approaching a maximum near breaking. Thompson and Vincent displayed the results in terms of a nomogram (Figure II-1-40). For steep waves, $H_{1/3}/H_{m0}$ increased from 1 to about 1.1, then decreased to less than 1 after breaking. For low steepness waves, the ratio increased from 1 before breaking to as much as 1.3-1.4 at breaking, then decreased afterwards. Thompson and Vincent explained this systematic variation in the following way. As low steepness waves shoal prior to breaking, the wave shape systematically changes from being near sinusoidal to a wave shape that has a very flat trough with a very pronounced crest. Although the shape of the wave is significantly different from the sine wave in shallow water, the variance of the surface elevation is about the same, it is just arranged over the wave length differently from a sine wave. After breaking, the wave is more like a bore, and as a result the $H_{1/3}$ can be smaller (by about 10 percent) than H_{m0}.

(c) The critical importance of this research is in interpreting wave data near the surf zone. It is of fundamental importance for the engineer to understand what estimate of significant height he is using and what estimate is needed. As an example, if the data from a gauge is actually H_{m0} and the waves are near breaking, the proper estimate of H_s is given by $H_{1/3}$. Given the steepness and relative depth, $H_{1/3}$ may be estimated from H_{m0} by Figure II-1-40. Numerically modelled waves near the surf zone are frequently equivalent to H_{m0}. In this case, H_s will be closer to $H_{1/3}$ and the nomogram should be used to estimate H_s.

(6) Parametric spectrum models.

(a) In general, the spectrum of the sea surface does not follow any specific mathematical form. However, under certain wind conditions the spectrum does have a specific shape. A series of empirical expressions have been found which can be fit to the spectrum of the sea surface elevation. These are called *parametric spectrum models*, and are useful for routine engineering applications. A brief description of these follows.

(b) There are many forms of wave energy spectra used in practice, which are based on one or more parameters such as wind speed, significant wave height, wave period, shape factors, etc. Phillips (1958) developed an equation for the equilibrium range of the spectrum for a fully-developed sea in deep water, which became the basis of most subsequent developments. Phillips' equilibrium range is often written in terms of the *angular frequency* ω and is of the form

$$E(\omega) = \alpha g^2 \omega^{-5} \tag{II-1-153}$$

where α is the *Phillips' constant* (= .0081) and g the gravitational acceleration.

(c) One commonly used spectrum in wave hindcasting and forecasting projects is the single-parameter spectrum of *Pierson-Moskowitz PM* (Pierson and Moskowitz 1964). An extension of the PM spectrum is the *JONSWAP* spectrum (Hasselmann et al. 1973, 1976); this is a five-parameter spectrum, although three of these parameters are usually held constant. The relationship between PM and JONSWAP spectra is shown in Figure II-1-38. Other commonly used two-parameter wave spectra forms, including those proposed by Bretschneider (1959), ISSC (1964), Scott (1965), ITTC (1966), Liu (1971), Mitsuyasu (1972), Goda (1985a), and Bouws et al. (1985) are essentially derivatives of the *PM* and *JONSWAP* spectra. A six-parameter wave spectrum has been developed by Ochi and Hubble (1976). The utility of this spectrum is that it is capable of describing multi-peaks in the energy spectrum in a sea state mixed with swell (Figure II-1-39). Only the parametric wave spectra forms most often used in coastal engineering will be briefly discussed here.

(d) The equilibrium form of the *PM* spectrum for fully-developed seas may be expressed in terms of wave frequency f and wind speed U_w as

$$E(f) = \frac{0.0081 g^2}{(2\pi)^4 f^5} \exp\left(-0.24 \left[\frac{2\pi U_w f}{g} \right]^{-4} \right) \tag{II-1-154}$$

where U_w is the wind speed at 19.5 m above mean sea level. The *PM* spectrum describes a *fully-developed sea* with one parameter, the wind speed, and assumes that both the fetch and duration are infinite. This idealization is justified when wind blows over a large area at a constant speed without substantial change in its direction for tens of hours.

(e) The *JONSWAP* spectrum for *fetch-limited seas* was obtained from the Joint North Sea Wave Project - JONSWAP (Hasselmann et al. 1973) and may be expressed as

$$\begin{gathered} E(f) = \frac{\alpha g^2}{(2\pi)^4 f^5} \exp\left[-1.25 \left(\frac{f}{f_p} \right)^{-4} \right] \gamma^{\exp\left[-\frac{\left(\frac{f}{f_p} - 1 \right)^2}{2\sigma^2} \right]} \\ f_p = 3.5 \left[\frac{g^2 F}{U_{10}^3} \right]^{-0.33} \quad ; \quad \alpha = 0.076 \left[\frac{gF}{U_{10}^2} \right]^{-0.22} \quad ; \quad 1 \le \gamma \le 7 \\ \sigma = 0.07 \quad for\ f \le f_p \qquad and \quad \sigma = 0.09 \quad for \quad f > f_p \end{gathered} \tag{II-1-155}$$

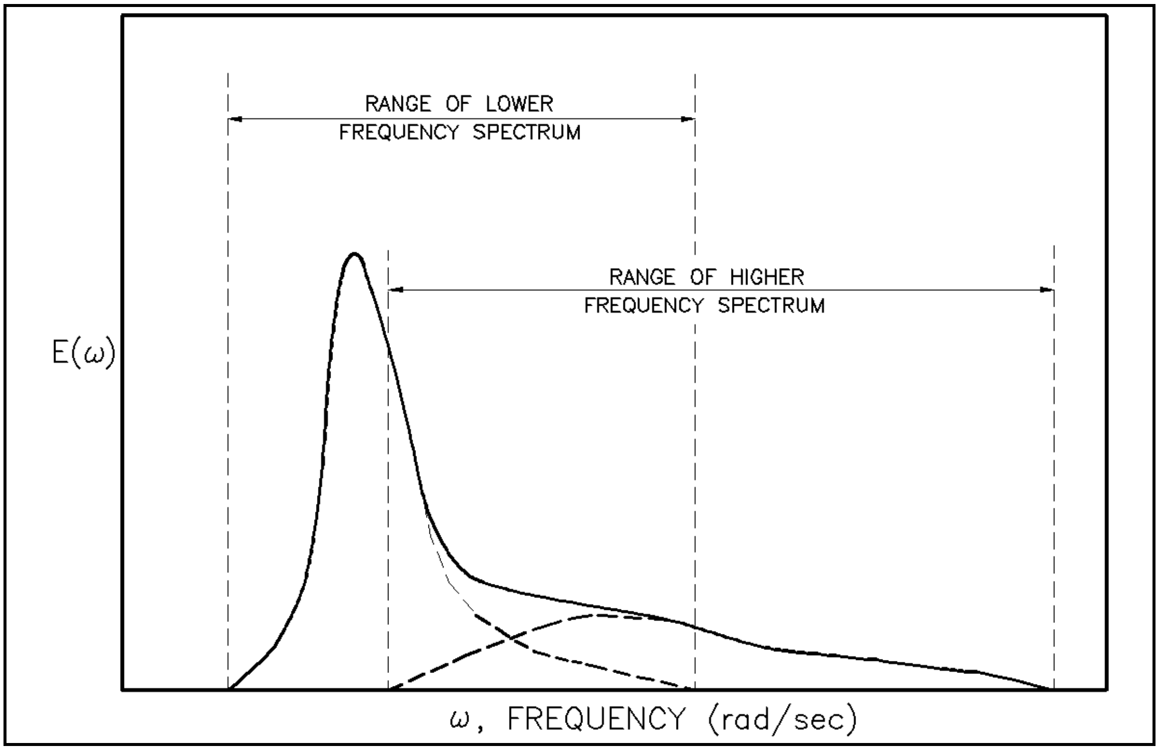

Figure II-1-39. Definition sketch for Ochi-Hubble spectrum (Ochi and Hubble 1976)

(f) In this equation, α is the scaling parameter, γ the peak enhancement factor, f_p the frequency at the spectral peak, U_{10} the wind speed at the elevation 10 m above the sea surface, F the fetch length. Figure II-1-38 qualitatively illustrates the relationship between JONSWAP and PM spectra. The JONSWAP spectrum can also be fitted mathematically to observed spectra by iteratively solving for d, γ, f_m , and σ.

(g) A six-parameter spectrum developed by Ochi and Hubble (1976) is the only wave spectrum which exhibits two peaks (Figure II-1-39), one associated with underlying swell (lower frequency components) and the other with locally generated waves (higher frequency components). It is defined as

$$E(\omega) = \frac{1}{4} \sum_{j=1}^{2} \frac{\left(\frac{4\lambda_j + 1}{4} \omega_{0j}^4 \right)^{\lambda_j}}{\Gamma(\lambda_j)} \frac{H_{sj}^2}{\omega^{4\lambda_j + 1}} \exp\left[-\frac{4\lambda_j + 1}{4} \left(\frac{\omega_{0j}}{\omega} \right)^4 \right] \qquad \text{(II-1-156)}$$

where H_{s1}, ω_{01}, and λ_1 are the significant wave height, modal frequency, and shape factor for the lower-frequency components while H_{s2}, ω_{02}, and λ_2 correspond to the higher frequency components (Figure II-1-39). The value of λ_1 is usually much higher than λ_2. For the most probable value of ω_{01}, it can be shown that λ_1 = 2.72, while λ_2 is related to H_s in feet as

$$\lambda_2 = 1.82 \; e^{(-0.027 H_s)} \qquad \text{(II-1-157)}$$

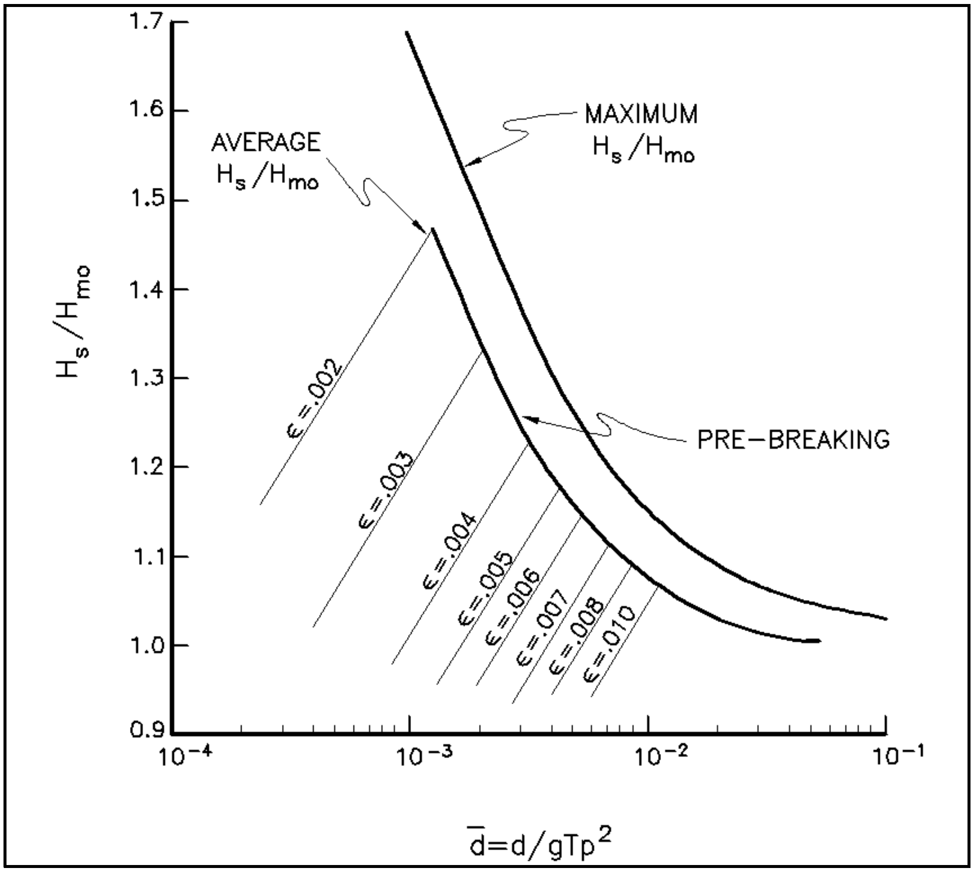

Figure II-1-40. Variation of H_s/H_{mo} as a function of relative depth $\bar{d}$ and significant steepness (Thompson and Vincent 1985)

(h) The parameters λ_j control the shape and the sharpness of the spectral peak of the Ochi-Hubble spectral model if in either spectral component (i.e., sea or swell) the values of H_{sj} and ω_{0j} are held constant. Therefore, λ_1 and λ_2 are called the *spectral shape parameters*. On the assumption of a narrow-bandedness of the entire Ochi-Hubble spectrum, an equivalent significant wave height may be calculated by

$$H_s = \sqrt{H_{s1}^2 + H_{s2}^2} \tag{II-1-158}$$

Note that for $\lambda_1 = 1$ and $\lambda_2 = 0$, the *PM* spectra may be recovered from this equation.

(i) In shallow water, the wave spectrum deviates from the standard spectra forms presented so far, and at frequencies above the peak, the spectrum no longer decays as f^{5}. Kitaigorodoskii et al. (1975) showed that the equilibrium range is proportional to *-3* power of the wave number, and thus, the form of the spectrum is of f^{3} in the high-frequency range. This change is attributed to the effect of water depth on wave spectrum and to the interaction between spectral components. Bouws et al. (1984) proposed a variation to the *JONSWAP* energy spectrum for representing wave spectra in finite-depth water. The spectrum so obtained, the product of *JONSWAP* and the *Kitaigorodoskii depth function* accounting for the influence of the water depth, is called the *TMA spectrum* after the names of three sources of data used in its development (Texel, Marsen, and Arsloe).

(j) Kitaigorodoskii et al. (1975) obtained the form of depth dependence as

$$\Phi(\omega, d) = \frac{\left[k^{-3}\frac{\partial k}{\partial \omega}\right]_{d=finite}}{\left[k^{-3}\frac{\partial k}{\partial \omega}\right]_{d=\infty}} \text{s} \tag{II-1-159}$$

(k) Thus, *Φ* is a *weighing factor* of the quantity in the bracket, which is determined from the ratio of the quantity evaluated for finite and infinite water depth cases. Using the linear wave theory, the above equation has been approximated by Kitaigorodoskii et. al. (1975) as

$$\Phi(\omega,d) \approx \begin{pmatrix} \frac{1}{2}\omega^2 & \textit{for}\ \omega \le 1 \\ 1 - \frac{1}{2}(2 - \omega)^2 & \textit{for}\ \omega > 1 \end{pmatrix} \tag{II-1-160}$$

(l) The *TMA* spectrum was intended for wave hindcasting and forecasting in water of finite depth. This spectrum is a modification of the *JONSWAP* spectrum simply by substituting Kitaigorodoskii's expression for effects of the finite depth equilibrium function□By using the linear wave theory, we find the following complete form of the *TMA* spectrum:

$$S_{TMA}(\omega,d) = S_{JONSWAP}(\omega)\ \Phi(\omega^*,d)$$

$$\Phi(\omega^*,d) = \frac{1}{f(\omega^*)}\left[1 + \frac{K}{\sinh\ K}\right] \quad ; \quad \omega^* = \omega\sqrt{\frac{d}{g}} \tag{II-1-161}$$

$$f(\omega^*) = \tanh^{-1}[k(\omega^*)d] \quad ; \quad K = 2\omega^{*2} f(\omega^*)$$

(m) In effect, this substitution transforms the decay or slope of the spectral density function of the *JONSWAP* spectrum in the high-frequency side from ω^{-5} to ω^{-3} type dependence during the shoaling process approximated by linear wave theory. Bouws et. al (1984) present equations for α, γ, and σ. As with the JONSWAP, the equation may be iteratively fit to an observed spectrum and α, γ, f_m , and σ may be estimated.

(n) The *PM*, *JONSWAP*, and *TMA* spectra can be estimated if something about the wind, depth and fetch are known. Furthermore, these spectral equations can be used as target spectra whose parameters can be varied to fit observed spectra which may have been measured. In the first situation, the value of the parameterization is in making an educated guess at what the spectrum may have looked like. The value in the second case is for ease of analytical representation. However, very often today engineering analyses are made on the basis of numerical simulations of a specific event by use of a numerical model (see Part II-2). In this case, the model estimates the spectrum and a parametric form is not required.

(7) Directional spectra.

(a) The wave spectra described so far have been one-dimensional frequency spectra. Wave direction does not appear in these representations, and thus variation of wave energy with wave direction was not considered. However, the sea surface is often composed of many waves coming from different directions. In addition to wave frequency, the mathematical form of the sea state spectrum corresponding to this situation should therefore include the wave direction θ. Each wave frequency may then consist of waves from different directions θ. The wave spectra so obtained are *two-dimensional*, and are denoted by $E(f,\theta)$. Figures II-1-33 and II-1-34 display directional spectra.

(b) Measurement of a directional spectrum typically involves measurement of either the same hydrodynamic parameter (such as surface elevation or pressure) at a series of nearby locations (within one to tens of meters) or different parameters (such as pressure and two components of horizontal velocity) at the same point. These records are then cross-correlated through a cross-spectral analysis and a directional spectrum is estimated. In general, the more parameters or more locations involved, the higher the quality of the directional spectrum obtained. The procedures for converting measurements into estimates of the directional spectrum are outside the scope of this chapter. Part VII-3 of the CEM and Dean and Dalrymple (1991) provide some additional details on this subject.

(c) The major systems routinely employed at the present time for measuring directional spectra include directional buoys, arrays of pressure or velocity gauges, and the p-U-V technique. With directional buoys, pitch-roll-and-heave or heave-and-tilt methods are used. Most directional buoys are emplaced in deeper water. Arrays of pressure gauges or velocity gauges arranged in a variety of shapes (linear, cross, star, pentagon, triangle, rectangle, etc.) are also used, but these are usually restricted to shallower water. The p-U-V technique uses a pressure gauge and a horizontal component current meter almost co-located to measure the wave field. This can be used in shallow or in deeper water if there is something to attach it to near the surface. Other techniques include arrays of surface-piercing wires, triaxial current meters, acoustic doppler current meters, and radars.

(d) A mathematical description of the directional sea `state is feasible by assuming that the sea state can be considered as a superposition of a large number of regular sinusoidal wave components with different frequencies and directions. With this assumption, the representation of a spectrum in frequency and direction becomes a direct extension of the frequency spectrum alone, allowing the use of *FFT* method. It is often convenient to express the wave spectrum $E(F,\theta)$ describing the angular distribution of wave energy at respective frequencies by

$$E(f,\theta) = E(f)\ G(f,\theta) \qquad \text{(II-1-162)}$$

where the function $G(f,\theta)$ is a dimensionless quantity, and is known as the *directional spreading function*. Other acronyms for $G(f,\theta)$ are the *spreading function*, *angular distribution function*, and the *directional distribution*.

(e) The one-dimensional spectra may be obtained by integrating the associated directional spectra over θ as

$$E(f) = \int_{-\pi}^{\pi} E(f,\theta)\ d\theta \qquad \text{(II-1-163)}$$

(f) It therefore follows from the above last two equations that *G(f,θ)* must satisfy

$$\int_{-\pi}^{\pi} G(f,\theta)\ d\theta \ = \ 1 \tag{II-1-164}$$

(g) The functional form of *G(f,θ)* has no universal shape and several proposed formulas are available. In the most convenient simplification of *G(f,θ)*, it is customary to consider *G* to be independent of frequency *f* such that we have

$$G(\theta) \ = \ \frac{2}{\pi}\ \cos^2\theta \quad for \quad |\theta| \ < \ 90^\circ \tag{II-1-165}$$

(h) This cosine-squared distribution is due to St. Denis and Pierson (1953), and testing with field data shows that it reproduces the directional distribution of wave energy. Longuet-Higgins (1962) found the cosine-power form

$$\begin{aligned} G(\theta) \ &= \ C(s)\ \cos^{2s}\frac{\theta \ - \ \bar{\theta}}{2} \\ C(s) \ &= \ \frac{\sqrt{\pi}}{2\pi}\ \frac{\Gamma(s \ + \ 1)}{\Gamma\left(s \ + \ \dfrac{1}{2}\right)} \end{aligned} \tag{II-1-166}$$

where θ is the principal (central) direction for the spectrum, s is a controlling parameter for the angular distribution that determines the peakedness of the directional spreading, *C(s)* is a constant satisfying the normalization condition, θ is a counterclockwise measured angle from the principal wave direction, and Γ is the Gamma function.

(i) Mitsuyasu et al. (1975), Goda and Suzuki (1976), and Holthuijsen (1983) have shown that for wind waves, the parameter s varies with wave frequency and is related to the stage of wave development (i.e., wind speed and fetch) by

$$s \ = \ \begin{cases} s_{\max}\left(\dfrac{f}{f_p}\right)^{5} & for\ f \ \le \ f_p \\ s_{\max}\left(\dfrac{f}{f_p}\right)^{-2.5} & for\ f \ > \ f_p \end{cases} \tag{II-1-167}$$

where s_{max} and f_p are defined as

$$\begin{aligned} s_{\max} \ &= \ 11.5\left(\frac{2\pi f_p U}{g}\right)^{-2.5} \\ \frac{2\pi f_p U}{g} \ &= \ 18.8\left(\frac{gF}{U^2}\right)^{-0.33} \end{aligned} \tag{II-1-168}$$

(j) In the above equations, U is the wind speed at the 10-m elevation above the sea surface and F is the fetch length. These equations remain to be validated with field data for wind waves. The parameter s for shallow-water waves may also vary spatially during wave transformation. This is due to refraction. A large value greater that 50, may be necessary if dependence of s_{max} on refraction is of concern. For deepwater applications where wind waves are jointly present with swells in deep water, Goda and Suzuki (1976) proposed the following values for s_{max} : 10 for wind waves, and 25 for swells present with wind waves of relatively large steepness, and 75 for swells with wind waves of small steepness. Under simple wind wave conditions, the spreading function may be approximated by the equations provided. They are typical of deepwater wind seas for which the wind has been constant. If the wind has shifted in direction, if there is significant swell, or if the waves are in shallow water, the directional distribution may be different than the shape functions presented.

(8) Wave groups and groupiness factors.

(a) Measurements of waves usually show a tendency of grouping between waves that is; high waves; often seem to be grouped together. Examination of the sea surface profile records indicates that wave heights are not uniform and they occur in successive groups of higher or lower waves. The interest in wave groups is stimulated by the fact that wave grouping and associated nonlinear effects play an important role in the long-period oscillation of moored vessels (Demirbilek 1988, 1989; Faltinsen and Demirbilek 1989), surf beats, irregular wave runup, resonant interaction between structures (Demirbilek and Halvorsen 1985; Demirbilek, Moe, and Yttervoil 1987;), and other irregular fluctuations of the mean water level nearshore (Goda 1985b; 1987). Unfortunately there is no way to predict grouping.

(b) Wave grouping is an important research topic and there are several ways to quantify wave grouping. These include the smoothed instantaneous wave energy history analysis (Funke and Mansard 1980), the concept of the run of wave heights (Goda 1976), and the Hilbert transform. A short exposition of the wave grouping analysis is provided here.

(c) The length of wave grouping can be described by counting the number of waves exceeding a specified value of the wave height which could be the significant, mean, or other wave height. The succession of high wave heights is called *a run* or *a run length* with an associated wave number j_1. The definition sketch for two wave groups is shown in Figure II-1-41 with the threshold wave height limit set at $H = H_c$. The recurrence interval or repetition length above the threshold value of wave height is called the *total run* denoted by j_2.

(d) The group occurrence for N waves with k number of lags between waves in a sequence in a record may be defined in terms of a correlation coefficient. The correlation coefficient R_H so defined will describe the correlation between wave heights as a function of the mean μ and standard deviation σ and is given by

$$\begin{aligned} R_H &= \frac{1}{\sigma_0}\,\frac{1}{N-k}\sum_{i=1}^{N-k}(H_i-\mu)(H_{i+k}-\mu) \\ \sigma_0 &= \frac{1}{N}\sum_{i=1}^{N}(H_i-\mu)^2 \end{aligned} \tag{II-1-169}$$

(e) Thus, R_H varies with the number of lags k between waves. If the succeeding waves are uncorrelated, then $R_H \rightarrow 0$ as $N \rightarrow \infty$. Real wave data indicate that $R_H(1) \approx 0.20$ to 0.40 while $R_H(k) \approx 0$ for $k > 1$. Furthermore, a positive value of R_H suggests that large waves tend to be succeeded by large waves, and small waves by other small waves.

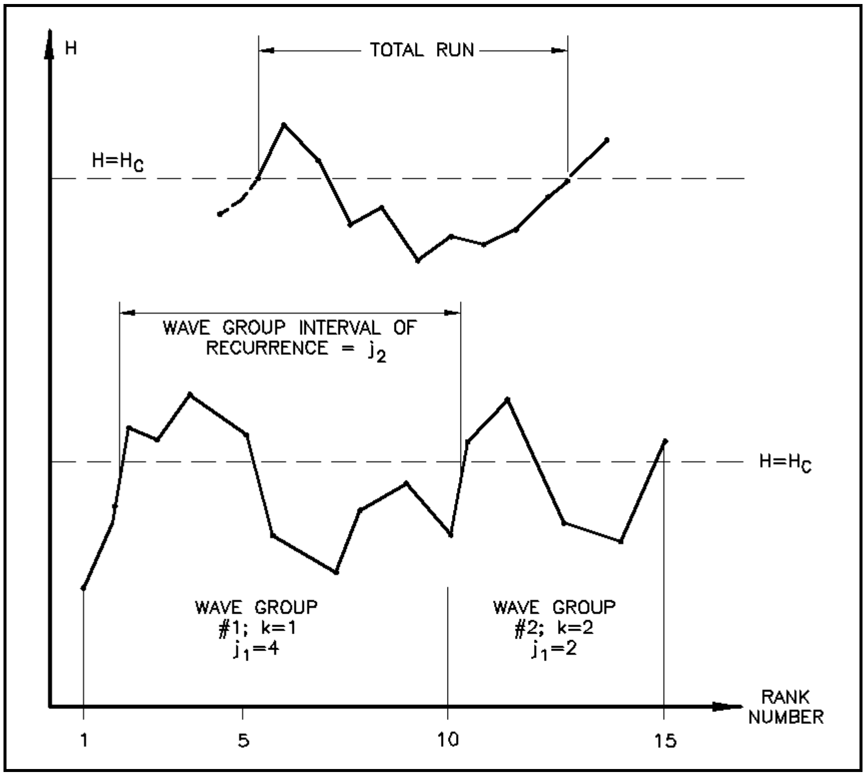

Figure II-1-41. Identification and description of wave groups through ordered statistics (Goda 1976)

(f) Assuming that successive wave heights are uncorrelated, the probability of *a* run length j_1 is (Goda 1976)

$$P(j_1) = p^{(j_1-1)}\,(1-p) \tag{II-1-170}$$

in which p is the occurrence probability for $H > H_c$. The mean and standard deviation of j_1 are

$$\begin{aligned} \mu_{j_1} &= \frac{1}{q} \quad ; \quad q = 1 - p \quad ; \quad \sigma_{j_1} = \frac{\sqrt{1-q}}{q} \\ p &= p(H{>}H_c) = \exp\left[-\frac{1}{8}\eta_c^2\right] \quad ; \quad \eta_c = \frac{H_c}{\sigma_\eta} \end{aligned} \tag{II-1-171}$$

(g) The probability of a *total run* with the length j_2 can be derived by mathematical induction as

$$\mu_{j_2} = \frac{1}{p} + \frac{1}{q} \quad ; \quad \sigma_{j_2} = \sqrt{\frac{p}{q^2} + \frac{q}{p^2}} \tag{II-1-172}$$

where it has been assumed that successive wave heights are uncorrelated. Successive wave heights of the real ocean waves are mutually correlated, and the degree of correlation depends on the sharpness of the spectral peak. The effect of spectral bandwidth on wave height distribution has been considered by Kimura (1980), Tayfun (1983a), and Longuet-Higgins (1984). Tayfun has shown that the parameter that best describes the spectral peakedness is the correlation coefficient of the wave envelope, relating wave height variation between successive wave heights. This coefficient R_{HH} may be calculated as (Tayfun 1983)

$$R_{HH} = \frac{E(\lambda) - (1-\lambda^2)\,\dfrac{K(\lambda)}{2} - \dfrac{\pi}{4}}{1 - \dfrac{\pi}{4}}$$

$$\lambda(\overline{T}) = \frac{1}{m_0}\sqrt{A^2 + B^2} \qquad \text{(II-1-173)}$$

$$A = \int_0^{\infty} E(f)\,\cos\,2\pi f \overline{T}\,df \quad ; \quad B = \int_0^{\infty} E(f)\,\sin\,2\pi f \overline{T}\,df$$

(h) By further assuming that Rayleigh distribution is suitable for the consecutive wave heights, the joint probability density function $p(H_1, H_2)$ for two successive wave heights H_1 and H_2 in the wave group may then be established. See Tayfun (1983) for details.

(i) The correlation coefficient R_{HH} takes a value of about 0.2 for wind waves and 0.6 or greater for swells (Goda 1976), a clear indication that wind waves rarely develop significant grouping of high waves. Su (1984) has shown that the wave group containing the highest wave in a record is often longer than the ordinary groups of high waves, and that the extreme wave usually consists of three high waves with the highest greater than the significant wave height. Wave groups and their characteristics have been investigated by analyzing the successive wave groups (Goda 1976 and Kimura 1980).

(j) Wave grouping and its consequences are of significant concern, but there is little guidance and few practical formulae for use in practical engineering. The engineer needs to be aware of its existence and, for designs that would be sensitive to grouping-related phenomena, attempt to evaluate its importance to the problem of concern. This may involve performing numerical simulations or physical model simulations in which a wide variety of wave conditions are tested and are designed to include those with high levels of groupiness. The procedures for this lie beyond the scope of the CEM.

(9) Random wave simulation.

(a) Given a one-dimensional parametric spectrum model or an actual wave energy density spectrum, it is sometimes necessary to use these quantities to calculate the height, period, and phase angle of a wave at a particular frequency. Such an approach for simulating random waves from a known wave spectra is sometimes termed the *deterministic spectral amplitude method*, since individual wave components in this superposition method are deterministic (Borgman 1967). The method is also called the *random phase method* because the phases of individual components are randomly chosen (Borgman 1969). Random waves simulated by this approach may not satisfy the condition of a Gaussian sea unless $N \rightarrow \infty$ in the limit. In practice, for $200 \leq N \leq 1200$ components, the spectrum can be duplicated accurately.

(b) The wave profile generated by simulation methods is used in a number of engineering applications in spite of requiring a large number of components and considerable computer time. For example, random wave simulation is frequently used during modeling studies in a wave tank for duplicating a required target wave energy density spectrum. Random wave profiles are also extensively used in numerical models for calculating structural loads and responses due to a random sea. The simulation method permits direct prediction of the wave particle kinematics at any location in a specified water depth for given wave height-period pair and random phase angle. The ARMA algorithms (Spanos 1983) and digital simulation methods

(Hudspeth and Chen 1979) are two alternatives for simulating random waves from a given one-dimensional spectrum.

(c) There are two ways for simulating wave surface profiles from known wave spectra; deterministic and non-deterministic spectral amplitude methods. In the deterministic spectral simulation method, the wave height, period, and phase angle associated with a frequency f_1 whose corresponding energy density is $E(f_1)$ may be obtained from

$$
\begin{aligned}
H(f_1) &= H|_{f_1} = 2\sqrt{2E(f_1)\,\Delta f} \\
T(f_1) &= T|_{f_1} = \frac{1}{f_1} \\
\varepsilon(f_1) &= \varepsilon|_{f_1} = 2\pi r_N
\end{aligned}
\tag{II-1-174}
$$

where the phase angle ε is arbitrary since r_N is a random number between zero and one. The time series of the wave profile at a point x and time t may be computed by (Tucker et al. 1984)

$$
\eta(x,t) = \sum_{n=1}^{N} H(n)\cos\left[k(n)x - 2\pi f(n)t + \varepsilon(n)\right] \tag{II-1-175}
$$

where $k(n) = 2\pi/L(n)$, and $L(n)$ is the wavelength corresponding to the n^{th} frequency $f(n)$; N the total number of frequency bands of width Δf. It is not required to divide the spectrum curve equally, except that doing so greatly facilitates computations. The value of wave height is sensitive to the choice of Δf, but as long as Δf is small, this method produces a satisfactory random wave profile. The use of the equal increments, Δf, requires N to be greater than 50 to assure randomness and duplicating the spectrum accurately.

(d) In the non-deterministic spectral amplitude method, the wave surface profile is represented in terms of two independent Fourier coefficients. These Gaussian distributed random variables a_n and b_n with zero mean and variance of $E(f)\,\Delta f$ are then obtained from

$$
\begin{aligned}
\eta(x,t) = &\sum_{n=1}^{N} a_n \cos\left[k(n)x - 2\pi f(n)t\right] \\
&+ \sum_{n=1}^{N} b_n \sin\left[k(n)x - 2\pi f(n)t\right]
\end{aligned}
\tag{II-1-176}
$$

(e) In essence, an amplitude and a phase for individual components are replaced by two amplitudes, the coefficients of cosine and sine terms in the wave profile. This random coefficient scheme may yield a realistic representation of a Gaussian sea, provided that N is large for a true random sea. This method differs from the deterministic spectral amplitude approach by ensuring that sea state is Gaussian. Elgar et al. (1985) have considered simultaneous simulation of both narrow and broad-banded spectra using more than 1000 Fourier components, and concluded that both simulation methods yield similar statistics. These approaches may be extended to the two-dimensional case. This is beyond the scope of the CEM.

(10) Kinematics and dynamics of irregular waves. In the above sections of the CEM we have considered definition of irregular wave parameters and development of methods to measure them and use them analytically. Velocities, pressures, accelerations, and forces under irregular waves are estimated analytically in three ways. In the first, an individual wave is measured by either a wave-by-wave analysis or constructed synthetically (such as choosing, H_s, T_z, and a direction) and monochromatic theory is used to estimate the

desired quantities at a given wave phase (Faltinsen and Demirbilek 1989). In the second, pressure, velocity, and acceleration spectra are estimated by applying linear theory to translate the surface elevation spectra to the desired parameter (Dean and Dalrymple 1991). Finally, the random wave simulation technique may be used to synthetically generate a surface time history and corresponding kinematic and dynamic properties (Borgman 1990). Of the three methods, the last may provide the most realistic results, but it is also the most complex approach. These methods lie beyond the CEM and generally require the assistance of a knowledgeable oceanographic engineer.

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II-1-5. Definitions of Symbols

α	Dimensionless scaling parameter used in the JONSWAP spectrum for fetch-limited seas
α	Phillips' constant (= 0.0081) (Equation II-1-153)
α_x , α_z	Fluid particle accelerations [length/time^2]
γ	Peak enhancement factor used in the JONSWAP spectrum for fetch-limited seas
Γ	Gamma function
Δp	Difference in pressure at a point due to the presence of the solitary wave [force/length^2]
Δt	Sampling interval (Equation II-1-144) [time]
ε	Dimensionless pertubation expansion parameter
ε	Spectral bandwidth used in the probability distribution for waves (Equation II-1-150)
ε	Wave steepness (= H/L)
ζ	Vertical displacement of the water particle from its mean position (Equation II-1-27) [length]
η	Displacement of the water surface relative to the SWL [length]
$\eta(t)$	Sea state depicted in time series of the wave profile [length]
$\eta(x,t)$	Time series of the wave profile at a point x and time t (Equation II-1-175) [length]
$\overline{\eta}$	Mean or expected value of the sea state (Equation II-1-119) [length]
$\eta_{envelope}$	Envelope wave form of two or more superimposed wave trains (Equation II-1-48) [length]
η_{rms}	Root-mean-square surface elevation [length]
θ	Angle between the plane across which energy is being transmitted and the direction of wave advance [deg]
θ	Principal (central) direction for the spectrum measured counterclockwise from the principal wave direction [deg]
$\lambda_{1,2}$	Spectral shape parameters controlling the shape and the sharpness of the spectral peak of the Ochi-Hubble spectral model
μ_η	Mean or expected value of the sea state (Equation II-1-119) [length]
ν	Dimensionless Spectral width parameter
ξ	Horizontal displacement of the water particle from its mean position (Equation II-1-26) [length]

ρ	Mass density of water (salt water = 1,025 kg/m^3 or 2.0 slugs/ft^3; fresh water = 1,000kg/m^3 or 1.94 slugs/ft^3) [force-time^2/length^4]
ρ_η	Autocorrelation coefficient (Equation II-1-122)
σ_η	Standard deviation or square root of the variance
Φ	Velocity potential [length^2/time]
Φ	Weighing factor (Equation II-1-159)
Ψ	Stream function
ω	Wave angular or radian frequency (= $2\pi/T$) [time^{-1}]
a	Wave amplitude [length]
A, B	Major- (horizontal) and minor- (vertical) ellipse semi-axis of wave particle motion (Equations II-1-34 and II-1-35) [length]. The lengths of A and B are measures of the horizontal and vertical displacements of the water particles (Figure II-1-4).
B_j	Dimensionless Fourier coefficients (Equation II-1-103)
C	Phase velocity or wave celerity ($= L/T = \omega/k$) [length/time]
$C(s)$	Dimensionless constant satisfying the normalization condition
C_g	Wave group velocity [length/time]
cn	Jacobian elliptic cosine function
d	Water depth [length]
E	Total wave energy in one wavelength per unit crest width [length-force/length^2]
$E(\omega)$	Phillips' equilibrium range of the spectrum for a fully-developed sea in deep water (Equation II-1-153)
$E(f)$	One-dimensional spectrum or frequency energy spectrum or wave energy spectral density (Equation II-1-144)
$\overline{E}$	Total average wave energy per unit surface area or specific energy or energy density (Equation II-1-58) [length-force/length^2]
$\overline{E_k}$	Kinetic energy per unit length of wave crest for a linear wave (Equation II-1-53) [length-force/length^2]
$\overline{E_p}$	Potential energy per unit length of wave crest for a linear wave (Equation II-1-55) [length-force/length^2]
$E[\eta]$	Mean or expected value of the sea state (Equation II-1-119) [length]
F	Fetch length [length]
f_p	Frequency of the spectral peak used in the JONSWAP spectrum for fetch-limited seas [time^{-1}]

g	Gravitational acceleration [length/time^2]
$G(f,\theta)$	Dimensionless directional spreading function
H	Wave height [length]
$\overline{H}$	Mean wave height [length]
$H_{1/3}$	Significant wave height [length]
$H_{1/n}$	The average height of the largest 1/n of all waves in a record [length]
H_d	Design wave height [length]
H_j	Ordered individual wave heights in a record (Equation II-1-115) [length]
H_{max}	Maximum wave height [length]
H_{rms}	Root-mean-square of all measured wave heights [length]
H_s	Significant wave height [length]
k	Modulus of the elliptic integrals
k	Number of lags between waves in a sequence in a record (Equation II-1-169)
k	Wave number (= $2\pi/L = 2\pi/CT$) [length^{-1}]
$K(k)$	Complete elliptic integral of the first kind
K_z	Pressure response factor (Equation II-1-43) [dimensionless]
L	Wave length [length]
M	Dimensionless parameter which is a function of H/d used in calculating water particle velocities for a solitary wave (Equations II-1-92 & II-1-93) (Figure II-1-17).
$m_{0,1,2,4}$	Moments of the wave spectrum
N	Dimensionless correction factor in determination of η from subsurface pressure (Equation II-1-46)
N	Dimensionless parameter which is a function of H/d used in calculating water particle velocities for a solitary wave (Equations II-1-92 & II-1-93) (Figure II-1-17).
N	Number of waves in a record
N_c	Number of crests in the wave record
N_z	Number of zero-upcrossings in the wave record
$-_o$	The subscript 0 denotes deepwater conditions
p	Pressure at any distance below the fluid surface [force/length^2]
P	Probability

$p(x)$	Probability density
$P(x)$	Probability distribution function - fraction of events that a particular event is not exceeded (Equation II-1-124)
$\overline{P}$	Wave power or average energy flux per unit wave crest width transmitted across a vertical plane perpendicular to the direction of wave advance (Equation II-1-59) [length-force/time-length]
p_a	Atmospheric pressure [force/length^2]
p'	Total or absolute subsurface pressure -- includes dynamic, static, and atmospheric pressures (Equation II-1-39) [force/length^2]
$Q(x)$	Probability of exceedence (Equation II-1-128)
Q_p	Spectral peakedness parameter proposed by Goda (1974) (Equation II-1-151)
R	Bernoulli constant (Equation II-1-102)
R	Cross-correlation coefficient - measures the degree of correlation between two signals (Equation II-1-123)
R_η	Autocorrelation or autocovariance function of the sea state (Equation II-1-121)
R_H	Correlation coefficient describing the correlation between wave heights as a function of μ and standard deviation σ (Equation II-1-169)
R_{HH}	Correlation coefficient of the wave envelope, relating wave height variation between successive wave heights (Equation II-1-173)
s	Dimensionless controlling parameter for the angular distribution that determines the peakedness of the directional spreading
T	Wave period [time]
$\overline{T}$	Mean wave period [time]
$\overline{T}_c$	Mean crest period [time]
$\overline{T}_z$	Mean zero-upcrossing wave period [time]
T_c	Average wave period between two neighboring wave crests (Equation II-1-116) [time]
T_p	Wave period associated with the largest wave energy [time]
T_r	Sampling record length [time]
T_r	Total wave record length [time]
T_z	Zero-crossing wave period (Equation II-1-116) [time]
T_*^{max}	Most probable maximum wave period (Equation II-1-141) [time]
u	Fluid velocity (water particle velocity) in the x-direction [length/time]

U	Current speed [length/time]
U	Wind speed at the 10-m elevation above the sea surface [length/time]
$\overline{U}(z)$	Mass transport velocity (Equation II-1-69) [length/time]
u_{max}	Maximum fluid velocity in the horizontal direction [length/time]
U_P	Universal parameter for classification of wave theories
U_R	Dimensionless Ursell number (Equation II-1-67)
U_w	Wind speed at 19.5 m above mean sea level (Equation II-1-155) [length/time]
V	Volume of water within the wave above the still-water level per unit crest (Equation II-1-90) [length^3/length of crest]
w	Fluid velocity (water particle velocity) in the z-direction [length/time]
y_c	Vertical distance from seabed to the wave crest (Equation II-1-79) [length]
y_s	Vertical distance from seabed to the water surface (Equation II-1-77) [length]
y_t	Vertical distance from seabed to the wave trough [length]
z	Water depth below the SWL (Figure II-1-1) [length]

II-1-6. Acknowledgments

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Bernard LeMéhauté, Ph.D., University of Miami, Miami, Florida, (deceased).
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