Chapter 9
Preanalysis and Network Adjustment

9-1. General

This chapter discusses preanalysis and network adjustment techniques for processing deformation surveying observations. A basic problem in surveying is to determine coordinates for a network of points using various types of measurements that establish a known geometrical relationship between them. Points with unknown spatial coordinates are connected into the network by the measurements. Surveying observation equations provide a mathematical model that organizes the measurements into a consistent form where methods for finding a unique solution for the unknown coordinates are possible. Instrumentation surveys should always be designed to gather more data than is absolutely necessary to determine station coordinates because this improves the reliability of the results. With extra measurements, unavoidable random errors create discrepancies depending on which set of measurements is used, and there is no unique solution for the coordinates. When this is the case, network adjustment techniques are used to estimate the most accurate set of possible coordinates by the least squares principle of minimizing errors in the measurements. Network adjustment permits all of the available survey measurements to be processed together to determine a weighted mean value for the coordinates. Coordinate accuracy is determined by the application of error propagation to the observation equations. A pre-determined uncertainty (standard deviation) is assigned each measurement, which then propagates to the coordinates during the adjustment. The probable error in the coordinates is reported by the point confidence ellipse for each point or by the relative confidence ellipse between two points. It is essential to determine the positioning accuracy, and without adequate knowledge of the probable error in coordinates the survey should be considered incomplete.

9-2. Theory of Measurements

a. *Random variables.* Survey measurements are geometrical quantities with numerical values assigned to them with a certain accuracy. The 'observable' is a term used to indicate the type of surveying measurement, such as direction, distance, azimuth, coordinate difference, and height difference. An 'observation' refers to the specific number assigned to the observable. Surveying observations always contain random deviations where each observation error is called an instance of a random variable. Random variables have a well-known expected frequency distribution (Gaussian or normal) that can be rigorously described by simple parameters.

b. Measures of central tendency. The influence of random error is minimized by computing the mean value of a series of observations, which is also the most probable estimate of the unknown true value. The mean value of a sample can be computed as follows:

$$\mathbf{x} = \sum (x_i) \; / \; n \qquad \text{(Eq 9-1)}$$

where

$\mathbf{x}$ = sample mean
x_i = observations (where i = 1 through n)
n = number of observations

The accuracy of the sample mean is very sensitive to bias or systematic error in the measurements. An incorrect value for the sample mean, due to measurement bias, is shifted away from the true population mean.

c. Measures of dispersion. The variance of a sample of measurements is an estimator of precision or repeatability. The variance describes how closely the measurements are grouped around the sample mean. The sample variance is computed from the average of the squares of the measurement deviations about the mean. A large variance implies lower precision and greater dispersion. The standard deviation or unbiased root mean square (RMS) error is the positive square root of the variance. The sample variance (s^2), or population variance (σ^2) are calculated as follows:

Sample Variance:

$$s^2 = \left(\sum \left(x_i - \mathbf{x} \right)^2 \right) / (n - 1) \quad \text{(Eq 9-2)}$$

Population Variance:

$$\sigma^2 = \left(\sum \left(x_i - \mu \right)^2 \right) / N \quad \text{(Eq 9-3)}$$

where

x_i = observations (where i = 1 through n)

s^2 = sample variance
$\mathbf{x}$ = sample mean = $(\sum x_i) / n$
n = number of observations

σ^2 = population variance
μ = population mean = $(\sum x_i) / N$
N = number of elements within the population

When the population mean (μ) is unknown, the sample variance (s^2) is computed using the sample mean ($\mathbf{x}$). Another measure of dispersion is the range (R) of a data sample:

$$R = | x_{MAX} - x_{MIN} | \quad \text{(Eq 9-4)}$$

where

R = range
x_{MAX} = maximum value
x_{MIN} = minimum value

The range R is the absolute value of the difference between the minimum and maximum value.

d. Blunders. Blunders are the result of mistakes by the user or inadvertent equipment failure. For example, an observer may misread a level rod by a tenth of a foot or a malfunctioning data recorder may cause erroneous data storage. Blunders are minimized by adopting consistent measurement procedures that contain self-checks. Blunders must be detected and removed before a final usable set of data can be compiled. Techniques used to identify blunders in the data include:

- calculation of loop and traverse closures to check whether the misclosure is within tolerances
- standard deviation of a series of measurements to check if the spread is within tolerance
- comparison of misclosures to a well-determined or to an assumed true position

e. *Systematic error*. Systematic error is the result of an inadequate mathematical model that omits some necessary physical or mathematical parameter that is necessary to exactly describe the relationship between measurements and coordinates. Systematic error is removed through calibrations and data reductions that are made before entering the data into the network adjustment software. Unremoved systematic errors are detected statistically by an examination of the observation residuals and using the Chi-square Goodness-of-Fit test.

f. *Random error.* Random error is an inherent result of the measurement process. Least squares processing requires the assumption that only random errors exist within the data. If all systematic errors and blunders have been removed, then observations will contain only random error.

9-3. Least Squares Adjustment

a. *General.* The Least Squares principle is widely applied to the adjustment of surveying measurements because it defines a consistent set of mathematical and statistical procedures for finding unknown coordinates using redundant observations. If the number of available measurements exceeds the minimum number required for a unique solution, then an adjustment is used to optimally fit a solution to all measurements. Application of the least squares principle relies on the condition that the weighted sum of the squares of the residuals is a minimum. The least squares adjusted coordinates are unique and have both maximum probability of being correct and minimum probable error.

b. *Observation weighting.* Not all surveying data will be collected with the same level of precision. Therefore, the measurements are weighted relative to each other according to their different precisions. Weights are based on a standard deviation prescribed to each measurement, and these are calculated (by the adjustment software) as the inverse of the measurement variance as follows:

$$w_i = 1 / \sigma_i^2 \qquad \text{(Eq 9-5)}$$

where

w_i = observation weight value
σ_i = measurement standard deviation

Observation weighting gives greater influence to the most precise measurements during the network adjustment process. Large standard deviations mean greater measurement uncertainty and lower precision for the measurements, which are then given less weight in the adjustment.

c. *Error propagation.* Formulas for propagation of variances assume that standard deviations of observations are small enough to be approximated by the squared differential changes of the observables: $\sigma_x^2 = dxdx$, and covariances by their products: $\sigma_{xy} = dxdy$. For a function x (a,b) of observations a and b, the squared differential of the function x is:

$$(dx)^2 = (¶x/¶a)^2\, da^2 + (¶x/¶b)^2\, db^2 + 2\,(¶x/¶a)(¶x/¶b)\, dadb$$

which may be generalized to any number of observables. For another function y (a,b) of the same observables a and b, the differentials of the functions x and y are:

$$dx = (¶x/¶a)\, da + (¶x/¶b)\, db$$
$$dy = (¶y/¶a)\, da + (¶y/¶b)\, db$$

which are multiplied to calculate the covariance $dxdy$.

$$dxdy = (¶x/¶a)(¶y/¶a)\, da^2 + (¶x/¶b)(¶y/¶b)\, db^2 + [(¶x/¶a)(¶y/¶b) + (¶x/¶b)(¶y/¶a)]\, dadb$$

Substitution of standard deviations of *a* and *b* for the differentials gives the following error propagation result in *x*:

$$\sigma_x{}^2 = (\partial x/\partial a)^2\, \sigma_a{}^2 + (\partial x/\partial b)^2\, \sigma_b{}^2 + 2\, (\partial x/\partial a)(\partial x/\partial b)\, \sigma_{ab}$$

and in the case of uncorrelated measurements, the variances and covariance in *x* and *y* are:

$$\sigma_x{}^2 = (\partial x/\partial a)^2\, \sigma_a{}^2 + (\partial x/\partial b)^2\, \sigma_b{}^2$$
$$\sigma_y{}^2 = (\partial y/\partial a)^2\, \sigma_a{}^2 + (\partial y/\partial b)^2\, \sigma_b{}^2$$
$$\sigma_{xy} = (\partial x/\partial a)(\partial y/\partial a)\, \sigma_a{}^2 + (\partial x/\partial b)(\partial y/\partial b)\, \sigma_b{}^2$$

(Eq 9-6)

d. *Covariance matrix of observations.* Error propagation formulas are used to calculate a standard deviation for each measurement in the adjustment (see Chapter 4). Once the measurements have been individually assigned a standard deviation, they are assembled into the covariance matrix of observations, and the adjustment software converts to measurement weights by finding the matrix inverse.

e. *Covariance matrix of parameters.* Before the network adjustment process is completed, the probable error in positioning is computed for each point. Entries of the covariance matrix of parameters contain the position accuracy information. The covariance matrix of parameters is derived from covariance matrix of observations by error propagation using a math model supplied by the adjustment software. Some degree of correlation of position error will likely exist between different stations in the network where the points have been tied together by redundant measurements of the same type.

f. Standard error ellipse. The geometric representation of the entries in the covariance matrix of parameters is through error ellipses describing the boundary of probable error around each point position. The maximum uncertainty in position is equivalent to the magnitude of the major semi-axis of this ellipse (i.e., its greatest dimension) for a given probability level used for reporting results. Its orientation and shape are also determined from the numerical entries of the covariance matrix of parameters. The error ellipse concept is illustrated by Figure 9-1, which depicts the intersection of two lines-of-position.

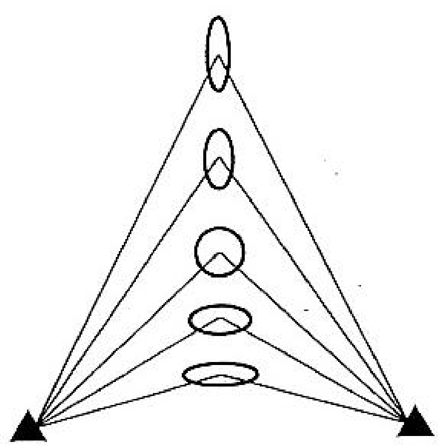

Figure 9-1. Error ellipses resulting from two lines of position at varying angles of intersection

g. *Confidence level.* For measurements made with the same level of precision, “one standard error” represents an uncertainty equivalent to the expectation that 67% of the measurements will fall within a distance of plus or minus one standard deviation from the mean. The 95% confidence level represents the probability that a data value lies within approximately two standard deviations from the mean. The probability of 95 percent (0.05 significance level) is usually accepted for the assessment of deformation measurements, meaning that the true position of an object point lies within a distance of 1.96-σ from the computed mean with 95% probability. Other multipliers can be used depending on the confidence required in the final positioning accuracy. For example:

1.96-σ corresponds to 95% probability,
2.58-σ corresponds to 99% probability,
3.00-σ corresponds to 99.7 % probability.

A probability value of 99% is usually accepted in practice as certainty.

h. *Degrees of freedom.* The number of redundant measurements in an adjustment is expressed as the degrees of freedom (*df*) of the adjustment. It is calculated as the difference between the number of independent observations and the number of unknown coordinate components in the math model.

$$df = n - u \qquad \text{(Eq 9-7)}$$

where
n = number of observations
u = number of unknowns

i. *Residuals.* The residual (*v*) is the difference between the adjusted value of an observation (i.e., as it was fit to the adjusted station position) and the actual input measurement value. Each measurement will have a residual value after the adjustment process. The residual value represents the amount by which the measurement failed to match to the adjusted position.

j. *Statistical testing.* Statistical tests are widely used to determine if a given quantity (e.g., residual) is compatible with, or significantly different from, some other quantity (e.g., the mean or variance of a set of related residuals). For example, one might test whether a particular measurement is compatible with the mean, or whether it should be removed from the adjustment as a blunder, based on the statistical testing of residuals. Statistical tests indicate whether one should accept or reject the null hypothesis. A null hypothesis (H_o) is a statement that is assumed to be true until proven otherwise, conversely the alternative hypothesis (H_a) will be true if the null hypothesis is false. For example:

H_o : An observation is compatible with the mean,
H_a : An observation is an outlier.

Statistical tests on the residuals would determine which of the above statements is supported by the data. For network adjustments, statistical testing is widely used for data quality assessment.

9-4. Adjustment Input Parameters

a. *General.* Background information is provided below for building network adjustment files. These computer files typically have a standard list of inputs that are needed to produce results from the adjustment software. Introductory geodesy textbooks or an adjustment software user's guide can provide further background for understanding of the principles and practice of network adjustments.

b. Adjustment input data. The following sections describe both necessary and optional input data for a typical network adjustment. Refer to the adjustment data sample for Yatesville Lake Dam at the end of this chapter for supporting illustrations.

(1) Authorized project name. This record contains basic information for organizing and indexing the project, such as, project name, type of network, date of survey, user comments, etc.

(2) Ellipsoid parameters. The major and minor semi-axes dimension for the reference ellipsoid are required to define the geodetic reference system. These parameters describe the size and shape of the reference ellipsoid to be used for 3D adjustments with geodetic coordinates. Usually a menu-based list of common reference ellipsoids are presented and indexed (by name) to be selected by the user.

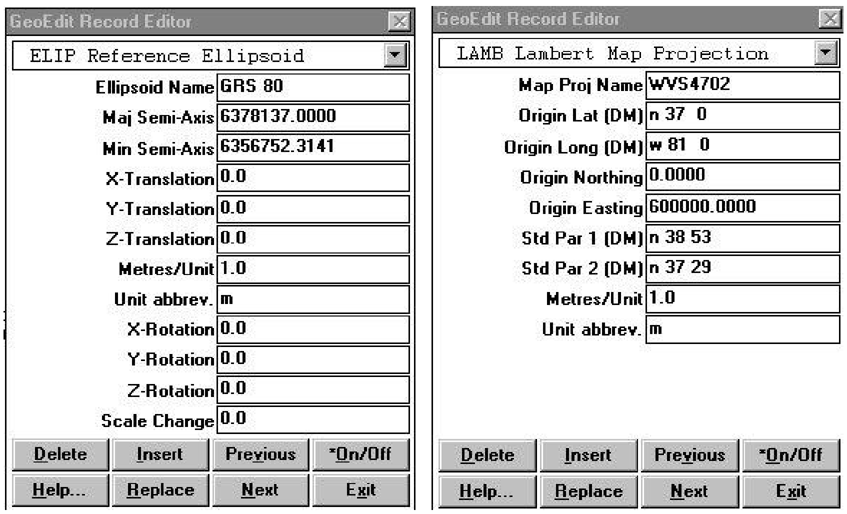

Figure 9-2. User option screens to select the reference ellipsoid and map projection for the adjustment. GRS80 is the ellipsoid that corresponds to NAD83 coordinates. Major and Minor Semi-Axes define the dimensions of the ellipsoid. Units are in meters with translations, rotations, and scale change set to zero. Under the map projection record screen, the projection type and name is selected from a pick list and then automatically populated with the standard Lambert Map Projection values (e.g., West Virginia South Projection in this example).

(3) Map projection parameters. Most software will require the user to select a standard map projection when adjusting plane coordinates. The parameters defining the projection are specified in the input file by, for example, projection type, name, units of measurement, standard parallel(s), central meridian, false easting, false northing, as typical categories. Actual parameter values used will depend on the type of projection chosen and the project's geographic location. See Figure 9-2.

(4) Computation mode. Many adjustment software applications can process either preanalysis (survey design) files or the actual data in an adjustment. Therefore, either adjustment or preanalysis mode is selected by the user. The least squares processing algorithm is identical for both functions but the computational mode must be specified in the input file. The major difference between these modes is that preanalysis does not require actual observations for its computations of expected position error.

(5) Measurement units. All data and constants must be entered in the correct linear and angular units. Never input one variable (e.g., coordinates) in feet and another variable (e.g., measurements) in meters. Most software packages cannot accommodate mismatched units.

(6) Statistical confidence level. For adjustment computations use 95 percent confidence level (significance level of 0.05) for computing adjustment statistics. It is recommended to use 99 percent confidence for preanalysis and design of field surveys. The significance level defines how error magnitudes are statistically tested and reported during the adjustment.

(7) Residual rejection criteria. This criteria defines the probability distribution to be used for data quality assessment and the critical values needed for outlier detection. The Tau distribution will be used to calculate statistics for most adjustments. Tau test statistics apply to data sets with prior unknown mean and variance values (which is the typical case for survey data). This means that normal distribution probability values are converted to Tau values by the software.

(8) Approximate station coordinates (Figure 9-3). In order to process data in an adjustment, each station in the network must be given an estimated position. Approximate coordinates accurate to one (1) meter are sufficient for most networks, although with deformation networks, accuracies less than 0.1 meter should be available. It is imperative that these values are realistic, otherwise the adjustment may not converge to the correct solution. Adjustments use iterative methods to correct the initial coordinates until the change between successive computations falls below a certain tolerance (this adjustment convergence limit is usually set at 0.1 mm). For monitoring networks, the station coordinates coming from previous instrumentation surveys are usually well known, and should be used as approximate coordinates on the current adjustment. If for some reason there are no existing station coordinates for a given control point, such as for new monuments, then traverse sideshot data or plotting from large scale maps can be used to roughly determine new approximate coordinates.

(9) Network constraints. Network constraints provide information to the adjustment software about the absolute position and orientation of the network. In practice, all of the stations in the monitoring network will have some form of position constraint (by their approximate coordinates) that defines their relationship to the project datum. In most adjustment software packages each coordinate component can be fixed separately, which permits breaking the network down into separate 2D and 1D adjustment schemes. There are several different types of network constraints available for adjustment processing, each having different advantages and uses.

(a) Minimum constraint. Any station in the reference network can be held fixed for a minimally constrained adjustment, although usually there is a “master” reference station on each project that is selected to serve as its main control point. For 3D networks, the coordinates of the selected constraint point are fixed along with the orientation and scale of the three axes of the network coordinate system. A minimally constrained adjustment is carried out mainly to validate the measurement data, check for blunders and systematic errors, and to look at the internal consistency of the measurements. The results from a minimum constraint adjustment will show only errors due to measurements without adding in any potential errors coming from inaccurate control station coordinates. Table 9-1 lists the ordinary minimum constraints for adjustment of conventional survey observations.

(b) Fully constrained. In a fully constrained adjustment all stations in the reference network are assumed to have well-known coordinates (i.e., are stable points), and these are fixed with zero error in the adjustment input file. With fully constrained adjustments only the monitoring point stations are

```
*
 PLH   111 SG2          n 38  8 44.850910 w 82 22 53.155010      158.1660
 PLH   111 EL1          n 38  8 39.238950 w 82 22 56.596230      189.8584
 PLH   000 D-2          N 38 08  44.96390 W 82 23   6.98524      213.5943
 PLH   000 SG1          N 38 08  38.33506 W 82 22  59.71445      203.0432
*
 NEO   000 U1                 127914.204        478737.790       220.0 WVS4702
 NEO   000 U2                 127939.886        478709.217       220.0 WVS4702
 NEO   000 U3                 127973.044        478677.195       220.0 WVS4702
 NEO   000 U4                 128008.556        478647.869       220.0 WVS4702
 NEO   000 U5                 128033.501        478629.927       220.0 WVS4702
 NEO   000 U6                 127893.902        478665.303       202.0 WVS4702
 NEO   000 U7                 127930.722        478629.834       202.0 WVS4702
 NEO   000 U8                 127970.066        478597.315       202.0 WVS4702
 NEO   000 D1                 127921.990        478744.299       220.0 WVS4702
 NEO   000 D2                 127947.196        478716.215       220.0 WVS4702
 NEO   000 D3                 127979.791        478684.729       220.0 WVS4702
 NEO   000 D4                 128014.747        478655.858       220.0 WVS4702
 NEO   000 D5                 128039.215        478638.251       220.0 WVS4702
 NEO   000 D6                 127987.591        478754.896       193.0 WVS4702
 NEO   000 D7                 128017.056        478726.426       193.0 WVS4702
 NEO   000 D8                 128048.472        478700.450       193.0 WVS4702
```

Figure 9-3. Approximate coordinates for each survey station in an example network. PLH stands for latitude, longitude, and ellipsoid height. NEO stands for Northing, Easting, and Orthometric height. The code 111 in the second column means each coordinate is fixed, and the code 000 means each coordinate is unfixed. The third column contains an abbreviated station name. The next three columns contain the approximate coordinate values for each station. The last column stands for West Virginia State plane projection 4702.

allowed to float and adjust in position. The drawback to a fully-constrained network adjustment is that any errors due to inaccurate reference station coordinates will be transferred to the monitoring points. Therefore, it is important that the reference network stations be surveyed independently with higher precision, and then checked against previous reference network surveys for stability.

(c) Weighted constraints. With a weighted constraints adjustment every station in the monitoring network, both the reference stations and the monitoring points, are assigned weights. No station is fixed absolutely with zero error, but the reference stations are usually given higher weights. The weights are assigned according to prior knowledge of their positioning uncertainty (i.e., point confidence ellipses) obtained from the results of a previous network adjustment.

- Reference stations are given a weight based on the covariance matrix of parameters resulting from the most recent project adjustment, or the adjustment of an independent network survey.

- Monitoring point stations are generally given a lower or essentially zero weight in relation to the reference network stations.

Coordinates for each station are assigned a separate weight matrix for position (e.g., a diagonal matrix constructed from the standard deviation of each coordinate component). A weighted constraints adjustment provides the most rigorous form of adjustment error propagation.

(d) Specialized constraints. When GPS survey observations are combined with photogrammetric surveys, the localized 3D coordinates (e.g., x, y, z coordinates) and associated variance-covariance matrix from the photogrammetric survey observations and subsequent bundle adjustment will be included. When only using photogrammetric surveys, free network constraints (i.e., inner constraints) will be used to define the datum. When photogrammetric surveys are combined only with conventional surveys, the

Table 9-1. Minimum Constraints[1]

Network Type	Minimum Constraint
1D (i.e., z)	z of 1 point held fixed
2D (i.e., x, y) with distance	x and y of 1 point held fixed azimuth of 2nd point held fixed (standard deviation of azimuth = 0.1")
2D (i.e., x, y) without distance	x and y of 2 points held fixed
3D (i.e., x, y, z) with distance	x, y, and z of 1 point held fixed azimuth and zenith angle to 2nd point held fixed, zenith angle to 3rd point held fixed (standard deviation of azimuth and zenith angles = 0.1")
3D (i.e., x, y, z) without distance	x, y, z of 3 points held fixed

[1]x = x horizontal value
y = y horizontal value
z = z vertical value (i.e., elevation)
NOTE: minimum constraints applied to opposite sides of network.

datum will be defined by the constraints used in a conventional survey adjustment. If photogrammetric surveys are combined only with GPS survey observations, the GPS survey observations will be used to define the datum (e.g., location, orientation, scale). In each of these examples where a covariance matrix is required, it is an example of using a weighted constraints approach.

(10) Observation type. The type of observation must be declared for each measurement. Standard observation equations are built into the software for each different type of measurement that defines the adjustment math model. The level of detail and rigor used in defining the observation equations determines the quality of the adjustment software. Examples of survey observation types include; distance, angle, azimuth, direction, absolute coordinates, 2D and 3D coordinate differences, elevation, height difference, geoid height, and others. See Figure 9-4.

(11) Station connections. Network geometry identifies how the measurements are connected to each other in relation to distance, height, and orientation. Station names (or other point identifiers) are referenced to each observation, and are required for every measurement used in the adjustment.

(12) Measurement value. Every observation record will contain the final reduced mark-to-mark measurement value in its prescribed units. The coordinate system expected for most conventional observations is the Local Astronomic System. This system is defined to correspond with a level reference plane, as used by most conventional instruments, and a horizontal reference alignment (i.e., using the local plumbline and Astronomic North for vertical and horizontal orientation respectively).

(13) Measurement standard deviation. Every observation data record will contain an estimated standard deviation. Its actual value is pre-computed from variance formulas prescribed for each observation type (Chapter 4).

```
*
* Baseline tie-in
*
 DSET
 DIR      EL1          SG1                               0   0     0.00        0.58
 DIR      EL1          SG1                             135  59    38.54        0.58
*
 DSET
 DIR      SG1          D-2                               0   0     0.00        2.69
 DIR      SG1          SG2                              79  22    58.17        2.69
 DIR      SG1          EL1                             110  44    34.01        2.69
*
 DSET
 DIR      SG2          EL1                               0   0     0.00        1.37
 DIR      SG2          SG1                              12  38    48.45        1.37
 DIR      SG2          D-2                              64  45    17.11        1.37
*
 DIST     EL1          SG1                                       81.9538      0.001
 ZANG     EL1          SG1                              80  44    28.10         1.0
*
 DIST     EL1          SG2                                      194.8543      0.001
 ZANG     EL1          SG2                              99  21    36.79         1.0
```

Figure 9-4. Adjustment input example showing conventional observations used to tie between reference stations. Each record contains the type of observation, station names, measurement, and standard deviation value.

9-5. Adjustment Output Parameters

a. Adjustment output. The following sections describe typical output data from an adjustment--refer to the sample adjustment of Yatesville Lake dam.

(1) Degrees of freedom. The degrees of freedom describes the level of redundancy for a given survey adjustment. Greater degrees of freedom generally means greater statistical reliability of the solution. If possible, the degrees of freedom should be more than twice the number of unknown parameters (coordinates) in the adjustment.

(2) Flagged outliers. A measurement is flagged and rejected as an outlier if an observation residual turns out to be larger than the statistical confidence interval established for the set of observation residuals as a whole.

(3) Standardized residuals. Higher values for a standardized residual means a low degree of fit, and indicates the measurement associated with it may be suspect. The value of the standardized residual for each observation is compared to the standardized residuals of similar measurements to determine relative data quality.

(4) Confidence ellipse. The point confidence ellipse represents the accuracy of the adjusted position stated at the probability (significance) level selected for the adjustment. Its dimensions and orientation (size and shape) are described by:

- Major semi-axis,
- Minor semi-axis,
- Vertical confidence interval,
- Azimuth or orientation of major semi-axis.

The magnitude of the major semi-axis of the point confidence ellipse represents the maximum expected error in horizontal position. The orientation of the confidence ellipse represent the principal direction of the maximum position error. The vertical error bar represents the maximum expected vertical positioning error. See examples at Figure 9-5.

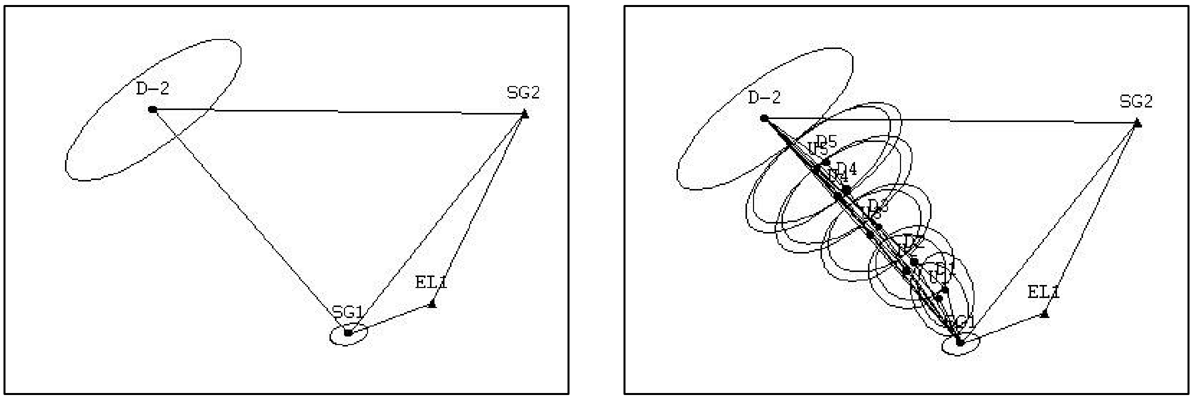

Figure 9-5. Adjustment output plots showing the reference network and monitoring network. The left-hand plot shows only the reference network stations with their error ellipses. The right-hand plot shows both the reference stations and the structure monitoring points with their error ellipses.

(5) Misclosures. (Figure 9-6). Large misclosures can signal problems with initial approximate coordinates. If an adjustment processing does not converge on a solution, then the approximate coordinates should be checked for possible data entry blunders. The station coordinates corresponding to the largest misclosure should be checked first as the most likely source of error.

```
================================================================================
                                          ECREST1
GeoLab V2.4d                 WGS 84              UNITS: m,DMS              Page 0002
================================================================================
Misclosures (pass 1):
TYPE AT             FROM          TO                       OBSERVATION STD.DEV.      MISC
---- ------------ ------------ ------------ ----------------- -------- --------
GROUP: 00002231.SSF,obs#:   16 day  98  OPT                          98 0  17:
DXCT                SG2           D4                         -252.4681  0.0008   0.0008
DYCT                SG2           D4                          -94.7024  0.0017  -0.0022
DZCT                SG2           D4                          -30.0746  0.0016   0.0039
GROUP: 00002235.SSF,obs#:   20 day  98  OPT                          98 0  16:
DXCT                SG2           U3                         -227.3046  0.0007   0.0014
DYCT                SG2           U3                         -117.2940  0.0018   0.0008
DZCT                SG2           U3                          -62.5023  0.0016   0.0010
GROUP: 00002227.SSF,obs#:   21 day  98  OPT                          98 0  17:
DXCT                SG2           U4                         -259.7704  0.0008   0.0009
DYCT                SG2           U4                          -99.6715  0.0017  -0.0023
DZCT                SG2           U4                          -34.9894  0.0016   0.0037
```

Figure 9-6. Adjustment output showing an example of misclosures for some observations. All misclosures shown are only a few centimeters, which indicates the approximate coordinates and the observations are a close match.

(6) Adjusted coordinates. (Figure 9-7). The main result of an adjustment is the adjusted coordinates for each station in the network. The output coordinates should be converted to a Cartesian System (x, y, z) for directly calculating linear displacements.

```
================================================================================
                                     ECREST1
GeoLab V2.4d                 WGS 84              UNITS: m,DMS           Page 0004
================================================================================
Adjusted PLH Coordinates:
                                      LATITUDE          LONGITUDE  ELIP-HEIGHT
 CODE FFF STATION                      STD DEV            STD DEV      STD DEV
 ---- --- ------------ ---------------------- -----------------  ------------
 PLH  000 D1           N 38 08 39.907802 W 82 23  0.259213       186.9275
                                 0.0005             0.0004         0.0010
 PLH  000 D2           N 38 08 40.711910 W 82 23  1.427794       187.0958
                                 0.0004             0.0003         0.0009
 PLH  000 D4           N 38 08 42.873908 W 82 23  3.947207       187.0954
                                 0.0003             0.0003         0.0009
 PLH  000 D5           N 38 08 43.658989 W 82 23  4.685115       186.8972
                                 0.0006             0.0004         0.0018
 PLH  111 EL1          N 38 08 39.238950 W 82 22 56.596230       189.8584
                                 0.0000             0.0000         0.0000
 PLH  111 SG2          N 38 08 44.850910 W 82 22 53.155010       158.1660
                                 0.0000             0.0000         0.0000
 PLH  000 U1           N 38 08 39.652228 W 82 23  0.521696       186.9883
                                 0.0006             0.0004         0.0010
 PLH  000 U2           N 38 08 40.471523 W 82 23  1.710551       187.1150
                                 0.0004             0.0003         0.0009
 PLH  000 U3           N 38 08 41.531445 W 82 23  3.045857       187.2968
                                 0.0005             0.0004         0.0011
 PLH  000 U4           N 38 08 42.669274 W 82 23  4.271490       187.1721
                                 0.0003             0.0003         0.0009
 PLH  000 U5           N 38 08 43.469769 W 82 23  5.023436       186.9739
                                 0.0006             0.0004         0.0019
```

Figure 9-7. Adjustment output showing the final adjusted coordinates for each station in the network.

(7) Goodness-of-fit. Residuals as a whole will either pass or fail the Goodness-of-Fit test. A Failed Chi-square test can indicate that there are measurement biases still remaining in the input data or that there are still some unremoved outliers. The Goodness-of-Fit test compares the shape of the actual distribution of residuals and the standard normal distribution to determine its degree of fit. If the test fails it indicates that the errors were not randomly distributed as should be expected in an adjustment. Separate Goodness-of-Fit tests can be made on the residuals from different types of measurements (e.g., distances, angles, or height differences, etc.). Partitioning the data into separate groups to make separate statistical tests is a procedure used to locate problems with particular types of measurements. A lack of fit between observations and coordinates can be determined for any particular group of measurements by examining the histogram of residuals. See also the sample output in Figure 9-8.

(8) A posteriori variance factor. The a posteriori variance factor is produced by the adjustment and indicates the precision for the results by incorporating the observation residuals into the assessment of coordinate accuracy. If the adjustment weighting scheme is too optimistic or too pessimistic, then the variance factor provides a scale factor to the adjustment covariance matrix. A posteriori variance factor values greater than one (1.0) indicate that the observation weights were overly-optimistic, values less than one (1.0) indicate that the observation weights were overly-pessimistic.

b. Test on the variance. The a posteriori variance factor is a global indicator of the quality of the adjustment and its weighting scheme. It is assessed by comparing its computed value to its expected value (i.e., 1.0) using a statistical test on the variance. The a posteriori variance factor is computed for an adjustment by dividing the quadratic form of the residuals by the degrees of freedom.

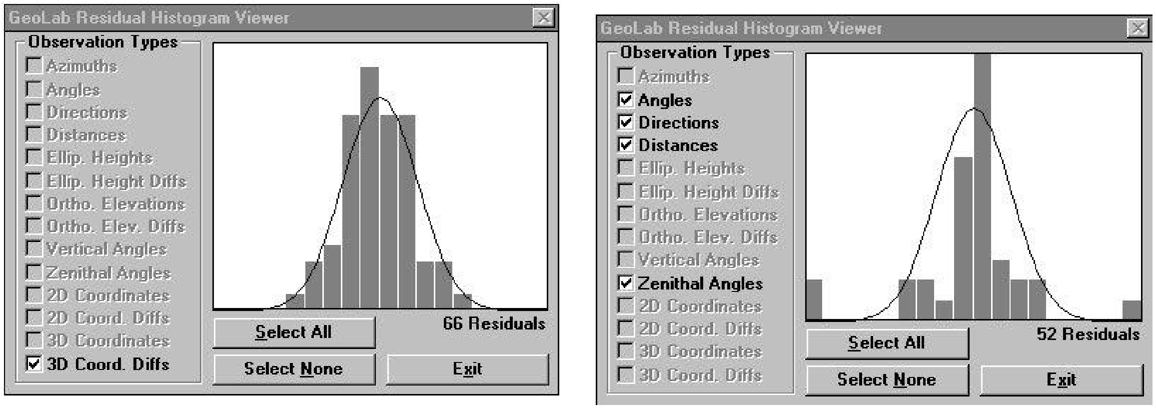

Figure 9-8. Histogram plots for the observation residuals. The left-hand plot shows normally distributed residuals for the GPS observations. The right-hand plot shows a few observations at the lateral margins indicating there are some outliers present in the conventional survey data.

$$\sigma^2 = (\mathbf{V}^t\mathbf{W}\mathbf{V})/df \qquad \text{(Eq 9-8)}$$

where

σ^2 = A posteriori variance factor
V = Residual vector
W = Covariance matrix of observations
df = Degrees of freedom

The null and alternative hypotheses for the test on the variance are as follows,

$$H_o : \sigma^2 = \sigma_o^2 \;\;\therefore\; \sigma^2 / \sigma_o^2 = 1$$
$$H_a : \sigma^2 \neq \sigma_o^2 \;\;\therefore\; \sigma^2 / \sigma_o^2 \neq 1$$

The test fails, and the null hypothesis is rejected, if:

$$df\,(\sigma^2) / \xi\chi^2_{df,\,1-\alpha/2} < \sigma_o^2 < df\,(\sigma^2) / \xi\chi^2_{df,\,\alpha/2}$$

where $\xi\chi^2_{df,\,1-\alpha/2}$ and $\xi\chi^2_{df,\,\alpha/2}$ are critical values from the Chi-square (χ) distribution tables based on a significance level alpha (α) and the degrees of freedom (*df*).

c. Detection and removal of outliers. An outlier is a measurement that is statistically incompatible with similar types of measurements from a given survey. Outlier detection is a quality control procedure essential to pre-processing and data cleaning. An observation is tested against the confidence level of the mean using a simple statistical test with a known or assumed variance.

$$\mu - (\sigma\,\xi) < obs_i < \mu + (\sigma\,\xi) \qquad \text{(Eq 9-9)}$$

The probability value ξ is determined from standard normal probability density tables using degrees of freedom equal to the number of observations and significance level (0.05).

d. *Statistical assessment of residuals.* Measurement outliers should be removed from the adjustment processing. Outliers can be identified by sequentially testing each standardized residual to determine if its value exceeds a defined rejection threshold. The Tau test uses the Tau distribution to compute rejection critical values.

(1) Tau Test probability. The Tau (τ) distribution can be derived from the Student-t distribution (t) using the following formula:

$$\tau = [(df)^{0.5} \ t_{df-1}] / [(df -1) + (t^{2}_{df-1})]^{0.5} \qquad \text{(Eq 9-10)}$$

df = degrees of freedom
t = Critical value from the Student-t distribution

Tau test critical values are computed for a significance value (α) and degrees of freedom (df).

(2) Standardized Residuals. A standardized residual (v') is defined as the observation residual divided by the standard deviation of the residual.

$$v' = v / \sigma_v \qquad \text{(Eq 9-11)}$$

where
v' = Standardized residual
v = Computed residual
σ_v = Standard deviation of the residual

Standardized residuals are computed to allow direct comparison between residuals of the same type. A much higher value for one of the standardized residuals indicates that it does not fit well compared to other standardized residuals, and its corresponding observation may be flagged as an outlier. When computing standardized residuals, the standard deviation of the residual (σ_r) can be replaced with the standard deviation of the corresponding observation (σ_r). However, because the residual standard deviation is smaller than the observation standard deviation,

$$|v| / \sigma_l \ < \ |v| / \sigma_v$$

an outlier may not be rejected. Therefore, it is recommended that the significance value (α) be increased for outlier detection in order to decrease the corresponding confidence level when using the observation standard deviation (σ_l).

(3) Outlier rejection. The rejection threshold for testing for measurement outliers is computed for a significance level (α) using the Tau distribution. The outlier rejection statistic for the standardized residual,

$$|v'| \ < \ (\xi_{\tau,df, 1-\alpha/2}) \ \sigma_v$$

determines if the standardized residual exceeds the rejection threshold, if so it is considered an outlier. The suggested significance value for the technique is $\alpha = 0.01$. The outlier rejection technique is based on univariate statistical testing which is most effective when only one significant outlier is present in the network.

e. Rejection criteria. The following table provides guidance on assessing rejection tolerances for various instruments.

Table 9-2. Rejection Criteria for Preprocessing of Deformation Survey Data

Type of Instrument	Type of Measurement	Test	Action to Follow if Data is Rejected
Theodolite[1] or Subtense Bar[1] or Theodolite[1] (Trigonometric Leveling)	Angle Angle Elevation	1. Reduced data must be less than 2 seconds from the mean reduced direction --> Otherwise, reject 2. Reduced zenith angle not being used to compute a height difference must be less than 4 seconds from the mean reduced direction --> Otherwise, reject 3. Reduced and corrected zenith angle not being used to compute a height difference must be less than 2 seconds from the mean reduced and corrected zenith angle --> Otherwise, reject	Reobserve the portion of the survey rejected
Steel or Invar Tape	Distance	1. Difference between two independently measured distances must be less than 2 mm --> Otherwise, reject	Remeasure the distance rejected
EDM Distance or EDM Elevation		1. Maximum difference among the four independent measured distances must be less than 5 mm --> Otherwise, reject	Remeasure the distance rejected
Automatic Level Setup	Elevation	1. Difference between readings on the left and right hand scale must be within 0.25 mm of rod constant --> Otherwise, reject 2. Difference between height difference determined from the foresight and backsight readings on the left rod scale and that determined from foresights and backsight readings from the rights scale must be less than 0.25 mm --> Otherwise, reject	Reobserve the portion of the survey rejected
Network of Level Setups	Elevation	1. Height difference misclosure in a loop must be less than 3 mm * _(K in km) --> (Minimum = 1 mm) Otherwise, reject	Formulate different loops to determine height differences between points common to loops which have been rejected; or, reobserve the portion of the survey rejected
Level and Meter Rule	HI	1. Difference between two independent readings must be less than 0.5 mm --> Otherwise, reject	Remeasure the distance rejected
Mono/stereo-comparator	Photo image coordinates	1. As applied by photogrammetry software for hardware used --> Otherwise, reject 2. Discrepancy between double measured image coordinates is less than 2 microns --> Otherwise, reject	Remeasure image coords
GPS Receivers	Horizontal coordinates and elevation	1. Tests as detailed in EM 1110-1-1003	Reoccupy baseline

[1] When performing these data reductions, no atmospheric, instrumental, standardization, and geometric corrections are necessary for angular observation made with a theodolite, except in the case of zenith angles which are observed for the purpose of determining height differences (in which case, earth curvature and refraction need be considered. Because a deformation survey is on a localized network, skew-normal, arc-to-chord, and normal section to geodetic correction need not be applied.

9-6. Adjustment Procedures

a. General. This section provides an overview of processing procedures and data requirements for using network adjustment software applications.

b. Coordinate system. Network adjustments that solve for coordinate positions do so in a defined coordinate system. Some software applications will require the use of geodetic coordinates. Geodetic coordinates are transferred to the reference network if non-geodetic coordinates are used on the project.

(1) Geodetic coordinates. NAD83 coordinates are recommended for new projects and projects that incorporate GPS measurements. The project reference network should be tied directly to established NAD83 control by a separate survey. This avoids potentially inaccurate coordinate transformations that might be needed when processing the network adjustment based on local coordinates.

(2) Map projection coordinates. Horizontal positions defined with coordinates based on a map projection are readily handled by most adjustment software applications. State Plane coordinates or Universal Transverse Mercator (UTM) coordinates still require having an underlying geodetic coordinate system related to a standard reference ellipsoid.

(3) Local coordinate system. Station and offset coordinates, based on a local construction datum, are often used for reporting survey results. Some software applications are available that work reasonably well using only simple plane coordinates. However, an arbitrary coordinate system may restrict user options for processing an adjustment.

c. Three dimensional networks. Measurement data combined from separate horizontal and vertical surveys are often used to determine 3D coordinates of network stations. Three dimensional networks yield the most robust adjustment results because there are a large number of redundant measurements needed to reliably interconnect the horizontal and vertical network components. Elevations are required on all reference stations for 3D networks, along with zenith angle measurement ties to common points in the vertical network.

d. Separate horizontal and vertical networks. Horizontal and vertical networks can be adjusted independently if they are setup in the proper manner. For example, with horizontal networks, the elevations for all horizontal network stations should be fixed to a constant, average height, for the entire network (project) and horizontal distances should be used instead of mark-to-mark slope distances. Vertical network stations require approximate horizontal coordinates for processing the vertical adjustment and for error propagation. Vertical networks should include zenith angle ties from the reference network stations where possible and establishing accurate elevations at the reference stations.

e. Configuration defects. Repeated deformation surveys should involve measurements made over the same station configuration using the same sub-sets of network stations, otherwise the comparison between epochs may fail to give adequate results due to configuration defects (i.e., missing observations). During processing and analysis of the adjustment, the same minimal and fixed constraints should be applied for both survey epochs and each survey should be adjusted with the same statistical tests and confidence level.

f. Observation weighting. Observations will be assigned weights according to the a priori estimation of variance for each measurement (i.e., using the standard deviation computed for each measurement).

(1) For conventional surveys, the standard deviations and error models applicable to each type of measurement are to be used in the survey data adjustment. The formulas for variance estimation are provided in Chapter 4.

(2) When GPS survey observations are used in the deformation survey, they will be adjusted with either the GPS based 3D coordinates or coordinate differences and their associated variance-covariance matrices according to EM 1110-1-1003, NAVSTAR Global Positioning System Surveying.

g. Survey adjustment. Final processing of the survey data should be made using least squares adjustment techniques and software. For each adjustment the following quantities will be determined.

- adjusted point coordinates,
- variance-covariance matrix of parameters,
- point confidence ellipse major semi-axis,
- standardized residuals for each observation,
- a posteriori variance factor,
- total redundancy of the network,

h. Data quality assessment. For each adjustment the following data quality indicators will be checked.

(1) Examination of misclosures. Computed misclosure values should not exceed one (1) meter, otherwise examine and correct initial approximate coordinates.

(2) Point confidence ellipse dimensions. The computed major semi-axis of each point confidence ellipse should not exceed the stated accuracy requirement for the survey. See sample outputs in Figures 9-9 and 9-10.

```
================================================================================
                                    ECREST1
GeoLab V2.4d                 WGS 84            UNITS: m,DMS          Page 0011
================================================================================
2-D and 1-D Station Confidence Regions (95.000 percent):
STATION            MAJOR SEMI-AXIS  AZ     MINOR SEMI-AXIS             VERTICAL
------------ -------------------- --- -------------------- --------------------
D1                        0.0015  28                0.0007               0.0019
D2                        0.0010  11                0.0007               0.0018
D4                        0.0009 157                0.0007               0.0019
D5                        0.0014 161                0.0009               0.0035
U1                        0.0015  28                0.0007               0.0019
U2                        0.0010  11                0.0007               0.0018
U3                        0.0012 162                0.0008               0.0022
U4                        0.0009 157                0.0007               0.0019
U5                        0.0015 161                0.0010               0.0037
```

Figure 9-9. Adjustment output showing confidence regions for each adjusted horizontal and vertical position of each station in the network. At the 95-percent confidence level, horizontal position uncertainty (MAJOR SEMI-AXIS) is between 0.9-1.5 mm, and vertical position uncertainty ranges between 1.8-3.7 mm.

(3) Goodness of fit test. The distribution of residuals should pass the Chi-square test for Goodness of Fit at the 0.05 significance level (95% confidence).

(4) Outlier detection. Standardized residuals should be within the tolerance limits for rejection as an outlier as established by the residual rejection critical value at the 95 percent confidence level. Observations flagged as outliers should be removed and the adjustment repeated. Only the single

observation associated with the greatest magnitude residual should be removed before reprocessing the adjustment. Outlier detection should be carried out only on a minimally constrained network.

(5) Chi-Square test on variance factor. The computed a posteriori variance factor should pass the Chi-square test. If the variance factor does not pass because of a value less than 0.5 or greater than 2.0, then observation weights should be verified, and if found to be realistic, then the covariance matrix of parameters should be multiplied by the estimated variance factor to scale its values.

(6) Redundancy number. The computed degrees of freedom should be no less than the number of unknown coordinate components, preferably two or more times greater.

i. Reference network stability. Examination of a separate adjustment of the reference network will be done to check whether the reference points were stable between epochs. Any reference points that are not found to be stable will be left unconstrained in the network adjustment. All reference network points found to be stable are held fixed.

j. Calculation of displacements. After a network adjustment is completed for two different instrumentation surveys, the adjusted coordinates for each monitoring point are extracted and differenced to calculate point displacements, and identify significant movement between the separate time epochs.

k. Required submittal documents. The contracting officer should require the contractor to supply the final adjustment for each project. The contractor should supply a list containing any observations that were removed due to blunders. The contractor must provide USACE with an analysis explaining the methodology used in the adjustment, assumptions, and possible error sources.

ECREST1

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2-D and 1-D Relative Station Confidence Regions (95.000 percent):

FROM	TO	MAJ-SEMI	AZ	MIN-SEMI	VERTICAL	DISTANCE	PPM
D1	EL1	0.0015	28	0.0007	0.0019	91.5964	15.95
D1	SG2	0.0015	28	0.0007	0.0019	232.3432	6.29
D1	U1	0.0013	28	0.0006	0.0017	10.1468	130.57
D2	EL1	0.0010	11	0.0007	0.0018	126.1441	7.97
D2	SG2	0.0010	11	0.0007	0.0018	240.2176	4.18
D2	U2	0.0008	11	0.0005	0.0014	10.1167	77.22
D4	EL1	0.0009	157	0.0007	0.0019	211.2129	4.15
D4	SG2	0.0009	157	0.0007	0.0019	271.3172	3.23
D4	U4	0.0008	157	0.0006	0.0016	10.1080	75.40
D5	EL1	0.0014	161	0.0009	0.0035	239.5408	6.01
D5	SG2	0.0014	161	0.0009	0.0035	284.6099	5.06
D5	U5	0.0015	161	0.0010	0.0038	10.0953	152.96
EL1	U1	0.0015	28	0.0007	0.0019	96.4764	15.60
EL1	U2	0.0010	11	0.0007	0.0018	130.2362	7.73
EL1	U3	0.0012	162	0.0008	0.0022	172.2461	6.99
EL1	U4	0.0009	157	0.0007	0.0019	214.7678	4.08
EL1	U5	0.0015	161	0.0010	0.0037	243.1797	6.25
SG2	U1	0.0015	28	0.0007	0.0019	242.2878	6.21
SG2	U2	0.0010	11	0.0007	0.0018	249.9477	4.03
SG2	U3	0.0012	162	0.0008	0.0022	263.3080	4.57
SG2	U4	0.0009	157	0.0007	0.0019	280.4268	3.12
SG2	U5	0.0015	161	0.0010	0.0037	293.5370	5.18

Figure 9-10. Adjustment output showing relative confidence regions for horizontal and vertical position between each station in the network. At the 95-percent confidence level, horizontal position uncertainty (MAJ-SEMI) is between 0.8-1.5 mm, and vertical position uncertainty is between 1.4-3.8 mm.

9-7. Sample Adjustment -- Yatesville Lake Dam

INPUT DATA

```
 TITL Yatesville Lake Dam 16th Observation
 ELIP GRS 80            6378137.0000  6356752.3141
 COMP ADJ
 PADJ YES NO  NO  NO  NO  NO
 PRES YES NO
 PSOL YES YES
 RTST TAU MAX
 PMIS YES YES
 CONV     0.0001
 MAXI         10
 VARF YES YES NO
 CONF YES YES NO  YES NO
 LUNT ft     0.3048006096
 CLEV       95.0
 LAMB KYN1601 n 37 30 w 84 15   0.0000  500000.0000 n 38 58 n 37 58  1.0 m

*Reference Station Coordinates

 NEO  111 R-1                231672.634      2087616.903    682.105  KYN1601
 NEO  111 R-2                231581.816      2086624.431    682.250  KYN1601
 NEO  111 R-3                231897.263      2087483.998    682.247  KYN1601
 NEO  111 R-4                231717.570      2086570.072    682.732  KYN1601

*Monitoring Station Coordinates

 NEO  000 C-1                231697.820      2087338.110    680.370  KYN1601
 NEO  000 C-2                231704.340      2087188.450    680.340  KYN1601
 NEO  000 C-3                231710.460      2087038.150    680.320  KYN1601
 NEO  000 C-4                231717.360      2086888.340    680.300  KYN1601
 NEO  000 C-5                231724.560      2086738.540    680.280  KYN1601
 NEO  000 D-1                231866.180      2087190.540    655.740  KYN1601
 NEO  000 D-2                231872.090      2087040.500    655.710  KYN1601
 NEO  000 D-3                231878.660      2086891.070    655.830  KYN1601
 NEO  000 U-1                231570.700      2087176.720    660.230  KYN1601
 NEO  000 U-2                231576.970      2087027.520    660.540  KYN1601
 NEO  000 U-3                231581.980      2086877.240    660.350  KYN1601

*Horizontal Angle Observations

 ANGL     R-1           R-4           U-1            344 30   18.50        1.97
 ANGL     R-1           R-4           U-2            348 19   23.00        1.75
 ANGL     R-1           R-4           U-3            350 33   15.30        1.64
 ANGL     R-1           R-4           C-1              2 42   19.50        2.69
 ANGL     R-1           R-4           C-2              1 46   28.70        2.01
 ANGL     R-1           R-4           C-3              1 16   55.10        1.76
 ANGL     R-1           R-4           C-4              1  3   16.80        1.65
 ANGL     R-1           R-4           C-5              0 55   31.30        1.59
 ANGL     R-1           R-4           D-1             21 57   27.50        2.26
 ANGL     R-1           R-4           D-2             16 37   48.20        1.74
 ANGL     R-1           R-4           D-3             13 23   18.90        1.64
 ANGL     R-4           R-1           U-1             11  9    3.70        1.71
 ANGL     R-4           R-1           U-2             14 37   36.10        1.91
```

ANGL	R-4	R-1	U-3	21 21	29.40	2.36
ANGL	R-4	R-1	C-1	359 0	53.20	1.62
ANGL	R-4	R-1	C-2	358 46	3.70	1.71
ANGL	R-4	R-1	C-3	358 24	40.50	1.92
ANGL	R-4	R-1	C-4	357 34	45.70	2.43
ANGL	R-4	R-1	C-5	355 9	53.30	3.18
ANGL	R-4	R-1	D-1	344 4	22.50	1.71
ANGL	R-4	R-1	D-2	339 21	32.60	1.88
ANGL	R-4	R-1	D-3	330 53	32.00	2.53

*Distance Observations

DIST	R-1	U-1	452.374	0.0051
DIST	R-1	U-2	597.508	0.0054
DIST	R-1	U-3	745.544	0.0057
DIST	R-1	C-1	279.948	0.0049
DIST	R-1	C-2	429.643	0.0050
DIST	R-1	C-3	580.016	0.0053
DIST	R-1	C-4	729.973	0.0057
DIST	R-1	C-5	879.943	0.0060
DIST	R-1	D-1	468.998	0.0049
DIST	R-1	D-2	610.542	0.0054
DIST	R-1	D-3	755.003	0.0059
DIST	R-2	U-1	552.872	0.0054
DIST	R-2	U-2	403.735	0.0051
DIST	R-2	U-3	253.770	0.0049
DIST	R-2	C-1	723.094	0.0058
DIST	R-2	C-2	577.216	0.0054
DIST	R-2	C-3	433.294	0.0051
DIST	R-2	C-4	296.703	0.0049
DIST	R-2	C-5	182.774	0.0047
DIST	R-3	C-1	247.126	0.0048
DIST	R-3	C-2	352.963	0.0050
DIST	R-3	C-3	483.427	0.0053
DIST	R-3	C-4	622.264	0.0057
DIST	R-3	C-5	765.236	0.0061
DIST	R-3	D-1	296.296	0.0049
DIST	R-3	D-2	445.017	0.0054
DIST	R-3	D-3	593.831	0.0058
DIST	R-4	U-1	624.623	0.0054
DIST	R-4	U-2	479.110	0.0051
DIST	R-4	U-3	336.535	0.0049
DIST	R-4	C-1	768.341	0.0058
DIST	R-4	C-2	618.551	0.0054
DIST	R-4	C-3	468.170	0.0051
DIST	R-4	C-4	318.295	0.0049
DIST	R-4	C-5	168.650	0.0047
DIST	R-4	D-1	638.628	0.0055
DIST	R-4	D-2	495.922	0.0052
DIST	R-4	D-3	360.176	0.0054

*Zenith Angle Observations

ZANG	R-1	U-1	92 46	23.50	2.00
ZANG	R-1	U-2	92 4	2.39	2.33
ZANG	R-1	U-3	91 40	25.10	2.75
ZANG	R-1	C-1	90 21	16.00	2.00

ZANG	R-1	C-2	90	14	8.70	2.03
ZANG	R-1	C-3	90	10	35.30	2.11
ZANG	R-1	C-4	90	8	35.70	2.23
ZANG	R-1	C-5	90	7	11.20	2.32
ZANG	R-1	D-1	93	13	22.60	2.20
ZANG	R-1	D-2	92	28	38.20	2.64
ZANG	R-1	D-3	91	59	38.90	2.85
ZANG	R-4	U-1	92	3	55.30	2.83
ZANG	R-4	U-2	92	39	15.70	2.57
ZANG	R-4	U-3	93	48	46.20	2.24
ZANG	R-4	C-1	90	10	37.60	2.42
ZANG	R-4	C-2	90	13	19.00	2.36
ZANG	R-4	C-3	90	17	44.50	2.24
ZANG	R-4	C-4	90	26	14.30	2.14
ZANG	R-4	C-5	90	49	54.70	2.06
ZANG	R-4	D-1	92	25	22.00	2.77
ZANG	R-4	D-2	93	7	23.20	2.56
ZANG	R-4	D-3	94	17	0.40	2.29

*Orthometric Height Difference Observations

OHDF	R-1	C-1	-1.733	0.006
OHDF	C-1	C-2	-0.027	0.003
OHDF	C-2	C-3	-0.025	0.003
OHDF	C-3	C-4	-0.014	0.003
OHDF	C-4	C-5	-0.022	0.003
OHDF	C-5	R-4	2.451	0.006
OHDF	R-1	U-1	-21.882	0.012
OHDF	U-1	U-2	0.3208	0.003
OHDF	U-2	U-3	-0.193	0.003
OHDF	U-3	R-2	21.883	0.012
OHDF	R-3	D-1	-26.484	0.012
OHDF	D-1	D-2	-0.027	0.003
OHDF	D-2	D-3	0.116	0.003
OHDF	D-3	R-4	26.886	0.012

END

ADJUSTMENT OUTPUT

```
================================================================================
Yatesville Lake Dam 16th Observation
GeoLab V2.4d      UNITS: ft,DMS     GRS 80     08:15:19, Thu Jun 16, 1998
================================================================================
INI file:    C:\WINDOWS\GEOLAB.INI
Input file:  C:\GEOLAB2\YATES\YLD16.IOB
Output file: C:\GEOLAB2\YATES\YLD16.LST
--------------------------------------------------------------------------------
              PARAMETERS                              OBSERVATIONS
--------------------------------------------------------------------------------
    Description          Number          Description          Number
--------------------------------------------------------------------------------
  No. of Stations           15          Directions                  0
  Coord Parameters          33          Distances                  38
  Free Latitudes            11          Azimuths                    0
  Free Longitudes           11          Vertical Angles             0
  Free Heights              11          Zenithal Angles            22
  Fixed Coordinates         12          Angles                     22
  Astro. Latitudes           0          Heights                     0
  Astro. Longitudes          0          Height Differences         14
  Geoid Records              0          Auxiliary Params.           0
  All Aux. Pars.             0          2-D Coords.                 0
  Direction Pars.            0          2-D Coord. Diffs.           0
  Scale Parameters           0          3-D Coords.                 0
  Constant Pars.             0          3-D Coord. Diffs.           0
  Rotation Pars.             0
  Translation Pars.          0
                          --------                                --------
Total Parameters          33        Total Observations          96
--------------------------------------------------------------------------------
Degrees of Freedom = 63

                        SUMMARY OF SELECTED OPTIONS
--------------------------------------------------------------------------------
    OPTION                                SELECTION
--------------------------------------------------------------------------------
    Computation Mode                      Adjustment
    Maximum Iterations                    10
    Convergence Criterion                 0.00010
    Confidence Level for Statistics       95.000
    Covariance Matrix Computation         Full
    Residual Rejection Criterion          Tau Max
    Confidence Region Types               1D 2D Station
    Variance Factor (VF) Known            Yes
    CMULT (Multiply Parm Cov With VF)     Yes
    RMULT (Multiply Res Cov With VF)      No
    Full Inverse Computed                 Yes
    Normals Reordered                     Yes
    Coordinates Generated                 No
    Geoid Interpolation Method            Bi-Linear
```

MISCLOSURES

TYPE	AT	FROM	TO	OBSERVATION			STD.DEV.	MISC
ANGL	R-1	R-4	U-1	344	30	18.5	2.0	-4.9
ANGL	R-1	R-4	U-2	348	19	23.0	1.8	-1.5
ANGL	R-1	R-4	U-3	350	33	15.3	1.6	1.5
ANGL	R-1	R-4	C-1	2	42	19.5	2.7	-4.7
ANGL	R-1	R-4	C-2	1	46	28.7	2.0	-1.3
ANGL	R-1	R-4	C-3	1	16	55.1	1.8	-1.9
ANGL	R-1	R-4	C-4	1	3	16.8	1.6	1.2
ANGL	R-1	R-4	C-5	0	55	31.3	1.6	-0.4
ANGL	R-1	R-4	D-1	21	57	27.5	2.3	-0.3
ANGL	R-1	R-4	D-2	16	37	48.2	1.7	-2.0
ANGL	R-1	R-4	D-3	13	23	18.9	1.6	-0.2
ANGL	R-4	R-1	U-1	11	9	3.7	1.7	1.8
ANGL	R-4	R-1	U-2	14	37	36.1	1.9	2.0
ANGL	R-4	R-1	U-3	21	21	29.4	2.4	5.3
ANGL	R-4	R-1	C-1	359	0	53.2	1.6	1.1
ANGL	R-4	R-1	C-2	358	46	3.7	1.7	-0.1
ANGL	R-4	R-1	C-3	358	24	40.5	1.9	3.7
ANGL	R-4	R-1	C-4	357	34	45.7	2.4	1.7
ANGL	R-4	R-1	C-5	355	9	53.3	3.2	4.7
ANGL	R-4	R-1	D-1	344	4	22.5	1.7	-0.8
ANGL	R-4	R-1	D-2	339	21	32.6	1.9	-2.4
ANGL	R-4	R-1	D-3	330	53	32.0	2.5	1.6
DIST		R-1	U-1	452.3740			0.0051	0.0102
DIST		R-1	U-2	597.5080			0.0054	0.0087
DIST		R-1	U-3	745.5440			0.0057	0.0101
DIST		R-1	C-1	279.9480			0.0049	0.0005
DIST		R-1	C-2	429.6430			0.0050	0.0078
DIST		R-1	C-3	580.0160			0.0053	0.0052
DIST		R-1	C-4	729.9730			0.0057	0.0023
DIST		R-1	C-5	879.9430			0.0060	0.0019
DIST		R-1	D-1	468.9980			0.0049	0.0046
DIST		R-1	D-2	610.5420			0.0054	-0.0023
DIST		R-1	D-3	755.0030			0.0059	0.0004
DIST		R-2	U-1	552.8720			0.0054	-0.0036
DIST		R-2	U-2	403.7350			0.0051	-0.0116
DIST		R-2	U-3	253.7700			0.0049	-0.0010
DIST		R-2	C-1	723.0940			0.0058	-0.0080
DIST		R-2	C-2	577.2160			0.0054	-0.0086
DIST		R-2	C-3	433.2940			0.0051	-0.0086
DIST		R-2	C-4	296.7030			0.0049	0.0010
DIST		R-2	C-5	182.7740			0.0047	-0.0060
DIST		R-3	C-1	247.1260			0.0048	-0.0009
DIST		R-3	C-2	352.9630			0.0050	0.0026
DIST		R-3	C-3	483.4270			0.0053	0.0026
DIST		R-3	C-4	622.2640			0.0057	0.0046
DIST		R-3	C-5	765.2360			0.0061	0.0089
DIST		R-3	D-1	296.2960			0.0049	0.0070
DIST		R-3	D-2	445.0170			0.0054	0.0100
DIST		R-3	D-3	593.8310			0.0058	0.0076

MISCLOSURES

TYPE AT	FROM	TO	OBSERVATION	STD.DEV.	MISC
DIST	R-4	U-1	624.6230	0.0054	-0.0114
DIST	R-4	U-2	479.1100	0.0051	-0.0030
DIST	R-4	U-3	336.5350	0.0049	-0.0092
DIST	R-4	C-1	768.3410	0.0058	-0.0049
DIST	R-4	C-2	618.5510	0.0054	0.0058
DIST	R-4	C-3	468.1700	0.0051	-0.0071
DIST	R-4	C-4	318.2950	0.0049	-0.0008
DIST	R-4	C-5	168.6500	0.0047	-0.0103
DIST	R-4	D-1	638.6280	0.0055	-0.0072
DIST	R-4	D-2	495.9220	0.0052	-0.0039
DIST	R-4	D-3	360.1760	0.0054	0.0002
ZANG	R-1	U-1	92 46 23.5	2.0	3.5
ZANG	R-1	U-2	92 4 2.4	2.3	-6.5
ZANG	R-1	U-3	91 40 25.1	2.8	1.8
ZANG	R-1	C-1	90 21 16.0	2.0	-3.7
ZANG	R-1	C-2	90 14 8.7	2.0	-0.8
ZANG	R-1	C-3	90 10 35.3	2.1	-2.3
ZANG	R-1	C-4	90 8 35.7	2.2	2.1
ZANG	R-1	C-5	90 7 11.2	2.3	-0.9
ZANG	R-1	D-1	93 13 22.6	2.2	-1.0
ZANG	R-1	D-2	92 28 38.2	2.6	-4.9
ZANG	R-1	D-3	91 59 38.9	2.9	-4.5
ZANG	R-4	U-1	92 3 55.3	2.8	-0.2
ZANG	R-4	U-2	92 39 15.7	2.6	-4.2
ZANG	R-4	U-3	93 48 46.2	2.2	-4.1
ZANG	R-4	C-1	90 10 37.6	2.4	-0.3
ZANG	R-4	C-2	90 13 19.0	2.4	-1.7
ZANG	R-4	C-3	90 17 44.5	2.2	-0.5
ZANG	R-4	C-4	90 26 14.3	2.1	-3.3
ZANG	R-4	C-5	90 49 54.7	2.1	-5.3
ZANG	R-4	D-1	92 25 22.0	2.8	-1.7
ZANG	R-4	D-2	93 7 23.2	2.6	-3.9
ZANG	R-4	D-3	94 17 0.4	2.3	-1.9
EHDF	R-1	C-1	-1.7330	0.0060	-0.0020
EHDF	C-1	C-2	-0.0270	0.0030	-0.0030
EHDF	C-2	C-3	-0.0250	0.0030	0.0050
EHDF	C-3	C-4	-0.0140	0.0030	-0.0060
EHDF	C-4	C-5	-0.0220	0.0030	0.0020
EHDF	C-5	R-4	2.4510	0.0060	0.0010
EHDF	R-1	U-1	-21.8820	0.0120	0.0070
EHDF	U-1	U-2	0.3208	0.0030	-0.0108
EHDF	U-2	U-3	-0.1930	0.0030	0.0030
EHDF	U-3	R-2	21.8830	0.0120	0.0170
EHDF	R-3	D-1	-26.4840	0.0120	-0.0230
EHDF	D-1	D-2	-0.0270	0.0030	-0.0030
EHDF	D-2	D-3	0.1160	0.0030	0.0040
EHDF	D-3	R-4	26.8860	0.0120	0.0160

ADJUSTED NEO COORDINATES (Reference Stations)

CODE	FFF	STATION	NORTHING STD DEV	EASTING STD DEV	O-HEIGHT STD DEV	MAPPROJ
NEO	111	R-1	231672.6340 0.0000	2087616.9030 0.0000	682.1050 0.0000	KYN1601
NEO	111	R-2	231581.8160 0.0000	2086624.4310 0.0000	682.2500 0.0000	KYN1601
NEO	111	R-3	231897.2630 0.0000	2087483.9980 0.0000	682.2470 0.0000	KYN1601
NEO	111	R-4	231717.5700 0.0000	2086570.0720 0.0000	682.7320 0.0000	KYN1601

ADJUSTED NEO COORDINATES (Monitoring Stations)

CODE	FFF	STATION	NORTHING STD DEV	EASTING STD DEV	O-HEIGHT STD DEV	MAPPROJ
NEO	000	C-1	231697.8239 0.0024	2087338.1127 0.0025	680.3734 0.0017	KYN1601
NEO	000	C-2	231704.3414 0.0026	2087188.4534 0.0023	680.3451 0.0018	KYN1601
NEO	000	C-3	231710.4664 0.0026	2087038.1552 0.0023	680.3205 0.0018	KYN1601
NEO	000	C-4	231717.3605 0.0025	2086888.3414 0.0023	680.3052 0.0016	KYN1601
NEO	000	C-5	231724.5637 0.0019	2086738.5470 0.0024	680.2838 0.0012	KYN1601
NEO	000	D-1	231866.1792 0.0030	2087190.5461 0.0024	655.7464 0.0025	KYN1601
NEO	000	D-2	231872.0906 0.0028	2087040.5051 0.0025	655.7199 0.0023	KYN1601
NEO	000	D-3	231878.6622 0.0030	2086891.0720 0.0027	655.8359 0.0023	KYN1601
NEO	000	U-1	231570.7087 0.0028	2087176.7277 0.0025	660.2268 0.0023	KYN1601
NEO	000	U-2	231576.9741 0.0028	2087027.5281 0.0025	660.5493 0.0022	KYN1601
NEO	000	U-3	231581.9820 0.0027	2086877.2482 0.0024	660.3560 0.0022	KYN1601

RESIDUALS

(critical value = 3.504):

TYPE	AT	FROM	TO	OBSERVATION	RESIDUAL	STD RES
ANGL	R-1	R-4	U-1	344 30 18.5	-1.8	-1.4
ANGL	R-1	R-4	U-2	348 19 23.0	-0.5	-0.4
ANGL	R-1	R-4	U-3	350 33 15.3	1.7	1.3
ANGL	R-1	R-4	C-1	2 42 19.5	-1.7	-1.0
ANGL	R-1	R-4	C-2	1 46 28.7	-0.5	-0.3
ANGL	R-1	R-4	C-3	1 16 55.1	0.5	0.4
ANGL	R-1	R-4	C-4	1 3 16.8	1.3	0.9
ANGL	R-1	R-4	C-5	0 55 31.3	0.6	0.4
ANGL	R-1	R-4	D-1	21 57 27.5	0.5	0.3
ANGL	R-1	R-4	D-2	16 37 48.2	-1.3	-1.0
ANGL	R-1	R-4	D-3	13 23 18.9	0.5	0.4
ANGL	R-4	R-1	U-1	11 9 3.7	-1.6	-1.2
ANGL	R-4	R-1	U-2	14 37 36.1	-0.8	-0.6
ANGL	R-4	R-1	U-3	21 21 29.4	2.2	1.6
ANGL	R-4	R-1	C-1	359 0 53.2	-0.0	-0.0
ANGL	R-4	R-1	C-2	358 46 3.7	-0.5	-0.4
ANGL	R-4	R-1	C-3	358 24 40.5	0.9	0.7
ANGL	R-4	R-1	C-4	357 34 45.7	1.4	1.0
ANGL	R-4	R-1	C-5	355 9 53.3	0.6	0.4
ANGL	R-4	R-1	D-1	344 4 22.5	-0.0	-0.0
ANGL	R-4	R-1	D-2	339 21 32.6	-1.9	-1.6
ANGL	R-4	R-1	D-3	330 53 32.0	0.9	0.6
DIST		R-1	U-1	452.37400	0.0009	0.2281
DIST		R-1	U-2	597.50800	-0.0003	-0.0556
DIST		R-1	U-3	745.54400	0.0015	0.3063
DIST		R-1	C-1	279.94800	-0.0019	-0.4906
DIST		R-1	C-2	429.64300	0.0045	1.0975
DIST		R-1	C-3	580.01600	0.0004	0.0859
DIST		R-1	C-4	729.97300	0.0009	0.1864
DIST		R-1	C-5	879.94300	-0.0050	-0.9516
DIST		R-1	D-1	468.99800	-0.0016	-0.4173
DIST		R-1	D-2	610.54200	-0.0074	-1.6738
DIST		R-1	D-3	755.00300	-0.0011	-0.2223
DIST		R-2	U-1	552.87200	0.0041	0.9087
DIST		R-2	U-2	403.73500	-0.0041	-0.9908
DIST		R-2	U-3	253.77000	0.0067	1.6943
DIST		R-2	C-1	723.09400	-0.0047	-0.9416
DIST		R-2	C-2	577.21600	-0.0050	-1.0598
DIST		R-2	C-3	433.29400	-0.0018	-0.3996
DIST		R-2	C-4	296.70300	0.0024	0.5768
DIST		R-2	C-5	182.77400	0.0012	0.2831
DIST		R-3	C-1	247.12600	-0.0057	-1.3969
DIST		R-3	C-2	352.96300	-0.0011	-0.2477
DIST		R-3	C-3	483.42700	-0.0046	-0.9979
DIST		R-3	C-4	622.26400	0.0031	0.6163
DIST		R-3	C-5	765.23600	0.0012	0.2193
DIST		R-3	D-1	296.29600	0.0005	0.1289
DIST		R-3	D-2	445.01700	0.0042	0.9480

RESIDUALS

(critical value = 3.504):

TYPE AT	FROM	TO	OBSERVATION	RESIDUAL	STD RES
DIST	R-3	D-3	593.83100	0.0053	1.1113
DIST	R-4	U-1	624.62300	-0.0058	-1.3055
DIST	R-4	U-2	479.11000	0.0031	0.7507
DIST	R-4	U-3	336.53500	-0.0030	-0.7783
DIST	R-4	C-1	768.34100	-0.0023	-0.4694
DIST	R-4	C-2	618.55100	0.0092	1.9861
DIST	R-4	C-3	468.17000	-0.0020	-0.4592
DIST	R-4	C-4	318.29500	0.0005	0.1364
DIST	R-4	C-5	168.65000	-0.0032	-0.8759
DIST	R-4	D-1	638.62800	-0.0017	-0.3722
DIST	R-4	D-2	495.92200	0.0006	0.1438
DIST	R-4	D-3	360.17600	0.0025	0.6040
ZANG	R-1	U-1	92 46 23.5	1.8	1.2
ZANG	R-1	U-2	92 4 2.4	-3.4	-1.6
ZANG	R-1	U-3	91 40 25.1	3.4	1.3
ZANG	R-1	C-1	90 21 16.0	-1.2	-0.9
ZANG	R-1	C-2	90 14 8.7	1.7	1.0
ZANG	R-1	C-3	90 10 35.3	-2.1	-1.1
ZANG	R-1	C-4	90 8 35.7	3.5	1.6
ZANG	R-1	C-5	90 7 11.2	-0.0	-0.0
ZANG	R-1	D-1	93 13 22.6	1.7	1.0
ZANG	R-1	D-2	92 28 38.2	-1.6	-0.6
ZANG	R-1	D-3	91 59 38.9	-2.9	-1.1
ZANG	R-4	U-1	92 3 55.3	-1.2	-0.4
ZANG	R-4	U-2	92 39 15.7	-0.0	-0.0
ZANG	R-4	U-3	93 48 46.2	-0.1	-0.1
ZANG	R-4	C-1	90 10 37.6	0.7	0.3
ZANG	R-4	C-2	90 13 19.0	0.0	0.0
ZANG	R-4	C-3	90 17 44.5	-0.3	-0.1
ZANG	R-4	C-4	90 26 14.3	0.1	0.1
ZANG	R-4	C-5	90 49 54.7	-0.5	-0.5
ZANG	R-4	D-1	92 25 22.0	0.4	0.2
ZANG	R-4	D-2	93 7 23.2	0.3	0.1
ZANG	R-4	D-3	94 17 0.4	1.6	1.0
OHDF	R-1	C-1	-1.73300	0.0014	0.2570
OHDF	C-1	C-2	-0.02700	-0.0014	-0.7042
OHDF	C-2	C-3	-0.02500	0.0004	0.2296
OHDF	C-3	C-4	-0.01400	-0.0013	-0.6835
OHDF	C-4	C-5	-0.02200	0.0006	0.2837
OHDF	C-5	R-4	2.45100	-0.0028	-0.4848
OHDF	R-1	U-1	-21.88200	0.0038	0.3277
OHDF	U-1	U-2	0.32080	0.0017	1.0556
OHDF	U-2	U-3	-0.19300	-0.0003	-0.1841
OHDF	U-3	R-2	21.88300	0.0110	0.9415
OHDF	R-3	D-1	-26.48400	-0.0166	-1.4262
OHDF	D-1	D-2	-0.02700	0.0005	0.3296
OHDF	D-2	D-3	0.11600	-0.0000	-0.0223
OHDF	D-3	R-4	26.88600	0.0101	0.8668

ADJUSTMENT STATISTICS

```
--------------------------------------------------------------------------------
S T A T I S T I C S     S U M M A R Y
--------------------------------------------------------------------------------
     Residual Critical Value Type                          Tau Max
     Residual Critical Value                                3.5042
     Number of Flagged Residuals                                 0
     Convergence Criterion                                  0.0001
     Final Iteration Counter Value                               3
     Confidence Level Used                                 95.0000
     Estimated Variance Factor                              0.6826
     Number of Degrees of Freedom                               63

--------------------------------------------------------------------------------
Chi-Square Test on the Variance Factor:

                   4.9528e-01  <  1.0000  <  1.0013e+00

                              THE TEST PASSES

 NOTE:  All confidence regions were computed using the following factors:
--------------------------------------------------------------------------------
     Variance factor used      =          0.6826
     1-D expansion factor      =          1.9600
     2-D expansion factor      =          2.4477

For relative confidence regions, precisions are computed from the ratio of
the major semi-axis and the spatial distance between the two stations.
--------------------------------------------------------------------------------
2-D and 1-D Station Confidence Regions (95.000 percent):
STATION         MAJOR SEMI-AXIS      AZMTH    MINOR SEMI-AXIS      VERTICAL
--------------------------------------------------------------------------------
C-1             0.0066               128      0.0052               0.0033
C-2             0.0067               149      0.0052               0.0036
C-3             0.0067               155      0.0052               0.0036
C-4             0.0066               148      0.0052               0.0032
C-5             0.0061               106      0.0045               0.0024
D-1             0.0074               178      0.0060               0.0049
D-2             0.0069                 3      0.0062               0.0045
D-3             0.0074                17      0.0065               0.0046
U-1             0.0068                11      0.0061               0.0046
U-2             0.0068               175      0.0060               0.0043
U-3             0.0068               162      0.0058               0.0043
```

HISTOGRAMS

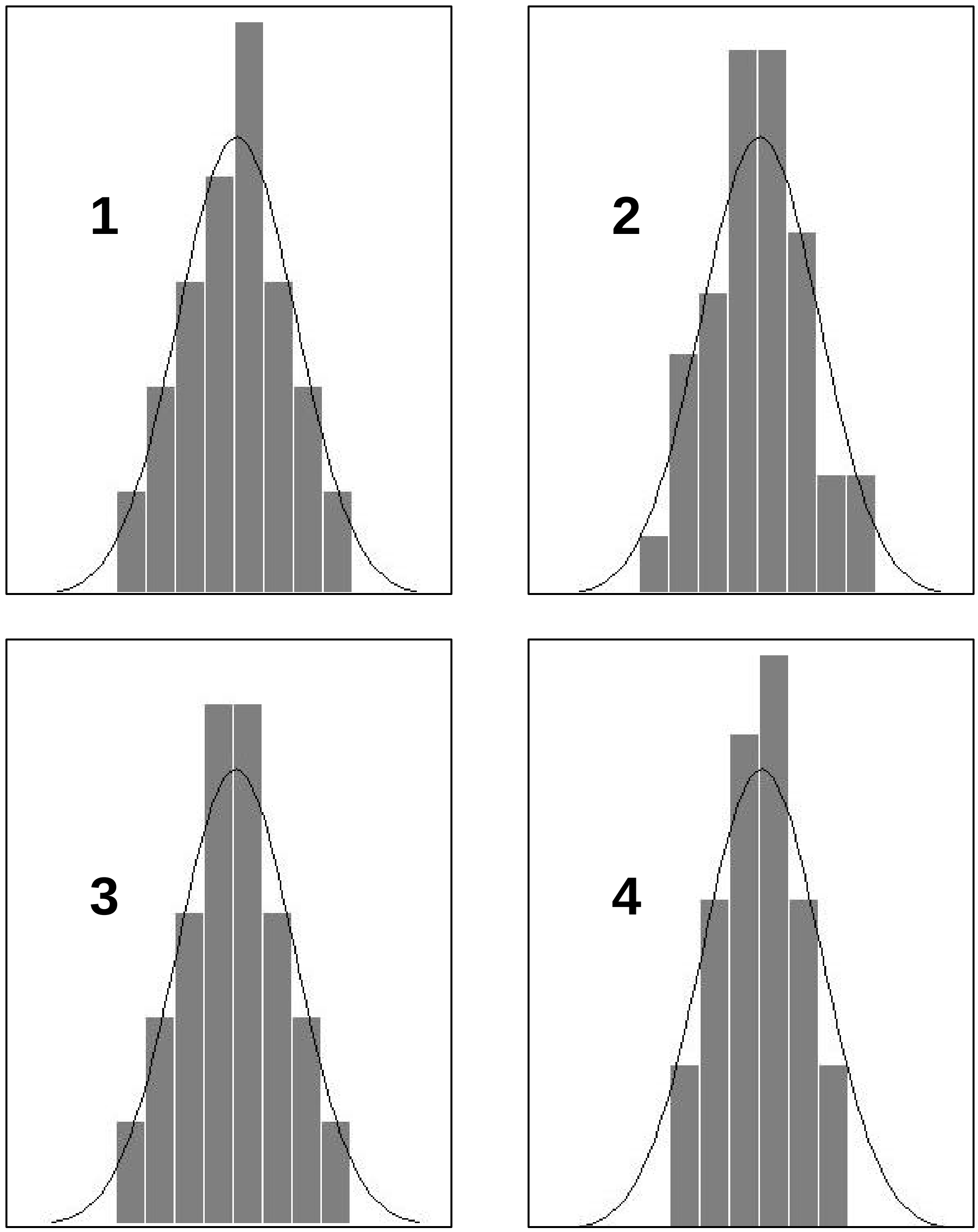

Figure 9-11. Adjustment histograms of the measurement residuals plotted for different observation types. (1) Histogram of horizontal angle observations. (2) Histogram of distance observations. (3) Histogram of zenith angle observations. (4) Histogram of height difference observations. Each example is normally distributed with no observation outliers present. The horizontal axis indicates the magnitude of the residual. Residuals are grouped into different classes, each covering a portion of the total range of observed values. The vertical axis indicates the relative frequency or number of residuals found in each class.

NETWORK MAPS

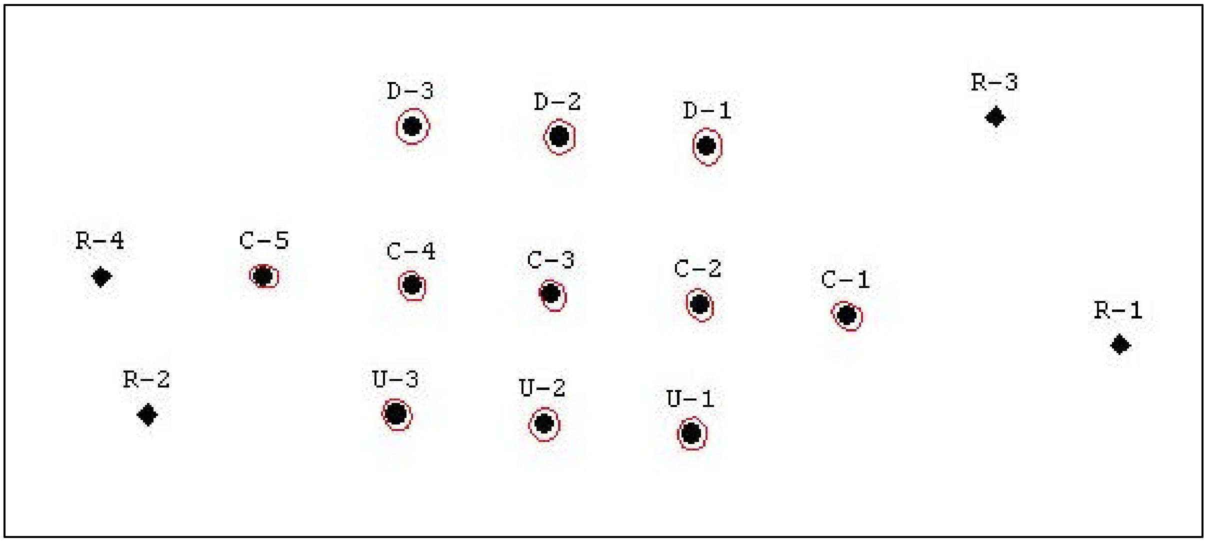

Figure 9-12. Adjustment network map (plan view) showing station names, relative locations, and the point confidence ellipse (95 percent) for horizontal positioning plotted around each point. Network map and confidence ellipses are plotted at different scales. Circles indicate monitoring points and diamonds represent reference stations.

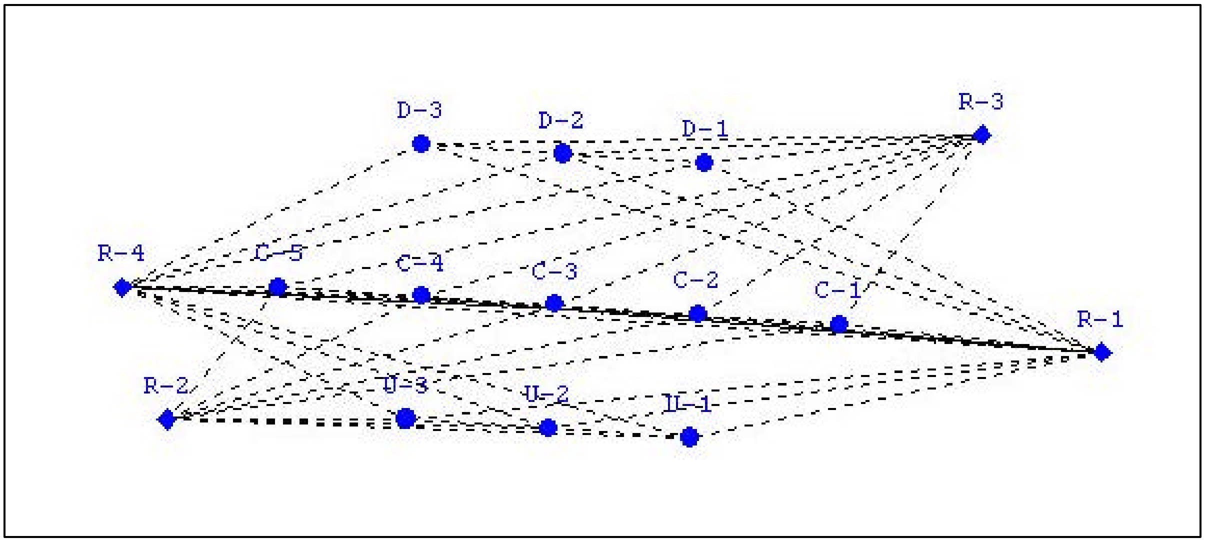

Figure 9-13. Adjustment network map showing station names and connections between points based on observations made during data collection. R-1 thru R-4 are reference stations, C-1 thru C-5 are centerline monitoring points, U-1 thru U-3 are upstream monitoring points at the base of the structure, D-1 thru D-3 are downstream monitoring points at the base of the structure.

9-8. Mandatory Requirements

The rejection criteria in Table 9-2 is considered mandatory.