Chapter 10
Relative Distance Ratio Assessment Methods

10-1. Introduction

Certain EDM biases such as refraction and scale error in EDM distance measurements can be minimized between two survey epochs, without calculating corrections, by application of "reference line ratio" methods. This method uses the fact that distance measurements made over similar line lengths under similar atmospheric conditions are affected equally by refraction (i.e., scale error). If measurements are made initially between two reference stations separated by a known distance, the ratio between the measured and known distance will provide a scale bias value for the network. The true distance to any other station will be proportional to the scale bias determined for the known baseline. This is also true for surveys conducted at any later epoch, where the atmospheric conditions will be different, but the ratio between a reference line and the measured line can be used to detect changes in the ratio of their distances. Thus, it is not necessary to explicitly determine EDM scale error or refractive index when using this technique. Although significant accuracy improvements are reported (when compared to results based on applying calculated refraction corrections), the disadvantage of using this method is that its accuracy is based on assumptions about uniform local atmospheric conditions. Techniques to reduce refractive index errors in measurements by using ratios, or reference lines, include two important rules:

Rule 1: Refractive index errors, resulting from end point measurements of temperature and pressure, tend to be the same for all lines measured from one point within a short period of time.

Rule 2: The ratios of observed distances, measured from one point within a short period of time, are constant.

Note: For both rules, a short period of time is 30 minutes or less.

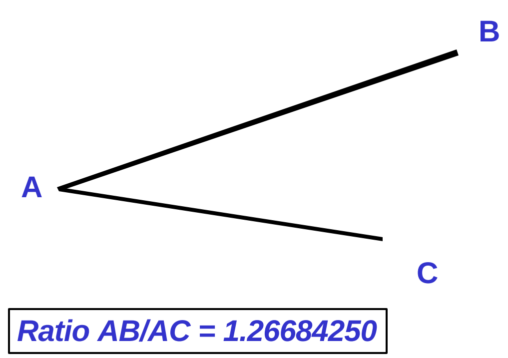

Figure 10-1. Ratio of two lines

a. General principle. In Figure 10-1, lines AB and AC are measured from a common point. Rule 1 states that if refractive index measurements are made at points A, B, and C within a short period, the errors in the measurements tend to be the same at all three points. If the true temperature along line AB is 20°C, but the mean of measurements made at A and B is 24°C (a condition typical of daytime), then the mean of temperature measurements at the end points of line AC would also be expected to be 4°C higher than the true temperature along that line. Because 1°C is approximately equivalent to 1 ppm of distance, both lengths will be in error by 4 ppm. However, if the measured length of AB is divided by the measured length of AC, the resulting ratio will equal the ratio of the true lengths. Thus, the *ratio* of two measured lengths will be more accurate than either of the lengths that were used to form the ratio. For example:

AB was measured to be 2839.611 meters,
AC was measured to be 2241.487 meters.

Their ratio is:

AB/AC = 1.26684250.

Both lines were in error by 4 ppm because of temperature-measuring errors; therefore, the true lengths were:

AB = 2839.611 + 0.0114 (4 ppm)
AC = 2241.487 + 0.0090 (4 ppm).

The ratio of the true lengths is:

2839.6224/2241.4960 = 1.26684250,

the same as the ratio of measured lengths.

b. Corrected and observed ratios. When ratios are formed from measurements that have had refractive index corrections applied, they will be called *corrected ratios*. The property of the corrected ratio is that it is very accurate. From corrected ratios, angles may be computed that are frequently within a few tenths of an arc second of their true values. A second set of ratios can be obtained from the same measurements by using the data before the application of the refractive index corrections. These are called *observed ratios*, and they have been formed from lines that have had no temperature of pressure corrections applied. Rule two states that the observed ratio is constant. This means that the observed ratio of two lines measured today will agree with the observed ratio of the same two lines measured months or years later. This will be true even though the observed lengths of the individual lines have changed greatly because of changes in atmospheric conditions between the two sets of measurements. The observed ratios will not, however, be the same as the corrected ratios unless certain conditions are met. To understand this, let us assume for a moment that an instrument has been set upon a hilltop. In the valley below, two points have been selected that are equidistant from the hilltop stations and are and the same elevation. The observed distances to the two points would appear the same because the distances are equal and both lines pass through roughly the same atmosphere. A point is then selected that is the same distance from the hilltop station as the other points, but with a higher elevation. When the observed distances are recorded, the two lengths to the valley points are the same, but the observed length to the higher elevation point is shorter. Because air density decreases with elevation, the light traversing the higher line travels faster and returns sooner. The instrument then shows the distance to be shorter. Two lessons can be learned from this. The first lesson is that if the mean elevations of two lines measured from a point are the same, the ratio of the observed distances is equal to the ratio of the corrected distances. In the example above, the observed distances to the valley points are the same, and the ratio of the two observed lengths is 1. The true lengths to the two points are the same so that the ratio of the corrected lengths is also 1. This is often the case with dams where the alignment markers along the crest of the dam are all within a few meters of the same elevation. This property of *observed ratios* will be used later on. The second lesson is that when the elevations of the end points to which measurements are being made are different, the ratio of observed lengths is not the same as the ratio of corrected (true) lengths because the refractive indices along the two lines are different. Even though it is not accurate, the observed ratio does not change with time and it may be used to detect changes in position. Furthermore, the observed ratio may be corrected by means of an atmospheric model. In many respects, ratios have properties similar to those of angles. In triangulation, the sum of the three angles of a triangle must equal 180 degrees, and a knowledge of two angles permits calculation of the third. Similarly, the product of three ratios obtained from a triangle must equal 1, and a knowledge of two ratios permits calculation of the third. For a triangle with sides A, B, and C as measured from vertices 1, 2, and 3 (Figure 10-2), the ratio measured from vertex 1 is A_1 / B_1, using a counterclockwise convention (A_1/B_1 rather than B_1/A_1) with the subscript designating the vertex from which the ratio was measured. Two other ratios, B_2/C_2 and C_3/A_3, may also be measured. If the measurements are perfect, then:

$$A_1 = A_3$$
$$B_1 = B_2$$
$$C_2 = C_3$$

and the ratios are simplified to:

$$(A_1 / B_1) (B_2 / C_2) (C_3 / A_3) = 1$$

If the measurements are not perfect (the usual case), the degree to which the product failed to equal 1 is a measure of the precision of the measurements. If only two ratios were measured, the third may be calculated. For example:

$$A_1 / B_1 = (C_2 / B_2) (A_3 / C_3)$$

Angles may be calculated directly from the ratios by using a modified cosine formula. The use of ratios yields angles as a result, and the angles determined from the ratios are more accurate than those determined from the lengths alone because a ratio is more accurate than either of the lengths from which it is derived. When the angles of a triangle do not sum to 180, the triangle may be adjusted by taking one-third of the difference between 180 and the sum of the angles and by applying it as a correction to each angle. With ratios, a correction may be made to each ratio. In measuring dams or other large structures, with the ratio method, refractive index errors are less important because relative displacement values are needed, and therefore relative, rather than absolute, distances may be used.

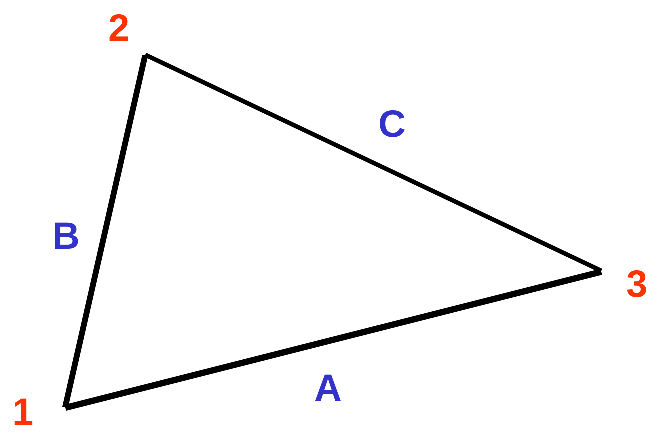

Figure 10-2. Ratios in a triangle

10-2. Deformation Monitoring Using Ratio Methods

This section provides guidance on performing deformation surveys using EDM ratio difference techniques. These surveys are done using an EDM or total station. Standard trilateration techniques are used to compute movements. This process requires measurement of the reference control network and the structure itself.

a. The reference control network. In monitoring possible movements of the structure with this technique, points on the structure, object points, must be related to points that that have been selected for stability, usually at some distance from the structure itself. All movements of the structure are related to one or more of these reference points. It is important that these reference points not move (i.e., they are stable), and for this reason, they should be placed in geologically stable positions. They should also afford a good geometry for trilateration measurements. Good geometry, in turn, consists of measuring along the line where movement is expected. For example, if measurements of upstream or downstream movements are required, the reference point should be located correspondingly either upstream or downstream. Also, the point should be at a sufficient distance from the structure so that the end points, as well as the center, can be monitored with good geometry. In Figure 10-3, a dam is shown with both an upstream and downstream control monument. Geometrically, measurements from both the upstream side of the dam will be poor, while those from the downstream side will be much stronger. If movement in two dimensions is desired, a point off the end of the dam should also be chosen. For best results, the angle of intersection should be as close to 90 deg as possible. Figure 10-4 depicts two acceptable selections of reference network control figures.

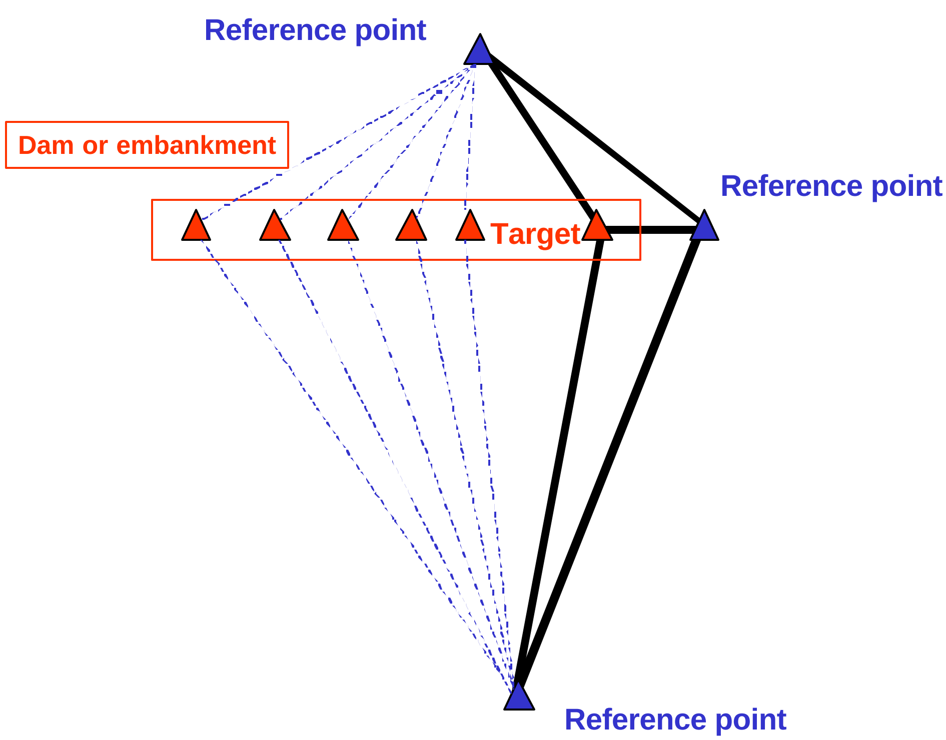

Figure 10-3. Control Monuments for a Dam

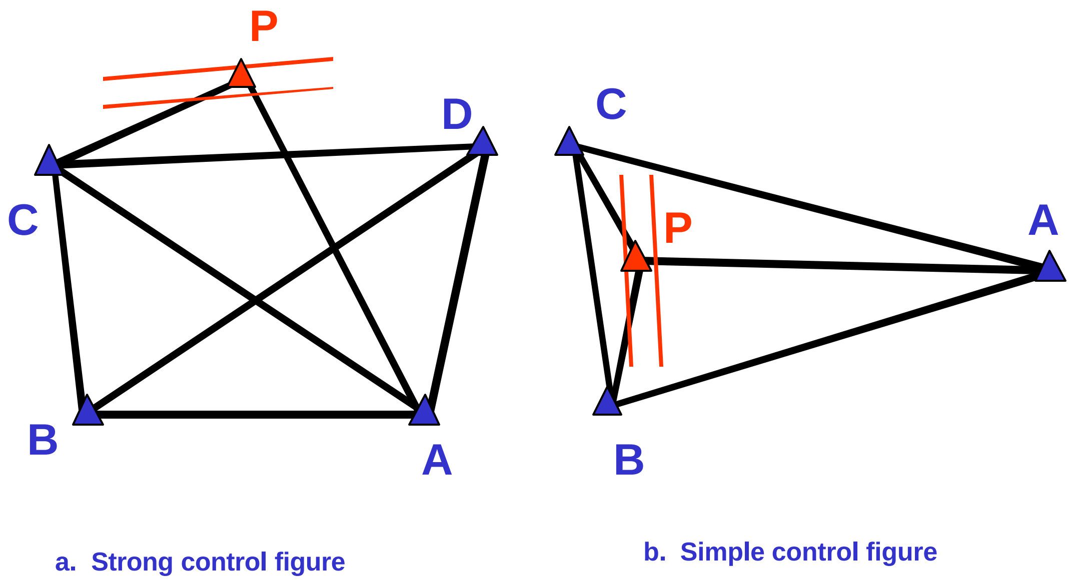

Figure 10-4. Strong and simple control figures around a structure

(1) A final criteria for the selection of reference control monuments is intervisibility. Because the control figure also provides a means of correcting for refractive index when optical electronic instrumentation is used, the points selected for control at the ends of the dam must be visible from the upstream and/or downstream points.

(2) In trilateration, lengths to an unknown station from each of two control points will give the position of the unknown station in two dimensions. Measurements from three control stations will give three positions of the unknown station and may be used as a check of survey accuracy. Figure 10-4a and 10-4b show good control figures for the measurement of a dam. In the figures, A, B, C, and D are control monuments, and all are intervisible. In Figure 10-4b, point P is an unknown station on the dam and is measured from control points A, B, and C. Positions of P are calculated from measurements of lines AP and BP, from lines BP and CP, and from lines AP and CP. The agreement between the three positions obtained for point P is a measure of the accuracy of the survey.

(3) When measurements are made of lines exceeding 600 meters, a major source of error is the inability to determine accurately the refractive index along the line. An error in temperature of 1 degree or in pressure of 2.5 mm (0.1 inches) of mercury will cause an error in length of one part per million. These errors may be minimized by considering the ratio of two lines that have been measured within 30 minutes of each other. The errors of each line tend to be the same so that taking a ratio greatly reduces the magnitude of the error. This may be shown by again referring to Figure 10-4a. Point D has been selected as a reference point. Its position was chosen so that it would be in stable ground, it would be visible from the other control points, and the lines to it from the other control points would pass through similar atmospheric conditions to those from the control points to unknown positions on the dam.

(4) The first time a structure is visited to make trilateration measurements, both ratios and conventional measurements are made to determine the shape and size of the control figure. The simplest example would be the triangular figure shown in Figure 10-4b. All control monuments should be

occupied by the EDM or total station. At each point, measurements should be made to all of the other control monuments within a short period of time. In the case of the triangle ABC in Figure 10-4b, monument A would be occupied and lengths AC and AB measured. Similar measurements should then be made as the EDM occupies stations B and C. Each line should then be reduced to the level or spheroid and the have the refractive index corrections applied. A typical set of measurements for triangle ABC is:

	Length (in m)	Ratio
A to	C 2547.447	AC/AB 0.9181349
	B 2774.589	
B to	A 2774.583	BA/BC 3.7776155
	C 734.480	
C to	B 734.478	CB/CA 0.2883212
	A 2547.430	

(AC/AB)*(BA/BC)*(CB/CA) = 1.0000018

	Adjusted Angles
A	15° 05' 47.84"
B	64° 35' 55.08"
C	100° 18' 17.08"

By way of comparison, angles calculated from the mean lengths would be:

A	15° 05' 47.59"
B	64° 35' 53.76"
C	100° 18' 18.65"

The adjusted angles determined from corrected ratios are more accurate than the angles determined from the means of the lengths of the sides because ratios are more accurate than the lengths of which they are composed.

(5) It may be seen from this example that the result of working with ratios is angles, and that in effect very accurate triangulation is being carried out using an EDM or total station. As in the case of triangulation, a baseline is necessary to determine the scale when ratios are used. Choose one of the sides of the triangle to serve as a baseline, and use the mean length as the scale for the triangle. In this example, AB has been chosen and its length is 2774.586 meters. Next, by using the sine formula and the angles determined from ratios, the other two sides may be determined:

$$\frac{2774.586}{\sin C} = \frac{BC}{\sin A} = \frac{AC}{\sin B}$$

BC = 734.481

AC = 2547.443

(6) The angles obtained by these methods are of the highest accuracy. The scale, however, is only as accurate as the mean of the two measurements of the baseline. Fortunately, this is not a serious

problem with measurements of dams because changes in lengths are desired rather than the absolute lengths themselves.

(7) The final task in establishing the control network is to assign coordinates to A, B, and C. These may be fitted into an existing network, or a local control net may be set up for the project.

(8) At a later date, the control figure may once again be occupied. The same procedure may be used, and the angles determined and compared with those obtained during the first survey. This, however, requires the use of temperature and pressure measuring devices each time the figure is surveyed.

(9) An easier method is to use the observed ratios, for these do not require knowledge of the refractive index. Remember that the observed ratios remain constant, and thus comparison of observed ratios from the first survey with observed ratios from the second survey are sufficient to determine whether any of the control monuments have moved. In fact, measurements of temperature and pressure need only be made of the control lines in order to give the proper scale to the figure. And these measurements need only be made the first time a project is surveyed. From that time on, only observed distances are required. In addition, all of the measurements from the control monuments to stations on the dam will be observed distances. Measurements of temperature and pressure are not necessary.

b. Points on the dam. When positions have been established for the monuments in the control figure, observed ratios will be used to determine the refractive index corrections for measurements of points on the dam. Referring again to Figure 10-4b, the lines AC, AB, and BC have been corrected for refractive index and may be used as reference lines. For measurements from control monument A, either AC or AB may be used as a reference line. A good reference line is one which traverses approximately the same atmosphere as is found along the lines to points on the dam and is almost the same length or longer. If we call the corrected length of the reference line R_{Corr} and the observed length of the same line R_{Obs}, the following equation may be written:

$$R_{Obs} \cdot k = R_{Corr} \qquad \text{(Eq 10-1)}$$

where k is a constant owing to the atmospheric conditions along the line at the time it was measured. Because the reference line has been selected to travel through approximately the same atmosphere as that to points on the dam, k is also the atmospheric constant for lines measured to the dam. If P_{Obs} is the observed length to a point on the dam, then P_{Corr} may be found from:

$$P_{Obs} \cdot k = P_{Corr} \qquad \text{(Eq 10-2)}$$

This technique enables the surveyor to correct for refractive index without using temperature and pressure measuring equipment. However, k is not really a constant because it changes slowly with time. For this reason, it must be remeasured at approximately 30 minute intervals, and it must be assumed it changes in a linear fashion.

(1) The following example will detail the previous phenomena. In Figure 10-5, the EDM has been set up at A. Measurements are made of AC, AP_1, AP_2, AP_3, and again AC.

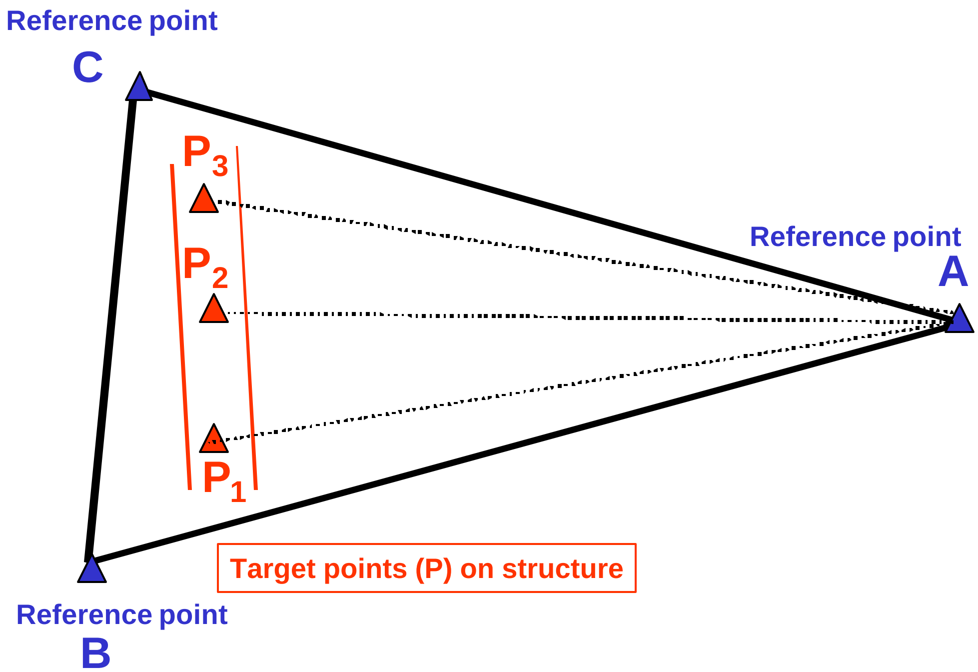

Figure 10-5. Use of a Reference Line

After the observed lengths have been reduced to the level or the spheroid, the measurements from control monument A were recorded as listed in Table 10-1.

Table 10-1. Measurement Taken (Example Deformation Survey)

To Station	Time	Observed Length (D_{Obs})	Refractive Index Constant (k)	Corrected Distance (D_{Corr})
C	1330	2547.326	1.0000459	2547.443
P_1	1335	2477.075	1.0000454	2477.187
P_2	1340	2407.354	1.0000449	2407.462
P_3	1345	2445.152	1.0000445	2445.261
C	1350	2547.331	1.0000440	2547.443*

Note: AC is the reference line

The first and last measurements are of AC. The length of AC is known and is used as a reference line to calculate the value of the refractive index constant. At first, the constant was 1.0000459 (2547.443/2547.326), but because of changes in the atmosphere, it changed to 1.0000440 (2547.443/2547.331). The value of k at intermediate times may be found by assuming that the change was linear. Thus, a value of k may be found for the times when P_1, P_2, and P_3 were measured. Applying the appropriate value of k to the observed length, D_{Obs}, of AP_1 gives 2477.075 * 1.0000454 = 2477.187 as its corrected length, D_{Corr}.

(2) Any length in a control figure may serve as a reference line, although some lines will be better than others. From A, AB would also serve. From B however, BC would be a better choice than BA because it passes through a similar atmosphere to that found in measuring from B to P_1, P_2, and P_3.

c. Reduction to the spheroid. Mention has been made of reducing lines either to the level or the spheroid. In very accurate work where lines exceed 1 km, the surface upon which a survey is being made can no longer be considered a plane. If distances are reduced to the level and used to calculate angles, the angles thus obtained may not agree with angles obtained from a theodolite. Further, the position of a point calculated from the lengths to two control monuments may not agree with the position of the same point when measured from two other control monuments. To prevent problems of this type, figures with line lengths in excess of 1 km should be reduced to the spheroid instead of the level.

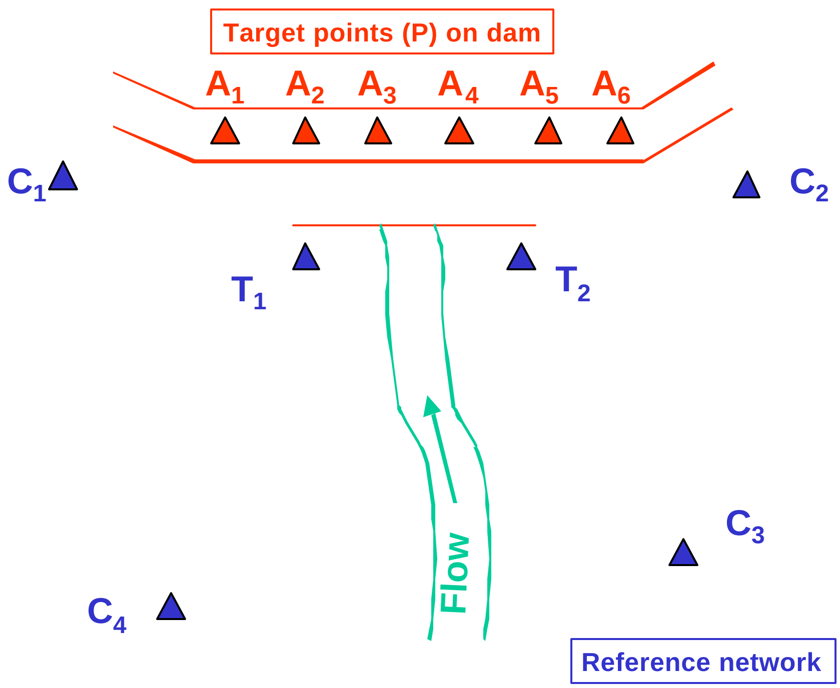

Figure 10-6. Example survey scheme on a concrete dam.

d. Example deformation survey. The example survey developed in the following paragraphs combines the principles developed for ratio lines. A diagram of the control setup and dam are shown in Figure 10-6. Control pedestals have been set at points C1, C2, C3, and C4. Markers A1 through A6 have been set along the crest of the dam, and T1 and T2 have been set near the toe of the dam. Elevations have been measured to obtain the list in Table 10-2.

(1) Each of the control monuments was occupied with an EDM, and measurements were made to the other three control monuments. Temperatures and pressures were also taken at both ends of the lines. After measuring the control lines, the lengths to stations on the dam were measured from three of the control monuments. Temperatures and pressures were not taken for these lines.

Table 10-2. Elevations for Example Deformation Survey

Point	Elevation (m above sea level)
A1	410.724
A2	410.718
A3	410.706
A4	410.721
A5	410.712
A6	411.245
C1	419.911
C2	413.275
C3	463.701
C4	521.537
T1	329.623
T2	329.394

(2) On a separate occasion, the following lengths were measured from C3:

Table 10-3. Measurements from C3 for Example Deformation Survey

#	To	Time	Observed Distance D_S (meters)	Mean Temp. (°C)	Mean Press. (inches Hg)
1	C1	0930	1081.105	16.4	28.26
2	C4	0935	945.03216.4	28.09	
3	C2	0940	703.78817.0	28.27	
4	C1	0945	1081.104	16.7	28.26
5	C1	1025	1081.101		
6	A1	1035	968.241		
7	A2	1045	924.456		
8	C1	1050	1081.103		
9	A3	1100	882.721		
10	A4	1115	843.323		
11	C1	1120	1081.104		
12	A5	1130	806.626		
13	A6	1145	772.950		
14	C1	1150	1081.104		
15	C1	1300	1081.100		
16	T1	1305	872.886		
17	T2	1315	836.021		
18	C1	1300	1081.097		

Measurements began with the control figure. Either C1 or C2 could have been used for a reference line, but in this case C1 has been chosen. Because it was the reference line, it was measured before and after the remaining control lines. This practice helped to check for both drift in the instrument and in the atmospheric conditions. When the control lines were completed, the operator next measured to points on the dam. Forty minutes had elapsed after completion of the contour line measurements before the field party with reflectors was set up on the dam. Because the reference line should be measured approximately every 30 minutes, the observed distance to C1 was again measured (measurement 5). A reflector was left unattended at C1 because it was no longer necessary to read the temperature and pressure. Remember temperature and pressure measurements are made only on the control lines and only when a study is made for the first time at a particular dam. The next time the dam is visited, perhaps 6 or

12 months later, it will not be necessary to measure refractive index. Possible movement in the control figure may be checked at that time by a comparison of ratios of observed distances.

(3) Measurements were made that same afternoon from C1. Only the control lines were measured. Three sets of positions will be obtained for the stations on the dam from C2, C3, and C4. Measurements from C1 would do little to improve the accuracy of these positions in the upstream-direction. Table 10-4 below gives the lengths from C1 recorded for that session.

Table 10-4. Measurements from C3 for Example Deformation Survey

#	To	Time	Observed Distance D_S (meters)	Mean Temp. (°C)	Mean Press. (inches Hg)
19	C2	1400	566.21219.0	28.28	
20	C3	1405	1081.095	18.8	28.20
21	C4	1410	989.41818.5	28.09	
22	C2	1415	566.21518.8	28.28	

(4) A week later, monument C4 was occupied and measurements were taken. These measurements are shown in Table 10-5 below.

Table 10-5. Measurements from C4 for Example Deformation Survey

#	To	Time	Observed Distance D_S (meters)	Mean Temp. (°C)	Mean Press. (inches Hg)
23	C1	0835	989.4466.1	28.85	
24	C2	0840	1138.277	6.1	28.87
25	C3	0845	945.0505.8	28.78	
26	C1	0850	989.4456.2	28.85	
27	C1	0900	989.444		
28	A1	0905	1031.587		
29	A2	0915	1042.973		
30	A3	0925	1057.756		
31	C1	0930	989.438		
32	A4	0940	1075.788		
33	A5	0945	1096.925		
34	A6	0955	1120.924		
35	C1	1000	989.432		
36	T1	1010	981.303		
37	T2	1020	987.682		
38	C1	1025	989.431		

(5) Later that day, monument C2 was occupied and measurements were taken. These measurements are shown in Table 10-6 below and completed the field measurement phase.

Table 10-6. Measurements from C2 for Example Deformation Survey

#	To	Time	Observed Distance D_S (meters)	Mean Temp. (°C)	Mean Press. (inches Hg)
39	C1	1230	566.2258.1	29.04	
40	C4	1235	1138.273	7.6	28.87
41	C3	1240	703.7997.8	28.97	
42	C1	1245	566.2258.3	29.04	
43	A1	1250	398.146		
44	A2	1300	337.350		
45	A3	1310	276.652		
46	C1	1315	566.225		
47	A4	1320	216.070		
48	A5	1330	155.828		
49	A6	1335	96.436		
50	C1	1345	566.224		

(6) The first step in the data reduction is to reduce all the lines (D_S) to the spheroid. This has been done and is shown in Table 10-7 below.

Table 10-7. Corrected Line Lengths

#	C3 To	Time	Observed Distance D_{Obs} (meters)	Corrected Distance (meters)
1	C1	0930	1080.143	1080.156*
2	C4	0935	943.188	943.201*
3	C2	0940	701.931	701.940*
4	C1	0945	1080.142	1080.155*
5	C1	1025	1080.141	(1080.155)
6	A1	1035	966.724	966.736
7	A2	1045	922.873	922.884
8	C1	1050	1080.141	(1080.154)
9	A3	1100	881.068	881.078
10	A4	1115	841.599	841.609
11	C1	1120	1080.142	(1080.154)
12	A5	1130	804.828	804.837
13	A6	1145	771.115	771.124
14	C1	1150	1080.142	(1080.154)
15	C1	1300	1080.138	(1080.154)
16	T1	1305	862.473	862.486
17	T2	1315	825.111	825.125
18	C1	1320	1080.135	(1080.154)
#	**C1 To**	**Time**	**Observed Distance D_{Obs} (meters)**	**Corrected Distance (meters)**
19	C2	1400	566.136	566.144*
20	C3	1405	1080.133	1080.149*
21	C4	1410	984.112	984.128*
22	C2	1415	566.139	566.147*

#	C4 To	Time	Observed Distance D_{Obs} (meters)	Corrected Distance (meters)
23	C1	0835	984.140	84.137*
24	C2	0840	1133.034	1133.030*
25	C3	0845	943.206	943.203*
26	C1	0850	984.139	984.136*
27	C1	0900	984.138	(984.134)
28	A1	0905	1025.543	1025.540
29	A2	0915	1036.993	1036.992
30	A3	0925	1051.857	1051.858
31	C1	0930	984.132	(984.134)
32	A4	0940	1069.987	1069.991
33	A5	0945	1091.232	1091.238
34	A6	0955	1115.403	1114.411
35	C1	1000	984.126	(984.134)
36	T1	1010	962.289	962.297
37	T2	1020	968.747	968.756
38	C1	1025	984.125	(984.134)

#	C2 To	Time	Observed Distance D_{O}bs(meters)	Corrected Distance (meters)
39	C1	1230	566.149	566.147*
40	C4	1235	1133.149	1133.027*
41	C3	1240	701.942	701.940*
42	C1	1245	566.149	(566.146)
43	A1	1250	398.112	398.110
44	A2	1300	337.318	337.316
45	A3	1310	276.622	276.621
46	C1	1315	566.149	(566.146)
47	A4	1320	216.041	216.040
48	A5	1330	155.797	155.796
49	A6	1335	96.408	96.408
50	C1	1345	566.148	(566.146)

Note: * - Denotes length corrected from temperature and pressure measurements.
( ) - Denotes true length.

(7) When the lines have been reduced to the spheroid, the next step is to define the size and shape of the control figure, in this case a doubly braced quadrilateral. There are several ways to do this. One way is that the figure contains four triangles, and these may be individually treated in the same manner as the triangle in Figure 10-4a. Another way would be to use the means of the six lines in the figure and adjust these by means of a quadrilateral adjustment. This is the technique that was used in the present case to obtain the following adjusted lengths:

C1 to C2	566.146 meters
C1 to C3	1080.154
C1 to C4	984.134
C2 to C3	701.940
C2 to C4	1133.029
C3 to C4	943.202

(8) The control figure may be fit into an existing coordinate system or a local system may be devised just for the dam. For the example dam, a local system was used. C4 was selected as a starting point was assigned coordinates of x = 1000.000 and y = 1000.000. The coordinates of C3 were then chosen to place C3 at a distance of 943.202 meters from C4; they are x = 1943.202 and y = 1000.000; The placement of C4 and C3 has determined the scale and orientation of the figure. Using the positions of C3 and C4 and the appropriate lengths, the positions of C1 and C2 can be determined to be:

C1: x = 1366.527 y = 1913.333

C2: x = 1890.936 y = 1699.991

(9) The establishment of the control figure needs be done only once. From that time on, it is only necessary to check for movements of the control monuments. This may be done by comparing observed ratios taken at some later time with the original set.

(10) Returning to Table 10-7, one may now calculate the corrected lengths D_C to the stations on the top and toe of the dam from the control monuments. This is done by using reference lines to make refractive index corrections.

(11) Measurements 15 through 18 from Table 10-7 are given in Table 10-8 below.

Table 10-8. Changes of Correction Factor with Time

#	C3 To	Time	D_O (meters)	Correction Factor	D_C* (meters)
15	C1	1300	1080.138	1.0000148	(1080.154)
16	T1	1305	862.473	1.0000155	862.486
17	T2	1315	825.111	1.0000169	825.125
18	C2	1320	1080.135	1.0000135	(1080.154)

* () denotes true length.

(12) At 1300, when the distance to C1 was measured, the observed distance, D_{Obs}, was found to be 1080.138 meters. This line, C3 to C1, is a part of the control figure, and its correct length has been determined to be 1080.154 meters. The atmospheric correction at 1300 may then be found by dividing. The correction is 1080.154/1080.138 = 1.0000148. Later, at 1320, the atmospheric correction has become 1.0000176. Assuming the change in correction has been linear as a function of time over the 20 minute interval, we may calculate the correction factor at 1305 and 1315 when observed distances were measured to T1 and T2. Multiplying the observed distance by the corresponding atmospheric correction gives the corrected distance, D_{Corr}, to T1 and T2. Thus in Table 10-7, the values in parenthesis in column 5 are the correct or true lengths of reference lines, and the values without an asterisk or parenthesis are the corrected lengths that have been calculated from reference lines.

(13) Finally, with the corrected lengths and the coordinates of the control monuments from which they were measured, it is possible to calculate the positions of the points on the dam. Because three lengths were measured to stations on the crest of the dam, three solutions will be obtained. Geometrically, some solutions will be superior to others. For stations at the toe of the dam, only one solution is possible.

(14) In Table 10-9, positions of the crest and toe markers are given for various line combinations, In the case of the crest markers, and adjusted position is also given.

Table 10-9. Crest and Toe Station Positions

Station	X	Y	From
A1	1533.713	1875.726	C2 to C3
	1533.710	1875.720	C2 to C4
	1533.705	1875.723	C3 to C4
	1533.709	1875.722	Adjusted
A2	1590.161	1852.688	C2 to C3
	1590.158	1852.682	C2 to C4
	1590.153	1852.685	C3 to C4
	1590.157	1852.684	Adjusted
A3	1646.583	1829.648	C2 to C3
	1646.588	1829.656	C2 to C4
	1646.594	1829.652	C3 to C4
	1646.589	1829.653	Adjusted
A4	1703.041	1806.615	C2 to C3
	1703.038	1806.609	C2 to C4
	1703.033	1806.613	C3 to C4
	1703.037	1806.612	Adjusted
A5	1759.465	1783.585	C2 to C3
	1759.467	1783.588	C2 to C4
	1759.470	1783.584	C3 to C4
	1759.468	1783.586	Adjusted
A6	1815.919	1760.547	C2 to C3
	1815.915	1760.542	C2 to C4
	1815.912	1760.545	C3 to C4
	1815.915	1760.544	Adjusted
T1	1568.152	1776.672	C3 to C4
T2	1608.187	1754.053	C3 to C4

(15) If desired, alignment may be determined from positions. Using the crest stations A1 and A6 as end points, the alignment of A2 through A5 is given in Table 10-10. T1 and T2 are also included in the alignment to help monitor any tilt in the dam. Alignment done from positions is not affected by curved dams, by bends, or by differences in elevations.

Table 10-10. Alignment

Station	Distance from A1 (meters)	Distance off Line (meters)*
A2	60.968	0.00
A3	121.919	- 0.001
A4	182.888	+ 0.001
A5	243.836	- 0.004
T1		+78.691
T2		+84.505

* + = Downstream
 - = Upstream

10-3. Mandatory Requirements

There are no mandatory requirements in this chapter.