Chapter 9
Route Surveying

9-1. General

Route surveys are most commonly used for levees, stream channels, highways, railways, canals, power transmission lines, pipelines, and other utilities. In general, route surveys consist of :

- Determining ground configuration and the location of objects within and along a proposed route.
- Establishing the alignment of the route.
- Determining volumes of earthwork required for construction.

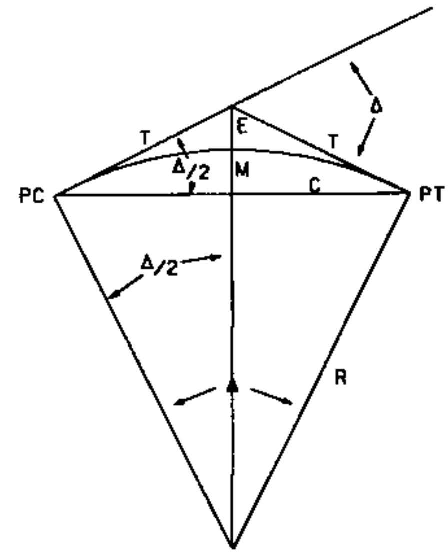
After the initial staking of the alignment has been closed through a set of primary control points and adjustments have been made, center-line/baseline stationing will identify all points established on the route. Differential levels are established through the area from two benchmarks previously established. Cross-sections in the past were taken left and right of center-line. Today digital terrain models (DTM) or photogrammetry is used to produce cross-sections for design grades. Surveys may be conducted to check these sections at intermittent stations along the center-line. Ground elevations and features will be recorded as required.

9-2. Horizontal Circular Curves

Route surveys often require layout of horizontal curves. The point of curvature (PC), point of intersection (PI), and point of tangency (PT) should be established on center line, identified, and staked including offsets to the center line. Figure 9-1 is a sketch of a horizontal curve. The traverse routes through the curve will be included into the closed traverse through two primary or secondary control points for closure and adjustment. Field layout/stakeout should be no more that 100 feet along the curve on even stationing. Points of curvature, angle points, and/or points of intersection should be referenced (line-of-sight) outside the clearing limits or the construction area.

9-3. Deflection Angles

The angles formed between the back tangent and a line from the PC to a point on the curve is the deflection angle to the curve. The deflection to the point on the curve is given by the equation:



where

Δ = Central and Deflection Angle
PC = Point of Curvature
PT = Point of Tangency
T = Tangent
R = Radius
C = Long Chord
M = Middle Ordinate
E = External

Figure 9-1. Horizontal curve

$$\Delta = \text{arc length} / \text{radius}$$

where

Δ = deflection angle or central angle

Arc length = the arc length found by subtracting the station number

Radius = distance from the radius point to the center line of the right-of-way alignment

Transportation engineers compute minimum/maximum allowable curvature based on weight and speed. Surveyors fit curves to existing alignments based on the minimum/maximum curvature limits. Curvature limits only apply to primary roads. Secondary roads or roads where speeds are low are based on radius and deflection angle. Two types of formulas have been used to fit curves based solely on curvature:

Degree of Curvature - Arc Definition
Degree of Curvature - Chord Definition

9-4. Degree of Curve - Arc Definition

The standard 100-foot steel highway chain used by surveyors was the basis of the amount of curvature developed in 100 feet of arc. The ratio of curvature over a 100-foot arc is equal to the total degrees in a circle over the total arc length in a circle.

$$D_C / 100 = 360^o/(2 * \pi * R)$$

or

$$D_C = 100' \; arc * 1/R * 180/\pi$$

Usually the curvature will be specified. The surveyor needs to solve for the radius. Rearranging the above formula yields:

$$R = 5729.578 / D_C$$

The computed radius and directions (azimuths or bearings) of the straight portions of the right-of-way, called tangents, are usually used to compute the curve in the field.

9-5. Degree of Curve - Chord Definition

The chord definition was popular in the railroad industry. Some USACE districts use this method. The definition is valid because the curvature is slight in railroad curves and the difference between 100 feet of arc and 100 feet of chord cannot be measured with a steel chain to the nearest 0.01 foot. The formula for any chord is

$$Chord = 2 * R * \sin(\Delta/2)$$

The method defines the amount of curvature found in a 100-foot chord. Substituting the value of 100 into the standard chord formula and rearranging gives the Degree of Curve - Chord Definition:

$$\sin(D_C/2) = 50 / R$$

where

50 = 100 / 2.

9-6. Curve Stakeouts

The first and most important point in a curve stakeout is the PI of two tangent sections of a right-of-way. This point is set in lieu of the radius because the radius may be too far away from an instrument station for curves with small curvature. All PI's are normally set from a cross-traverse which was designed to have stations close to where the PI's actually fall. The PI's are set from the traverse and checked for distance to the adjacent PI. If the distances are correct the lines are cut out and the PI's are referenced. Backsights for the references are set out of the construction areas as points on line (POL). Because the PI is not on the curve the PI does not have a center-line station number. The PI may have a station number during preliminary reconnaissance of a major transportation route. The PI stations are angle points with deflection angles between straight sections (tangents). Once the curves are determined, the entire center line is restationed. The distance to the PI from the curve/tangent intersection point is found from the tangent formula given in paragraph 9-7.

9-7. Curve Formulas

a. Required parameters. Usually only two parameters need be specified to totally lay off a curve. If the project drawings contain station numbers, subtract the station number of the PC from the PT station number when using the Arc Definition. This gives the amount of arc length. The radius or the intersection deflection angle can be used for all other calculations.

$$tangent \rightarrow T = R * \tan(\Delta/2)$$

$$chord \rightarrow ch = 2 * R * \sin(\Delta/2)$$

where

Δ = intersection deflection angle or the central angle at the radius for the entire curve.

The relationship of arc to angular measurement can always be used if the radius is known.

$$s = r * \theta$$

where

s = arc length

r = radius

θ = angle in radians, convert from degrees to radians by $\pi/180°$

NOTE: If the angle is being used to compute the arc or radius, the units must be in radians. Convert to radians by multiplying degrees by $\pi/180°$.

b. Relation between central angles and deflection angles. The deflection angle measured at the PC between the tangent and the line to the point is 1/2 the central angle subtended between the PC and the point. The relationship comes directly from the geometry of a circle.

9-8. Transition Spirals

The initial factor to determine the transition spiral is the velocity of the vehicle using the structure. Until further guidance becomes available, the formula to be used for determining the minimum length of the spiral will be the highway definition:

$$L_S = (1.6 * V^3) / R_C$$

where

L_s = the minimum length of the spiral

V = the design speed (mph)

R_c = radius of the circular curve

Figure 9-2 is a diagram of a spiral curve used for transition.

$$\Delta_S = (L_s * D_c)/200$$

where

Δ_s = central angle for the spiral

L_s = the length of the spiral used for the spiral design

D_c = degree of curve for the highway curve

The new circular curve will be reduced by

$$\Delta_c = \Delta - (2 * \Delta_s)$$

where

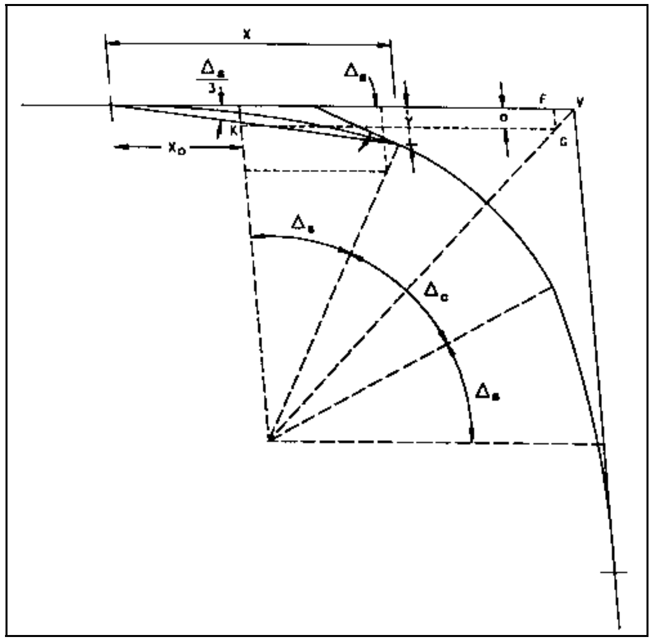


Figure 9-2. Spiral curve diagram

Δ_c = new central angle of the circular curve

Δ = old central angle of the circular curve

Δ_s = central angle for one spiral

Many procedures exist for computing spiral curves. The method recommended for use by USACE to compute spirals where Δ_s is less than 15 degrees is

$$X = L_s \left[1 - \frac{\Delta^2}{(5)(2!)} + \frac{\Delta^4}{(9)(4!)} - \frac{\Delta^6}{(13)(6!)} \right]$$

where

X = distance from the tangent to spiral (TS) and PC of the circular curve along the tangent.

$$Y = L_s \left[\frac{\Delta}{3} - \frac{\Delta^3}{(7)(3!)} + \frac{\Delta^5}{(11)(5!)} - \frac{\Delta^7}{(15)(7!)} \right]$$

where

Y = distance from the tangent to the PC

Refer to Figure 9-2.

$$o = Y - R(1-\cos(\Delta_s))$$

$X_0 \;= X - (R)(\sin(\Delta_s)$

$KG \;= (R)(\tan(\Delta/2))$

$FV \;= (o)(\tan(\Delta/2)$

$T_s \;= X_o + KG + FV$

$T_s \;= X - (R)(\sin(\Delta_s) + (R+o)(\tan(\Delta_c/2)$

9-9. Spiral Stakeout

The point T_s is set from the PI on both tangents. From this point forward, only one side of the spiral will be discussed. The other side is the same. The distance *X*, measured from the T_s is set as a POL. The instrument is moved to the POL, backsighted along the tangent and the perpendicular is turned to locate the horizontal curve center line at a distance *Y* from the tangent line. The deflection angles are computed by the formula

$$\delta_S = (I_S^2 / L_S^2) * \Delta_S$$

The stakeout of a spiral is much the same as a horizontal curve. The arc lengths in the spiral are assumed to be equal to the chords provided chaining is done between short stations. Fifty-foot stations are common. For an example of a stakeout, assume the T_s falls on station 164+68.21. Assume the Δ_s was 10 degrees, and L_s is 300.00 feet. To set the first deflection angle with the instrument located at T_s, backsighting the PI, subtract out the next even station. 165+00 - 164+68.21 = 31.79 feet (pull 31.79 as the chained distance from T_s). The deflection angle is

$$\delta_S = 31.79^2 / 300^2$$

$$\delta_S = 00^\circ\ 02'\ 15''$$

A curve (spiral) table is constructed until the last deflection computed before the PC. The deflection from the T_s to the PC is approximately $\Delta_s/3$. No angle in the table should exceed this value.

9-10. Vertical Curves

Vertical curves are not typically surveyed to a predetermined design involving topographic surveys. Basically, the same criteria apply for horizontal and vertical curves in preliminary design project phases which are highly dependent on topographic surveys. Two methods are traditionally used to compute a vertical curve. These are the direct equation method and the tangent offset method. Both methods are discussed. USACE recommends use of the equation method. The tangent offset method offers insight to the calculation of slope and rate of change of slope components to find the elevation on the vertical curve.

9-11. Vertical Curve - Tangent Offset Method

Figure 9-3 shows a planview of a straight vertical curve. Vertical curves can be applied to horizontal curves as well. The tangent offsets are computed and algebraically added to the slope elevation computed for the center-line station plus. In Figure 9-3, the tangent offsets reduce all the elevations computed along the tangent to an elevation on the vertical curve. This will not always be the case. Vertical curves have many shapes.

a. Two definitions are used for the tangent offset method:

(1) The parabola is defined as the locus of points equally distant from a point (focus) and a line (directrix).

(2) $y = x^2$ is the reduced form of the parabola equation.

b. An independent parabola is constructed from the known slopes and the length of the curve. Elevations are computed for the point of vertical curve (PVC) and the point of vertical tangent (PVT) (see Figure 9-3, part b). The average of these two elevations is the midpoint of the elevation on the parabola's axis of symmetry. This elevation is substituted for the parabola's focus. The slope intersection is substituted to be the intersection of the axis of symmetry and the directrix (see Figure 9-3, part c). The difference in elevation between the long chord midpoint and the slope intersection is "*d*" (see Figure 9-3, part b). A parabola is now constructed with no reference to the actual route alignment until later.

c. Using definition (1), the maximum tangent offset is found along the axis of symmetry between the vertex and the directrix (see Figure 9-3, part c). By definition, the value of this tangent offset is "*d/2*".

d. The vertical distance or tangent offset to any other point on the parabola varies as the square of the horizontal distance from the curve beginning. Both ends of the curve are used if the grades are not equal.

e. The minimum tangent offset distance is zero on both ends of the parabola. A proportion can be

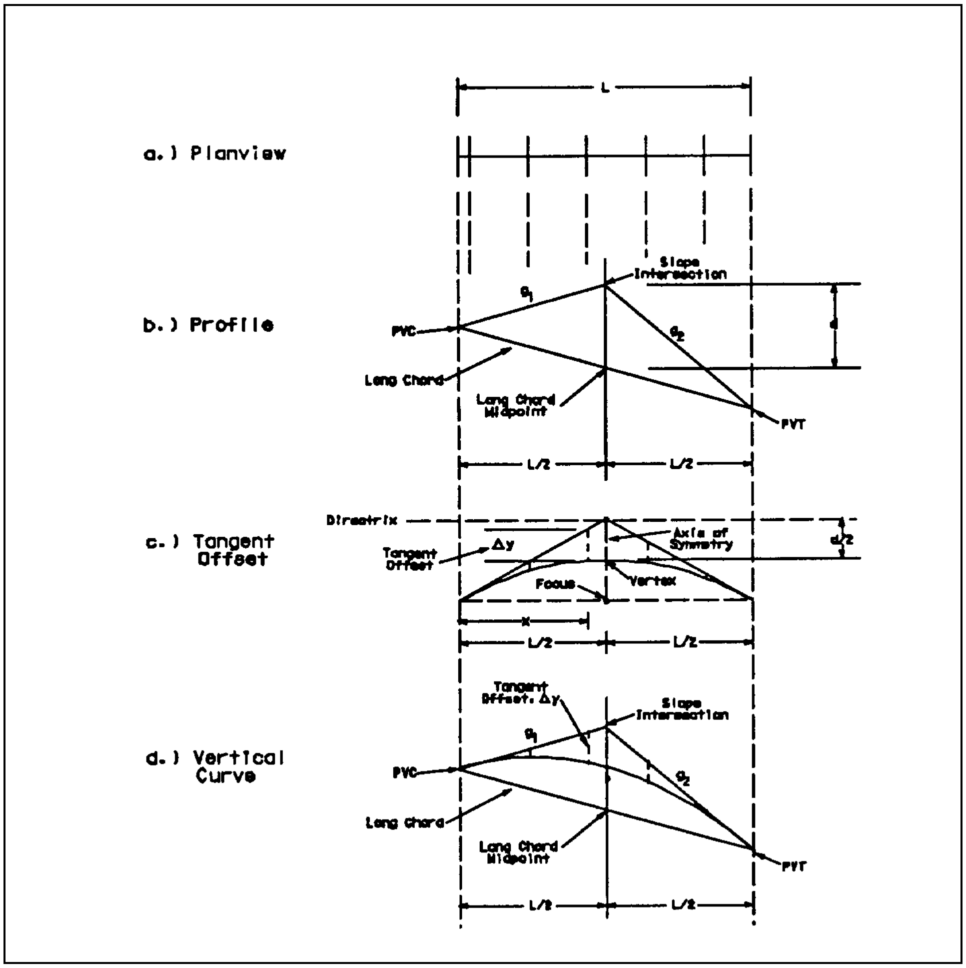


Figure 9-3. Vertical curve geometry

established based on the maximum tangent offset which is stationed at length divided by 2. Combining this with definition (2) gives

$$\Delta y \,/\, (d/2) = x^2 \,/\, (L/2)^2$$

See Figure 9-3, part *c*.

9-12. Vertical Curve - Equation Method

The rate of change of slope in a vertical curve is fixed. This constant is:

Slope Rate Equation

$$r = (g_2 - g_1) / L$$

where

r = rate of change of grade

g_2 = is the grade opposite the PVC

g_1 = is the grade adjacent the PVC

L = the length of the vertical curve

Normally the grades are entered into the equations as percents and the lengths are reduced to stations. This makes the rate of change in units of percent grade change per station.

a. The slope at any point can be found from the Slope Rate Equation as:

Slope Equation

$$g = rx + g_1$$

At the PVC, $x = 0$ and $g = g_1$.

At the PVT, $x = L$ and $g = g_2$.

b. The Slope Equation is used to find stations of no slope. These stations are either high points or low points in the curve. If slopes g_2 and g_1 are equal, the point of zero slope is on a vertical line with the slope intersection. Otherwise the Slope Equation is used to locate the route station as

$$rx + g_1 = 0$$

$$x = -g_1 / r$$

c. The elevation of any point along the vertical curve can be obtained from the Slope Equation as

$$y = (r/2)x^2 + g_1 x + Elev.\ of\ PVC$$

9-13. Vertical Curve Obstructions

Other criteria may impact the design of vertical curves. Obstructions may be the controlling factor in vertical curve design. USACE designs bridges over navigation channels. Shipping commerce must be accommodated in the waterways. A high water elevation in the same datum units as the highway elevations, design shipping clearance, and safety factor provide an obstruction elevation used to compute the length of a vertical curve. Figure 9-4 shows a sketch of a vertical curve and an obstruction elevation "*z*" at a horizontal distance "*s*" along the highway route from the slope intersection of two known grades. The tangent offsets from both ends of the curve to the elevation "*z*" are used to compute "*L*."

$$h_1 / (L/2 + s)^2 = h_2 / (L/2 - s)^2$$

$$j = i + s * g_1$$

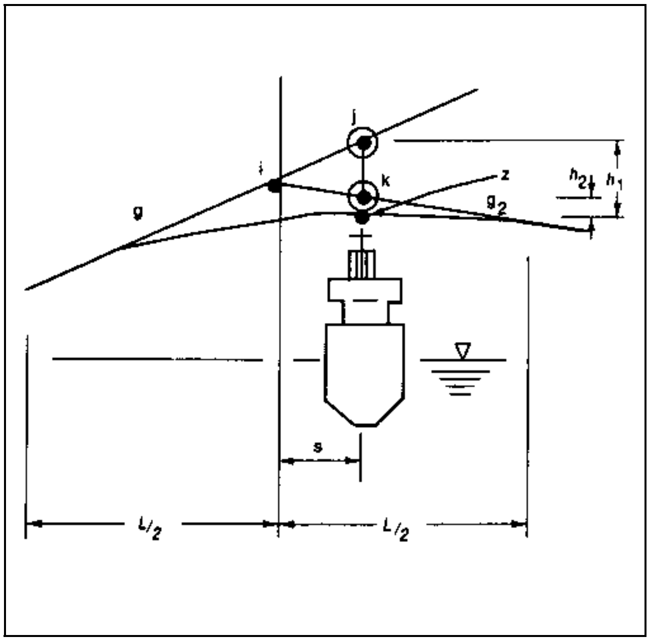


Figure 9-4. Vertical curve obstruction

$$h_1 = j - z$$

$$k = i + s * g_2$$ NOTE: g_2 is negative in the figure

$$h_2 = k - z$$

$$L = 2s \times \left[\frac{\sqrt{\frac{h_2}{h_1}} + 1}{1 - \sqrt{\frac{h_2}{h_1}}} \right]$$